



SGH

**COLLEGIUM OF ECONOMIC ANALYSIS
WORKING PAPER SERIES**

Randomness or stock-flow: Which mechanism
describes labour market matching in Poland?

Ewa Gałęcka-Burdziak

Randomness or stock-flow: Which mechanism describes labour market matching in Poland?

Abstract:

I compare random, stock-flow and job queuing models to determine which mechanism prevails in the Polish labour market. I use monthly registered unemployment data for the period 1999 – 2013 and econometrically correct for temporal aggregation bias in the data. I extend the known solutions to make them directly applicable to a job queuing model.

I find that stocks and inflows engage in a matching process. Job seekers (from the pool) seek work among old and new job posts, but only a small fraction of the newly unemployed individuals find work instantaneously. Vacancies are the driving force in aggregate hiring, but the inflow is more important than the stock. The random model has greater explanatory power, although the results do not negate the non-random model. The random model entails the need to improve information to facilitate matching. The stock-flow model implies that policy should aim at creating more job offers.

JEL classification: J63; J64

Keywords: Stock-flow matching; Random matching, Job queuing, Polish labour market, Temporal aggregation

♦ PhD, Department of Economics I, Warsaw School of Economics, ul. Madalińskiego 6/8, 02-513 Warsaw, Poland, [mailto: ewa.galecka-burdziak@sgh.waw.pl](mailto:ewa.galecka-burdziak@sgh.waw.pl), phone: +48 22 564 93 22. Ewa Gałęcka-Burdziak is a “START” Scholarship holder granted by the Foundation for Polish Science (FNP).

1. Introduction

I compare labour market matching mechanisms, specifically, random and stock-flow, to determine which one prevails in the Polish labour market. I explicitly address the nature of matching technology while econometrically addressing the temporal aggregation bias in the data. This approach produces robust results and indicates the relative importance of stocks and flows in the employment creation process. Different matching technologies offer competing explanations of how job seekers and job vacancies are matched. If we identify the matching technology, we can formulate policy recommendations to improve the efficiency of the matching process.

I use data on public employment intermediation in the Polish labour market. Poland is an exceptionally interesting case. This country has experienced substantial volatility in unemployment rates over the past twenty years (oscillating between less than 7% in 2008 and more than 20% in 2004). The labour market has undergone fundamental changes since the beginning of the transition from a centrally planned to a market-oriented economy. Lehmann (2012) indicates that the success of repeated attempts at reform and strong overall economic performance were, among other factors, determined by the fact that none of the important reforms were reversed, even following changes in government. However, various amendments and small modifications were regularly implemented in this period. Specific goals were partially or fully achieved at the cost of substantial labour market adjustment. Large fractions of workers who could not adapt to new conditions exited the labour market, which resulted in relatively low labour force participation rates (Lehmann 2012). I focus on public employment intermediation only. A relatively small fraction of companies publishes vacancies in the public employment offices (approximately 16.5% in 2012). Nevertheless, this remains the most common job search method among job seekers (approximately 70% of job seekers contact a public employment office).

I compare the random (expressed by stock-based and job queuing models) with the stock-flow approach. I use overidentified equations to identify the role of the inflow variables in the matching process. I consider the period 1999 – 2013. The database includes all of the necessary stocks and flows of workers and job postings that are required to conduct thorough empirical research. I use the results to formulate qualitative conclusions and policy recommendations.

The matching function is a well-known tool used to analyse aggregate matching processes. Models differ in their description of the matching mechanism, depending on the assumptions made regarding the impact of stocks and flows. In a random model (see, e.g., Blanchard and Diamond 1994) a match occurs once a job seeker is assigned to a particular job. Vacancies and unemployment coexist due to coordination failure among agents, even if demand equals supply. The stock-flow model (see, e.g., Coles and Smith 1998) assumes perfect information to reflect the fact that agents first consider numerous advertisements before applying for selected job offers, and once an offer has been rejected, reapplication is less likely than a search for new vacancies. Agents who remain in the job market lack a proper partner, as all trade options have been exploited. The job queuing model (see, e.g., Shapiro and Stiglitz 1984) is formulated to reflect the large discrepancies between demand and supply. The short side of the market clears in each period, but an insufficient number of vacancies means that workers must wait for new job postings.

Petrongolo and Pissarides (2001) provide the most thorough review of the literature on matching functions. They group the research according to the type of study, methods used or particular research areas. They refer, among other factors, to worker heterogeneity, mismatch, ranking and data aggregation. They present studies on the Beveridge curve and aggregate, sectoral and micro studies. They conclude that a matching function is a nice, but somewhat black box aggregate function, the existence of which is well documented but lacks

microfoundations. Additionally, certain papers refer to particular aspects of the empirical application of the matching function. For example, Borowczyk-Martins et al. (2011) refer to search endogeneity that may bias the estimated elasticities. This endogeneity results from the search behaviour of agents on both sides. Galuščák and Münich (2005) analyse how different worker flows (e.g., to and from inactivity) affect the matching function elasticities.

A few papers apply the matching function concept to Polish data (see Roszkowska (2009) for a literature review). These papers rarely explicitly define the type of technology that describes the trade process but usually assume matching between the unemployment stock and vacancy inflow. Often, the aim of such research is to identify the determinants of the efficiency of the matching process from a regional perspective. This is primarily achieved by estimating an augmented matching function (see, e.g., Kwiatkowski and Tokarski 1997). Tyrowicz (2011), in turn, applies the stochastic frontier model at the NUTS-4 level. Only Gałęcka-Burdziak (2012) considers temporal aggregation bias in the data.

I contribute to the literature in several ways. I use overidentified specifications of selected matching function models to identify which type of agents (stocks or flows) form pairs. I employ both temporal data aggregation solutions to compare them, but my primary aim is to obtain robust results. This approach allows me to identify the matching function elasticities with respect to stocks, flows and ‘at risk’ measures. I also extend these solutions to directly analyse the job queuing framework. Finally, I provide a thorough analysis of the matching process in the Polish labour market, particularly concerning public employment intermediation.

I find that stocks and inflows of agents engage in a matching process. Job seekers (from the pool) seek work among old and new job posts, but only a small fraction of the newly unemployed individuals enjoy a positive instantaneous re-employment probability. Demand is the driving force in aggregate hiring, but the vacancy inflow is more important

than the stock counterpart. The positive elasticity of the vacancy stock demonstrates that not all job offers are covered instantaneously, despite the large discrepancies between demand and supply. I find that the random model appears to be more relevant, but the results do not negate the non-random matching mechanism. If the random model prevails, this entails the need to improve information to facilitate the matching process. The stock-flow model justifies labour market policy actions intended to create more job offers.

2. Labour market matching models and the method

Different theoretical frameworks (stock-based random, job queuing and stock-flow) offer various explanations of the matching process. Matching is determined either by stocks or by a combination of stocks and flows. Appropriate specifications of the matching function are used to verify theoretical assumptions concerning the matching technologies. Typically, the form is a Cobb-Douglas function, $M = m(U, u, V, v)$, where M represents the number of matches, U is the unemployment stock, u is the unemployment inflow, V is the vacancy stock, and v is the vacancy inflow. Table 1 compiles the forms of an unemployed worker's hazard rate, according to particular model assumptions.

[Table 1]

In the empirical analysis, I extend these forms by assuming more general specifications. I attempt to determine the importance of particular matching mechanisms. Coles and Smith (1998) consider the outflow from unemployment to employment (disaggregated according to search duration) to be dependent on a vacancy stock and an inflow. Gregg and Petrongolo (2005) propose a test to verify stock-flow and random mechanisms by enabling an unemployment stock to match both the stock and the inflow of job offers, as do Coles and Petrongolo (2008). I extend this test (using overidentified specifications) to determine what types of agents form pairs.

I perform the analysis and econometrically correct for temporal aggregation bias in the data. Such bias arises when continuous economic processes are described using discrete data. Coles and Smith (1998) present an example. A job posting is published at the beginning of the month but is not matched until the end of the month. A job seeker arrives in a market at the end of the month. Monthly data present both agents as part of an inflow, although from a job seeker's perspective, the vacancy is part of a stock. Moreover, the literature suggests that large fractions of newcomers immediately match after entering the market. These agents are not reflected in end-of-period stocks, and hence, the stocks do not properly approximate job seeker or vacancy pools (Gregg and Petrongolo 2005; Petrongolo and Pissarides 2001). Temporal aggregation bias in the data leads to understatement of the importance of stocks and overstatement of the importance of inflows.

I address this problem using the frameworks of Gregg and Petrongolo (2005) and Coles and Petrongolo (2008), and proposing a slight modification that enables the direct estimation of a job queuing model. Gregg and Petrongolo (2005) and Coles and Petrongolo (2008) identify the number of agents available for matching. However, only the second framework fully accounts for temporal aggregation in the data. Coles and Petrongolo (2008) highlight differences between frameworks that mathematically address the temporal aggregation problem in the data. Studies adopt different conditions for the hazard rate of exiting unemployment. In Gregg and Petrongolo (2005), this rate depends on beginning-of-period stocks (in random matching) or on a stock and a relevant inflow (in stock-flow matching). Coles and Petrongolo (2008) condition the hazard rate on 'at-risk' measures¹. Thus, even in the random model, the vacancy inflow is operationalised through a vacancy 'at-risk' variable. This leads to the conclusion that Gregg and Petrongolo's model is biased

¹ An 'at-risk' measure of the unemployed individuals (or vacancies) presents a pool of unemployed workers (or vacancies) that are available for matching in every particular moment. This measure includes the respective share of a beginning-of-period stock and an inflow of new agents.

against the random mechanism because it does not fully reflect temporal aggregation in the data (Coles and Petrongolo 2008). However, Gregg and Petrongolo's model provides coefficients that enable calculations of the mean elasticities of the dependent variable with respect to stocks and flows. These elasticities are typically the most important outcome of the empirical analysis. Coles and Petrongolo's solution directly estimates elasticities of the dependent variable with respect to 'at-risk' measures of unemployment and vacancies as well as the inflows of unemployed individuals and job postings. Therefore, the quantitative results of the two frameworks are not explicitly comparable, but they do complement one another.

Gregg and Petrongolo² (2005) condition the number of matched unemployed job seekers on unemployment stocks and inflows and corresponding outflow rates. The specific models are verified using appropriate specifications of λ (unemployment stock hazard rate) and p (instantaneous unemployment inflow matching probability). In simple random matching, for example, λ depends on beginning-of-period stocks, but in full non-random matching, it depends on the unemployment stock and vacancy inflow, while p depends on the unemployment inflow and vacancy stock. The equation for the outflow from unemployment for a random model takes the following form:

$$M_t = U_t(1 - e^{-\lambda_t}) + u_t \left(1 - \frac{1 - e^{-\lambda_t}}{\lambda_t} \right)$$

where $\lambda = \lambda(U, V)$. For a stock-flow model, it takes the following form:

$$M_t = U_t(1 - e^{-\lambda_t}) + u_t \left(1 - (1 - p_t) \frac{1 - e^{-\lambda_t}}{\lambda_t} \right)$$

where $\lambda = \lambda(U, v)$, $p = p(u, V)$ and

M_t – the number of matched unemployed individuals during month t ,

U_t – beginning-of-month t unemployment stock,

² Appendix A contains the model derivation.

- u_t – unemployment inflow during month t ,
- λ_t – unemployment stock hazard rate, and
- p_t – instantaneous unemployment inflow matching probability.

A straightforward modification produces a job queuing model. We assume a random matching mechanism and that job seekers only match with vacancy inflow; hence, the total number of matches equals

$$M_t = U_t(1 - e^{-\lambda_t}) + u_t \left(1 - \frac{1 - e^{-\lambda_t}}{\lambda_t}\right)$$

where $\lambda = \lambda(U, v)$.

In Coles and Petrongolo's (2008) solution³, the total number of matched job seeker – vacancy pairs in random matching equals

$$M_t = \lambda_t \bar{U}_t$$

while in stock-flow matching, it equals

$$M_t = \lambda_t \bar{U}_t + p_t u_t$$

where \bar{U}_t is the unemployment 'at-risk' measure.

In a stock-based mechanism, the number of matches depends on the unemployment and vacancy 'at-risk' measures (\bar{V}_t). The non-random model assumes that members of the unemployment pool match the vacancy inflow, while individuals in the unemployment inflow match members of the vacancy pool.

Under a job queuing framework, we can assume a random mechanism:

$$M_t = \lambda_t \bar{U}_t$$

However, the unemployment pool matches the vacancy inflow:

$$\lambda_t \bar{U}_t = v_t \text{ with } \lambda_t = \lambda_t(\bar{U}_t, v_t)$$

³ Appendix B contains the model derivation.

3. *Data*

I analysed the period 1999 – June 2013 (using monthly⁴, seasonally adjusted registered unemployment data⁵). The labour market exhibited an anticlockwise loop around a downward-sloping Beveridge curve. The period 1999 – 2002 represents a recession. Since 2002, one can distinguish a positive aggregate activity shock, although the sub-period 2002 – 2004 is a jobless recovery primarily caused by labour productivity growth (Drozdowicz-Bieć 2012). In 2008, the *UV* curve began to reverse.

[Figure 1]

The values of the labour market tightness indices suggest that, on average, job seekers experienced difficulty in finding work⁶ and that enterprises found workers with relative ease. The best conditions for job seekers existed between 2007 and 2009 (partially prolonged to the end of 2010). The index based on vacancy inflow was constantly above that based on the vacancy stock, confirming the importance of a flow determinant. The former also exhibited greater volatility and short-term variations.

[Table 2]

Table 2 provides the summary statistics for the selected variables: total outflow from unemployment, outflow from unemployment to employment, unemployment stock, unemployment inflow, vacancy stock and vacancy inflow. Unit root tests for all variables in

⁴ Burdett et al. (1994) indicate that the temporal aggregation bias in monthly data can be small. However, time aggregated models more properly reflect particular agent pools, notably because they consider an inflow to be a determinant of the magnitude of the stock.

⁵ It would have been possible to conduct the analysis using LFS data on the number of job seekers. I elected to consider the registered data, as vacancies are primarily directed towards unemployed individuals registered at public employment offices.

⁶ The data only refer to job offers registered at public employment offices. The number of job offers is often underestimated. The extent of the possible bias can be approximated by analysing the difference between outflow from unemployment to employment during a month (registered data) and the sum of the monthly vacancy inflow and beginning-of-month vacancy stock. Positive values imply that more unemployed job seekers found jobs than there were available job offers at the employment agencies (for example, this situation occurred between 1999 and 2005). Those who found work might have done so without assistance from public employment offices. Only approximately 16.5% of companies list job offers at public employment offices (results refer to 2012, compare *Badanie Ankiety Rynku Pracy* NBP).

first differences reject the null hypothesis of the existence of a unit root⁷. Higher turnover is observed in vacancies than among the unemployed individuals. The degree of volatility in the monthly $\frac{\text{inflow}}{\text{stock}}$ ratios is much higher in the case of vacancy variables (from 1.3 to 5.0 for vacancies, versus 0.06 to 0.17 for unemployment). This result means that vacancies are both much more volatile and of lower expected duration than unemployment. All series display a high degree of persistence. Monthly autocorrelation coefficients are slightly higher for stock variables than for flow variables.

4. Results

I estimated unemployment outflow equations to analyse matching technologies prevailing in the labour market in Poland. I used the frameworks of Gregg and Petrongolo (2005) and Coles and Petrongolo (2008), as they produce complementary results. Table 3 presents results using Gregg and Petrongolo's (2005) framework for different specifications of λ and p . All of the estimated specifications are reported in Appendix C. The analysis was based on seasonally adjusted monthly registered unemployment data, where the outflow from unemployment to employment was an endogenous variable. The estimation was performed using non-linear least squares and including first-order serial correlation in the disturbance term to address autocorrelation. ADF tests indicated that, at the 5% significance level, the null hypothesis of the presence of a unit root in the disturbance term could be rejected. Therefore, although the time series were non-stationary, cointegration occurred, and the equations converged to the long-run equilibrium. Table 3 includes structural parameter estimates and summary statistics. The hazard rates for leaving the unemployment stock and unemployment

⁷ For a discussion of the use of both unit root and stationarity tests, see Charemza and Syczewska (1998), Gabriel (2001) or Harris and Sollis (2005).

inflow (instantaneous match) were estimated or counted on the basis of coefficients (the average values of model predictions are presented in square brackets).

I began the analysis using the most general specifications of λ and p . Most of the estimations produced statistically insignificant and/or incorrectly signed coefficients. I chose five equations that allow for qualitative inference. Model I (of table 3) reflects non-random matching, in which p is assumed to be constant, while the unemployed individuals seek work among old and new vacancies. Model II presents the results of the full stock-flow specification in which a stock on one side matches an inflow on the other. The random model assumes that $\alpha_1 > 0$ and $p = 0$, while the stock-flow framework assumes that $\alpha_1 = 0$ and $p > 0$. Models III and IV refer to the job queuing concept. In the third model, the unemployment stock matches both vacancy variables. Econometrically, this is a reduced-form version of the specification in column I (except for p), but it assumes a random mechanism. Model IV refers to the pure job queuing model. Model V reports results for the random stock-based model.

[Table 3]

The positive and significant estimate of α_1 (column V) confirmed the importance of a vacancy stock in the random model. Equations that included the vacancy inflow provided new insights into the significance of the stock and inflow variables. The stock coefficient remained statistically significant in all specifications, but its magnitude decreased sharply. The elasticities calculated with respect to stocks and inflows indicate that the unemployment stock remained the most important variable in generating outflow from unemployment to employment across all specifications. The elasticity of the vacancy stock decreased sharply once its inflow counterpart was incorporated into the analysis.

[Figure 2]

The exit rate from unemployment to employment had a mean value of 0.043 and fluctuated between 0.025 and 0.066. The mean predicted hazard rate of exiting unemployment was in the range (0.036; 0.041). The hazard rate was highest in the random and job queuing models. The small or statistically insignificant values of p did not create substantial variations in λ values. Models incorporating vacancy inflow better reflected short-term variations in the exit rate, but they more substantially underestimated the mean value. The goodness-of-fit of the predicted λ values deteriorated considerably since 2009.

Unemployed individuals spent, on average, at least 78 weeks in unemployment, with relatively comparable results across different specifications. Models that included the vacancy inflow yielded slightly higher values. The instantaneous matching probability for the unemployment inflow was positive and significant at the 5% level only in the full stock-flow equation, implying that only 5% of newcomers could obtain suitable job offers immediately after entering the market.

I based the second part of the analysis⁸ on Coles and Petrongolo's (2008) framework. These specifications (table 4) are analogous to those presented in table 3. The full stock-flow model (Model II) yielded statistically insignificant parameters (apart from $\hat{\alpha}_3(u_t)$), but I present it as a reference. Elasticities refer to 'at-risk' measures (or inflows).

[Table 4]

The statistically significant parameter estimates for the stock-based model demonstrated that a random mechanism was operating in the labour market. The vacancy pool had a slightly greater influence on the outflow from unemployment to employment than did unemployment. In the job queuing models, the unemployment pool had a higher elasticity

⁸ In this article, I used the algorithm presented in Álvarez de Toledo et al. (2008), who based their research on a version of the model presented in Coles and Petrongolo (2003). The job queuing framework equations were estimated assuming a random mechanism and adjusting the code for the stock-based model. I checked the robustness of the results by estimating the parameters on the basis of the unemployment 'at-risk' measure obtained from random and reduced-form stock-flow models. The point estimates were virtually identical.

than the vacancy pool. Equations incorporating the vacancy inflow confirmed its role in the matching process. A direct comparison of the vacancy pool and vacancy inflow indicates that the flow had a greater impact on generating matches (not in Model I; however, this stock-flow model also contains an insignificant estimate of p). The instantaneous matching probability of the newly unemployed was close to zero and statistically insignificant.

[Figure 3]

The highest mean re-employment probability was observed in the job queuing model predictions (columns III and IV) – 0.0414. The random model yielded a virtually identical value – 0.0413. Random mechanisms most accurately reflected the mean exit rate from unemployment. The reduced-form stock-flow specification yielded the most severely underestimated λ value (apart from the full stock-flow specification). The goodness-of-fit deteriorated slightly, which implies underestimated values of λ , particularly since 2009. The mean unemployment duration across the specifications was at least 78 weeks.

5. Discussion

The estimated elasticities indicate that most of the matches originate from the unemployment stock and vacancy inflow. This means that many job seekers are situated on the long side of the market and wait for new job postings to arrive. Matches between the unemployment inflow and the vacancy stock play a lesser role in generating the outflow from unemployment to employment. The instantaneous matching probability of the unemployment inflow was close to zero in most specifications. This finding confirms that the matching process is time consuming and only few workers have non-zero re-employment probability. The positive elasticity of the vacancy stock also demonstrates that not all vacancies are covered instantaneously and that job seekers consider both old and new job offers.

The results do not preclude any of the analysed matching technologies, although random matching seems to have some superiority over the stock-flow model. If a random mechanism prevails (in either the stock-based or job queuing form), this means that labour market policy should be directed at improving the information in the labour market to facilitate matching. The finding that the stock-flow model has some explanatory power implies that heterogeneous agents engage in search activity.

All estimates indicate a mean unemployment duration of at least 17 months (78 weeks). These results overestimate the true value of the mean unemployment duration, but the models assumed only that there is an outflow from unemployment to employment (the outflow from unemployment to employment constituted on average 45% of the total outflow from unemployment during the studied period). The LFS data show that the mean unemployment duration oscillated between 9 and 19 months during the period 2003 – 2013, averaging 14 months. However, job seekers can use various job search methods, and unfortunately, the outflow from unemployment to employment cannot be equated with public employment intermediation.

I attempted to check the robustness of the results in a few ways. The time series used in the analysis are non-stationary, but the residuals are stationary. Thus, the models converge to the long-run equilibrium. The application of Engle-Granger procedure would yield only one cointegrating vector, even if there were, in fact, more than one. Gałecka-Burdziak (2015a) analysed the aggregate labour market matching in Poland during an analogous period. She used time series techniques. Using the Johansen approach, she reported, among other findings, one cointegrating vector in majority of the matching functions models. Thus, I assume that the results presented in this paper reflect long-run equilibrium in the matching process.

As indicated above, various worker flows affect the matching function elasticities. Gałecka-Burdziak (2015b) mathematically and empirically analysed the direction of the bias for random, stock-flow and job queuing models when certain worker flows are omitted. She examined various sets of endogenous and exogenous variables from registered unemployment data in Poland and Spain. Only some of the theoretical implications were confirmed. Additionally, Gałecka-Burdziak (2015b) found that unemployment has a greater influence on matching than it does on discouragement.

The findings of this article are consistent with the previous results for the Polish labour market. The unemployment stock enjoys the highest elasticity in most of the results, but demand is the driving force in the job creation process. The exit rate from unemployment to employment appears to jointly depend on the stock and inflow of new job offers, whereas certain λ estimates reflect indices of labour market tightness. The point estimates in this paper are more quantitatively robust than those of previous contributions. They differ because I econometrically address the temporal aggregation problem in the data and do not adopt the augmented matching function concept, as the matching itself is of primary interest in this study. The analysis could be extended by considering data on the precise effects of public employment offices or data referring to the entire labour market. That, however, is precluded by data limitations.

6. Concluding remarks

I analysed the matching technology in the matching process between workers seeking jobs and companies seeking workers. The comparative macroeconomic analysis referred to the Polish labour market during the period 1999 – 2013. I used registered unemployment data. I estimated various model parameters while econometrically addressing

temporal aggregation bias in the data. I extended the known solutions to make them directly applicable to a job queuing model.

I found that stocks and inflows of agents engaged in a matching process. Job seekers (from the pool) sought work among old and new job posts. Only a small fraction of the newly unemployed individuals enjoyed positive instantaneous re-employment probability. The vacancy inflow was more important than the vacancy stock, but the positive elasticity of the vacancy stock demonstrated that not all job offers were covered instantaneously. The inflows did not match with one another.

The results indicated some superiority of the random matching mechanism, but the stock-flow explanation was not negated. Thus, the real matching process is complicated and time-consuming. If the random model prevails, this means that labour market policy should be focused on improving the information in the labour market. Better information should facilitate the matching process. The stock-flow model, in turn, demonstrates that agents are more aware of the search and recruitment process and that job search activities are systematic. Here, heterogeneous agents are better off if there are more potential partners to choose from; thus, labour market policy should have the goal of increasing the inflows in the labour market and creating more job offers. The shape of the exit rate from unemployment also emphasises the need to increase the number of the job offers. Certain model predictions resemble vacancy series properties. Thus, demand is the driving force in aggregate hiring in Poland.

Acknowledgements

I am grateful to researchers at the University of Seville – Fernando Núñez and Pablo Álvarez de Toledo – and Pablo de Olavide University, Seville – Carlos Usabiaga – for computational assistance and detailed comments on previous drafts of this paper. I am also grateful to Marek Góra, seminar participants at Pablo de Olavide University, Seville, and the Labour Economics Summer School held at Barcelona Graduate School of Economics, Simposio de la Asociación Española de Economía-Spanish Economic Association (SAEe) for useful comments on earlier drafts of the paper. Any remaining errors are mine.

I gratefully acknowledge financial support from the National Science Centre Poland, project no. UMO-2012/05/N/HS4/00194. Ewa Gałecka-Burdziak is a “START” Scholarship holder granted by the Foundation for Polish Science (FNP).

References

- Álvarez de Toledo, P., & Núñez, F. & Usabiaga, C. (2008). La función de emparejamiento en el mercado de trabajo español. *Revista de Economía Aplicada*, 48 (vol. XVI), 5–35
- Badanie Ankietowe Rynku Pracy. Raport 2012. Instytut Ekonomiczny National Bank of Poland, <http://www.nbp.pl>, access: 01.03.2013
- Blanchard, O. & Diamond, P. (1994). Ranking, Unemployment Duration, and Wages. *The Review of Economic Studies*, 61:3, 417–434
- Borowczyk-Martins, D. & Jolivet, G. & Postel-Vinay, F. (2011). Accounting For Endogenous Search Behavior in Matching Function Estimation, IZA DP NO. 5807.
- Burdett, K. & Coles, M. & van Ours, J. (1994). Temporal Aggregation Bias in Stock-Flow Models. Centre for Economic Policy Research Discussion Paper no. 967, May
- CSO, Central Statistical Office, <http://www.stat.gov.pl>, access: 03.09.2012.
- Charemza, W. & Syczewska, E. (1998). Joint Application of the Dickey-Fuller and KPSS Tests. *Economic Letters*, 70, 1–8
- Coles, M. & Petrongolo, B. (2003). A Test Between Unemployment Theories Using Matching Data. CEP (LSE) Discussion Paper no. 570
- Coles, M. & Petrongolo, B. (2008). A Test Between Stock-Flow Matching and the Random Matching Function Approach. *International Economic Review*, Vol. 49, No. 4 (November): 1113–1141
- Coles, G. & Smith, E. (1998). Marketplace and Matching. *International Economic Review*, Vol. 39, No. 1 (Feb.): 239–254
- Drozdowicz-Bieć, M. (2012). Cykle i wskaźniki koniunktury. Wydawnictwo Poltext, Warszawa
- Gabriel, V. (2001). Cointegration and the joint confirmation hypothesis. NIPE Working Paper Series 12
- Galušćák, K. & Münich, D. (2005). Structural and cyclical unemployment: what can we derive from the matching function? Czech National Bank Working Paper Series 2.
- Gałęcka-Burdziak, E. (2012). Labour market matching – the case of Poland. *Bank i Kredyt* 43 (3): 31–46
- Gałęcka-Burdziak, E. (2015a). Aggregate labour market matching in Poland. *International Journal of Economics and Business Research (IJEER)*, Vol. 9, No. 4: 393–402.
- Gałęcka-Burdziak, E. (2015b). Underestimated or overestimated: the matching function elasticities biased by worker inflows and outflows, manuscript.
- Gregg, P. & Petrongolo, B. (2005). Stock-flow matching and the performance of the labor market. *European Economic Review*, 49: 1987–2011
- Harris, R. & Sollis, R. (2005). Applied Time Series Modelling and Forecasting. John Wiley & Sons Ltd
- Kwiatkowski, E. & Tokarski, T. (1997). Efekty polityki państwa wobec rynku pracy w Polsce. Analiza na podstawie funkcji dostosowań. *Ekonomista*, 3, 345–372
- Lehmann, H. (2012). The Polish Growth Miracle: Outcome of Persistent Reform Efforts. IZA Policy Paper No. 40
- Petrongolo, B. & Pissarides, C. (2001). Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature*, vol. XXXIX (June), 390–431

- Registered unemployment 1999 – 2013. Public Employment Services <http://www.psz.praca.gov.pl>
access: 01.09.2013.
- Roszkowska, S. (2009). Aggregate Matching Function. The Case of Poland. *Central European Journal of Economic Modelling and Econometrics*, vol. 1, issue 2: 157–177
- Shapiro, C. & Stiglitz, J. (1984). Equilibrium Unemployment as a Worker Discipline Device. *American Economic Review*, Vol. 74, No. 3 (Jun.): 433–444
- Tyrowicz, J., 2011, Histereza bezrobocia w Polsce, Warszawa: Wydawnictwo Uniwersytetu Warszawskiego.

Appendix A: Gregg and Petrongolo (2005) model

Let $M_t = m(U_t, V_t)$ express the random matching technology, where M_t is the number of jobs created at time t , U_t is unemployed job seekers at time t , and V_t is the number of vacancies at time t . The data refer to the number of matches aggregated over time. The mean rate of leaving unemployment is $\lambda_t = \frac{M_t}{U_t}$. The unemployment exit rate λ_t by random matching is equal for all job seekers. Substituting the unemployment matching rate and normalising the length of the studied period to 1 yields the number of matches M_t at each moment.

$$M = \int_0^1 m(U_t, V_t) dt = \int_0^1 U_t \lambda_t dt \quad \text{A.1}$$

U_t , the unemployment pool available for matching at each moment, depends on the fraction of the beginning-of-period unemployment stock that has not formed any match during the period $(0, t)$ and the fraction of the unemployment inflow that entered the unemployment pool but has not yet matched during the interval $(0, t)$.

$$U_t = U_0 \cdot e^{-\int_0^t \lambda_s ds} + \int_0^t u_{t'} \cdot e^{-\int_{t'}^t \lambda_s ds} dt' \quad \text{A.2}$$

where U_0 is the beginning-of-period unemployment stock and u_t is the unemployment inflow during a period $(0, t)$. If we assume that the unemployment inflow and the unemployment stock hazard rate are constant, i.e., $u_t = u$ and $\lambda_t = \lambda$, respectively, we expect a uniform distribution of unemployment inflow during a month and relatively large month-to-month variation in regressors compared with a within-month variation and a constant hazard of leaving unemployment. Calculating integrals yields the aggregated number of job seekers available for matching at time t :

$$U_t = U_0 \cdot e^{-\lambda t} + u \cdot \frac{1 - e^{-\lambda t}}{\lambda} \quad \text{A.3}$$

It also yields the matching function:

$$M = U_0(1 - e^{-\lambda}) + u \left(1 - \frac{1 - e^{-\lambda}}{\lambda} \right) \quad \text{A.4}$$

The terms in brackets are the outflow rates of the unemployment stock and unemployment inflow, respectively. Analogous procedures may be used to derive an equation representing the total number of matched vacancies.

In the stock-flow matching mechanism, stocks match with inflows. A positive estimate of the p parameter indicates an instantaneous probability of leaving the unemployment pool by the unemployment inflow (instantaneous match after entering the unemployment pool). If the newly unemployed individuals do not find proper matching partners, they must wait with probability $(1 - p)$ for new vacancies to enter the market. Then, each agent matches according to a hazard rate λ . The unemployment outflow equation is

$$M = U_0(1 - e^{-\lambda}) + u \left[1 - (1 - p) \frac{1 - e^{-\lambda}}{\lambda} \right] \quad \text{A.5}$$

Appendix B: Coles and Petrongolo (2008) model

The authors define the equation presenting the total number of matches in month t . They assume constant inflows of new agents – u_t and v_t – which represent job seekers and vacancies, respectively. The first integral of equation B.1. shows the fraction of the unemployment stock that successfully matches in month t . The second integral shows the fraction of the newly unemployed who enter the market and also match during month t with the probability expressed in the bracket integral.

$$M_t = \int_0^1 U_t e^{-\lambda_t s} \lambda_t ds + \int_{x=0}^1 u_t dx \left[\int_{s=x}^1 e^{-\lambda_t(s-x)} \lambda_t ds \right] \quad \text{B.1}$$

which simplifies to

$$M_t = \lambda_t U_t \left[\frac{1 - e^{-\lambda_t}}{\lambda_t} \right] + \lambda_t u_t \left[\frac{e^{-\lambda_t} - 1 + \lambda_t}{\lambda_t^2} \right] \quad \text{B.2}$$

The “at risk” measure of unemployment⁹ includes the fraction of the beginning-of-period stock that failed to match until time t and the fraction of the inflow of new job seekers who also failed to match until time t .

$$\bar{U}_t = U_t \left[\frac{1 - e^{-\lambda_t}}{\lambda_t} \right] + u_t \left[\frac{e^{-\lambda_t} - 1 + \lambda_t}{\lambda_t^2} \right] \quad \text{B.3}$$

The expected number of matches in the random model is then

$$M_t = \lambda_t \bar{U}_t \quad \text{B.4}$$

where M_t is total number of matches in month t , \bar{U}_t is the “at risk” measure of unemployment, and λ_t is the average re-employment rate.

An analogous procedure applied to vacancies yields the number of matches:

$$M_t = \mu_t \bar{V}_t \quad \text{B.5}$$

where V_t is the “at risk” measure of vacancies, μ_t is the average vacancy matching rate, and the “at risk” measure of vacancies, \bar{V}_t , is as follows:

⁹ I assume no flows to and from inactivity and no withdrawals of job offers.

$$\bar{V}_t = V_t \left[\frac{1 - e^{-\mu_t}}{\mu_t} \right] + v_n \left[\frac{e^{-\mu_t} - 1 + \mu_t}{\mu_t^2} \right] \quad \text{B.6}$$

The expected number of job seekers must equal the expected number of vacancy matches. Equating B.4 and B.5 yields

$$\lambda_t \bar{U}_t = \mu_t \bar{V}_t \quad \text{B.7}$$

where

$$\lambda_t = \lambda(\bar{U}_t, \bar{V}_t) \quad \text{B.8}$$

The empirical strategy combines an estimation of the hazard function parameters and solving the set of equations B.3, B.6 – B.8, where the predicted number of matches is $M_t(\theta) = \lambda_t \bar{U}_t$.

In the stock flow-matching model, the fraction p_t of the newly unemployed is on the short side of the market and rematches immediately after arriving in the market. The remaining job seekers are on the long side of the market and match according to a hazard rate λ_t . The total number of matches equals

$$M_t = \int_0^1 U_t e^{-\lambda_t s} \lambda_t ds + p_t u_t + \int_{x=0}^1 \left[\int_{s=x}^1 e^{-\lambda_t(s-x)} \lambda_t ds \right] (1 - p_t) u_t dx \quad \text{B.9}$$

The first integral of equation B.9 refers to matches arising from the original stock of job seekers. These agents and the fraction $(1 - p_t)$ of the unemployment inflow (second integral) are situated on the long side of the market and match at the hazard rate λ_t . The term $p_t u_t$ expresses immediate matches from the unemployment inflow. Equation B.9 simplifies to the following equation:

$$M_t = \lambda_t U_t \left(\frac{1 - e^{-\lambda_t}}{\lambda_t} \right) + p_t u_t + \frac{\lambda_t (1 - p_t) u_t}{\lambda_t} \left(\frac{e^{-\lambda_t} - 1 + \lambda_t}{\lambda_t} \right) \quad \text{B.10}$$

B.10 leads to the “at risk” measure of job seekers situated on the long side:

$$\bar{U}_t = U_t \left[\frac{1 - e^{-(\lambda_n + \delta_U)}}{\lambda_t} \right] + (1 - p_t) u_t \left[\frac{e^{-\lambda_t} - 1 + \lambda_t}{\lambda_t^2} \right] \quad \text{B.11}$$

The expected number of matches is as follows:

$$M_t = \lambda_t \bar{U}_t + u_t p_t \quad \text{B.12}$$

An analogous procedure, applied to vacancies, yields the number of matches as follows:

$$M_t = \mu_t \bar{V}_t + q_t v_t \quad \text{B.13}$$

where q_t is the fraction of the vacancy inflow that matches immediately after arriving in the market, and the “at risk” measure of vacancies, \bar{V}_t , is

$$\bar{V}_t = V_t \left[\frac{1 - e^{-\mu_t}}{\mu_t} \right] + (1 - q_t) v_t \left[\frac{e^{-\mu_t} - 1 + \mu_t}{\mu_t^2} \right] \quad \text{B.14}$$

The expected number of job seeker matches must equal the expected number of vacancy matches. Stock-flow matching assumes that a respective stock matches a relevant inflow, leading to additional restrictions:

$$q_t v_t = \lambda_t \bar{U}_t \quad \text{B.15}$$

For unemployed individuals situated on the short side of the labour market,

$$p_t u_t = \mu_t \bar{V}_t \quad \text{B.16}$$

and

$$\lambda_t = \lambda(\bar{U}_t, v_t) \quad \text{B.17}$$

$$p_t = p(u_t, \bar{V}_t) \quad \text{B.18}$$

The empirical strategy combines estimation of the hazard function and instantaneous matching probability parameters and solving the set of equations B.11, B.14 – B.18, where the predicted number of matches is $M_t(\theta) = \lambda_t \bar{U}_t + u_t p_t$.

Appendix C:

Model I:

$$M_t = \left[1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right] U_{q-1} + \left\{ 1 - \frac{(1 - p_u) \left[1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right]}{e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right\} u_t + \varepsilon_t$$

Model II:

$$M_t = \left[1 - e^{-e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right] U_{q-1} + \left\{ 1 - \frac{\left[1 - e^{\gamma_0 + \gamma_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right)} \right] \left[1 - e^{-e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right]}{e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right\} u_t + \varepsilon_t$$

Model III:

$$M_t = \left[1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right] U_{q-1} + \left\{ 1 - \frac{\left[1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right]}{e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right) + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right\} u_t + \varepsilon_t$$

Model IV:

$$M_t = \left[1 - e^{-e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right] U_{q-1} + \left\{ 1 - \frac{\left[1 - e^{-e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right]}{e^{\alpha_0 + \alpha_2 \ln\left(\frac{v_t}{U_{t-1}}\right)}} \right\} u_t + \varepsilon_t$$

Model V:

$$M_t = \left[1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right)}} \right] U_{t-1} + \left[1 - \frac{1 - e^{-e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right)}}}{e^{\alpha_0 + \alpha_1 \ln\left(\frac{V_{t-1}}{U_{t-1}}\right)}} \right] u_t + \varepsilon_t$$

Table 1. An unemployed worker's hazard rate in particular matching mechanisms

Model	Unemployment stock	Unemployment inflow
Random model	$\lambda = \lambda(U, V)$	–
Stock-flow model	$\lambda = \lambda(U, v)$	$p = p(u, V)$
Job queuing model	$\lambda = \lambda(U, v)$	–

λ – unemployment stock hazard rate, p – instantaneous unemployment inflow matching probability, U – unemployment stock, V – vacancy stock, u – unemployment inflow, v – vacancy inflow.

Source: Author.

Table 2. Variables' main statistical properties, 1999 – 2013

	O	$O \rightarrow E$	U	u	V	v
Mean	219781	97605	2391337	222164	33211	69839
Median	221445	95643	2263031	218356	35547	68262
Stand. Deviation	30871	13080	548703	19230	18930	19074
Monthly autocorr.	0.919	0.888	0.996	0.808	0.986	0.957
Min	153270	69448	1455690	188022	8503	34853
Max	283298	123948	3203591	276894	83356	112303
ADF (p-value)	-1.78 (0.39)	-2.13 (0.23)	-1.60 (0.48)	-3.67 (0.00)	-1.12 (0.71)	-1.96 (0.30)
ADF ^a (p-value)	-13.44 (0.00)	-12.83 (0.00)	-2.74 (0.00)	-18.27 (0.00)	-7.68 (0.00)	-15.15 (0.00)
KPSS (p-value)	0.70 (0.01;0.05)	0.26 (>0.1)	0.87 (<0.01)	0.42 (0.05;0.1)	1.06 (<0.1)	0.74 (0.01;0.05)
KPSS ^a (p-value)	0.24 (>0.1)	0.16 (>0.1)	0.39 (0.05;0.1)	0.09 (>0.1)	0.16 (>0.1)	0.11 (>0.1)
Average turnover:						
unempl. inflow	–		0.10		–	
unempl. stock						
vacancy inflow	–		–		2.66	
vacancy stock						
No. of observations	174	174	174	174	174	174

^a – calculated for first difference

O – total outflow from unemployment, $O \rightarrow E$ – outflow from unemployment to employment, U – unemployment stock, u – unemployment inflow, V – vacancy stock, v – vacancy inflow

Source: Registered unemployment 1999 – 2013, seasonally adjusted data, Author's calculation.

Table 3. Estimates of time aggregated matching models, Gregg and Petrongolo's (2005) framework, 1999 – 2013

Independent variable/ statistics	Parameter estimates (Student's t-statistics)				
	Model I	Model II	Model III	Model IV	Model V
α_0	-1.839*** (0.084)	-2.085*** (0.174)	-2.023*** (0.072)	-1.917*** (0.067)	-2.158*** (0.052)
α_1	0.093*** (0.025)	–	0.076*** (0.027)	–	0.236*** (0.012)
α_2	0.271*** (6.09)	0.350*** (0.040)	0.264*** (0.048)	0.388*** (0.019)	–
γ_0	–	-1.996*** (0.441)	–	–	–
γ_1	–	0.494** (0.243)	–	–	–
λ	[0.0410]	[0.0364]	[0.0377]	[0.0377]	[0.0412]
p	0.004 (0.021)	[0.0508]	–	–	–
R^2 (adj. R^2)	0.881 0.878	0.877 0.875	0.878 0.876	0.873 0.871	0.856 0.854
ADF test for residuals (p-value)	-13.14 (0.00)	-13.23 (0.00)	-13.22 (0.00)	-13.30 (0.00)	-14.10 (0.00)
Log likelihood	-1688.21	-1690.89	-1690.49	-1690.49	-1704.92
Mean unemployment duration	79.3	84.8	86.1	86.2	78.9
Elasticities:					
$\frac{\partial M}{\partial U} \frac{U}{M}$	0.593	0.544	0.571	0.527	0.722
$\frac{\partial M}{\partial u} \frac{u}{M}$	0.047	0.099	0.043	0.043	0.047
$\frac{\partial M}{\partial V} \frac{V}{M}$	0.091	0.057	0.032	–	0.232
$\frac{\partial M}{\partial v} \frac{v}{M}$	0.264	0.306	0.238	0.351	–

Model I – overidentified stock-flow with $\lambda = \lambda(U, V, v)$ and $p = const$, Model II – full stock-flow with $\lambda = \lambda(U, v)$ and $p = p(u, V)$, Model III – overidentified job queuing with $\lambda = \lambda(U, V, v)$, Model IV – job queuing with $\lambda = \lambda(U, v)$, Model V – stock-based with $\lambda = \lambda(U, V)$. Dependent variable: outflow from unemployment to employment. Estimation method: non-linear least squares. * - significant at the 10 per cent level, ** - significant at the 5 per cent level, *** - significant at the 1 per cent level. Standard errors reported in brackets. Expected duration measured in weeks and counted as $\frac{13(1-p)}{\lambda}$. The matching elasticities are sample averages. The sample averages of λ and p are reported in square brackets.

Source: Registered unemployment 1999 – 2013, seasonally adjusted data, Author's calculation.

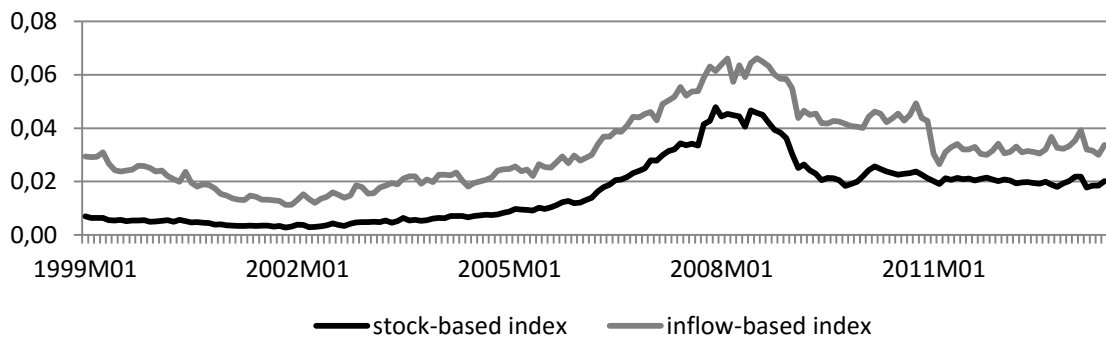
Table 4. Estimates of time-aggregated matching models, Coles and Petrongolo's (2008) framework, 1999 – 2013

Independent variable/ statistics	Parameters estimates (Student's t-statistics)				
	Model I	Model II	Model III	Model IV	Model V
const for (\bar{U}_t, v_t) pair	0.044 (0.063)	0.001 (0.007)	0.230 (0.221)	0.650 (0.547)	0.087 (0.097)
$\alpha_1(\bar{U}_t)$	-0.307*** (0.078)	-0.084 (0.619)	-0.428*** (0.051)	-0.487*** (0.043)	-0.353*** (0.056)
$\alpha_2(\bar{V}_t)$	0.224*** (0.069)	0.262 0.343	0.149** (0.073)	–	0.396*** (0.035)
$\alpha_3(u_t)$	–	-0.859*** (0.271)	–	–	–
$\alpha_4(v_t)$	0.171** (0.078)	0.452 (0.301)	0.261*** (0.072)	0.395*** (0.031)	–
const for (u_t, \bar{V}) pair	–	350.088 (2645.354)	–	–	–
p	0.024 (0.029)	[0.1631]	–	–	–
R^2 (adj. R^2)	0.881 0.877	0.876 0.872	0.876 0.873	0.873 0.871	0.867 0.865
ADF test for residuals (p-value)	-13.11 (0.00)	-9.02 (0.00)	-13.32 (0.00)	-13.31 (0.00)	-13.93 (0.00)
Log likelihood	-1688.58	-1691.67	-1691.94	-1694.03	-1697.93
λ	[0.0390]	[0.0258]	[0.0414]	[0.0414]	[0.0413]
Mean unemployment duration:	83.4	126.73	78.6	78.5	78.6

Equation describing outflow from unemployment to employment includes AR(1). Model I – overidentified stock-flow $\lambda = \lambda(\bar{U}, \bar{V}, v)$ and $p = \text{const}$, Model III – overidentified job queuing $\lambda = \lambda(\bar{U}, \bar{V}, v)$, Model IV – job queuing $\lambda = \lambda(\bar{U}, v)$, Model V – stock-based $\lambda = \lambda(\bar{U}, \bar{V})$. Estimation method: non-linear least squares. * - significant at the 10 per cent level, ** - significant at the 5 per cent level, *** - significant at the 1 per cent level. Standard errors reported in brackets. Expected duration measured in weeks and calculated as $\frac{13(1-p)}{\lambda}$. The sample averages of λ and p are reported in square brackets.

Source: Registered unemployment 1999 – 2013, seasonally adjusted data, Author's calculation.

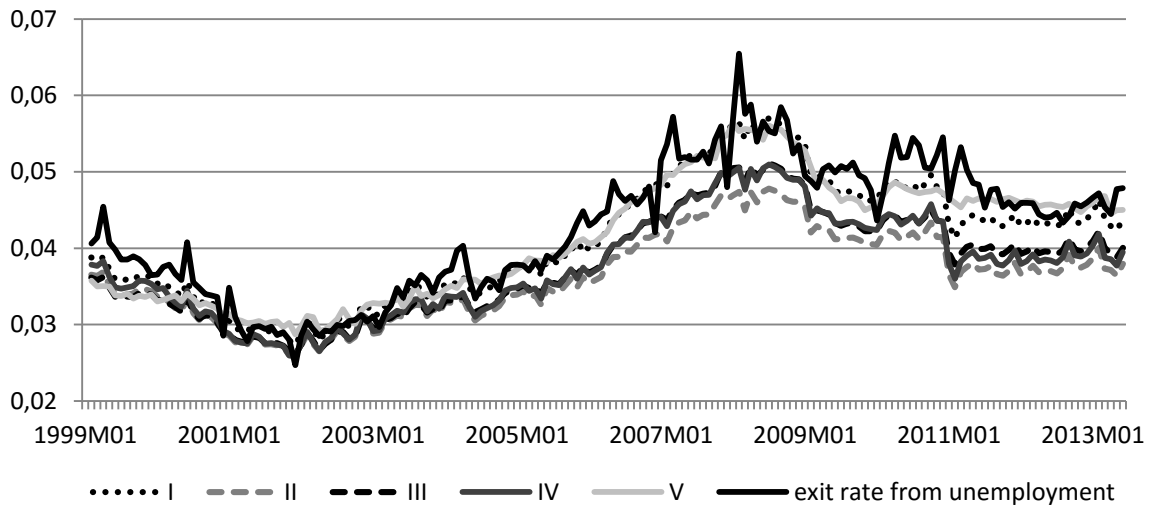
Figure 1. Labour market tightness indices, 1999 – 2013



$\frac{v}{u}$ – stock-based index, $\frac{v}{u}$ – inflow-based index

Source: Registered unemployment 1999 – 2013, seasonally adjusted data, Author's calculation.

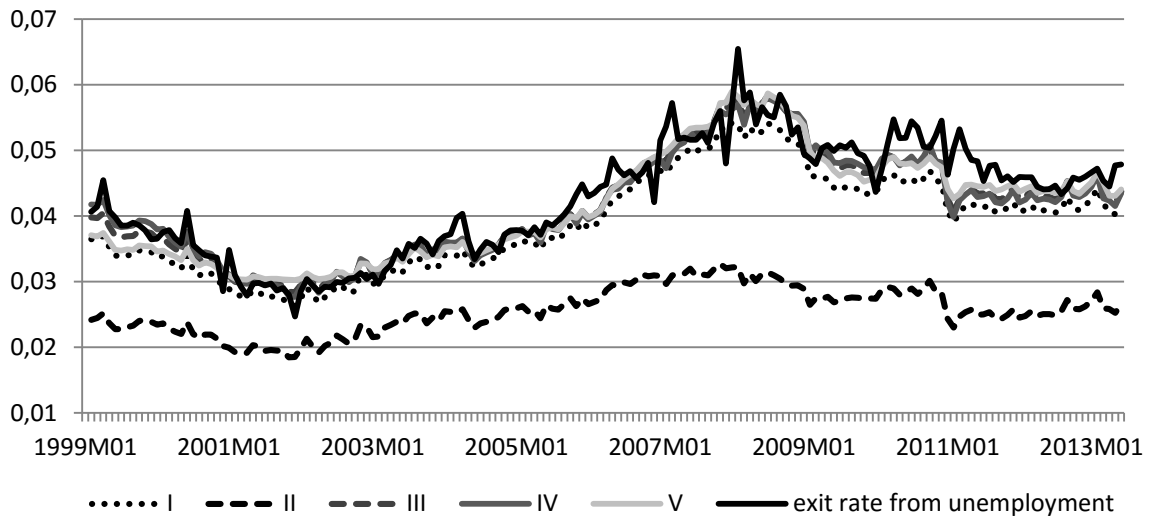
Figure 2. Predicted hazard rates (equations I – V) of leaving the unemployment pool and the exit rate from unemployment, 1999 – 2013



Model I – overidentified stock-flow, Model II – full stock-flow, Model III – overidentified job queuing, Model IV – job queuing, Model V – stock-based.

Source: Registered unemployment 1999 – 2013, data seasonally adjusted, Author's calculation.

Figure 3. Hazard rates of leaving the unemployment pool in the respective models and the exit rate from unemployment in Poland, 1999 – 2013



Source: Registered unemployment 1999 – 2013, seasonally adjusted data, Author's calculation.