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Abstract

Weaknesses of intertemporal optimisation approaches to consumption modelling include excess sensitivity of an individual's expenditure to interest rate changes and the inability to account for the documented behavioural aspects of decision-making, such as mental accounting or infrequent purchases and debt-taking. Credibly representing category-of-goods mental accounting in an intertemporal optimisation framework is notoriously difficult, as this modelling approach imposes interrelations between the demand for different categories through first-order conditions. This breaks the principle of nonfungibility, contrary to the rationale of mental-accounting theory. Thus, a behavioural-procedural framework is needed. This work applies such an approach in the form of a merger with categorisation theories, devised in a separate paper, to modelling consumer demand in a multimarket overlapping-generations agent-based income distribution model. Consumer decisions about spending on nondurable and frequently bought durable goods and infrequently-bought durable goods, such as houses and flats, are subject to different rules, which allows to model real-world features such as infrequent purchases and rare debt-taking. The devised single and multi-agent models of consumer behaviour are consistent both with microeconomic and macroeconomic evidence on consumption. Moreover, the results of the overlapping-generations agent-based income distribution model demonstrate that income changes are greatly enhanced by behavioural responses of consumers, thus creating high aggregate demand growth.

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1. Introduction

Is mental-accounting or any other psychology-based economic theory of consumer behaviour needed? If credibility and realism of microfoundations of economic models and the theoretical foundations of econometric studies of consumption matter, then the answer to this question is positive. After all, as Canova (2009) argues, if the estimated theoretical model does not reflect the structure of the real process, then the estimation will be biased. The same reasoning applies to drawing conclusions from theoretical models themselves. Mental-accounting behaviour – the set of cognitive rules used by investors or consumers to make financial decisions or choose the desired amount of consumption expenditure – has been documented in experimental and empirical studies, therefore it constitutes a good candidate for behavioural microfoundations of economic theory (Antonides et al. 2011; Thaler 1999; Thaler 1994; Thaler 1990; Thaler 1985; Thaler and Shefrin 1981).

Mental accounting and computational approaches provide an opportunity for exploring issues that cannot be answered in the standard framework. Households' housing loans and mortgages constitute the largest component of assets of the commercial banks' sectors in developed countries[1]. Decisions about making purchases of very expensive goods, often requiring taking out large amounts of debt, are likely to significantly affect the life-cycle expenditure of consumers and to cause cyclical behaviour at least on that particular market. Additionally, if consumers react and change their expenditure patterns in response to stimuli such as the changes of disposable income (here defined as income net of taxes and debt payments), then the rates of growth of consumer expenditure may surpass the growth of consumers' nominal income. These questions, however, require a simulations approach, due to the infrequency, state-dependence of such decisions, and the non-balanced character of economic growth that can appear in such a framework. All of the

[1] See, for example, the balance sheets of commercial banks in the United States: `https://www.federalreserve.gov/releases/h8/current/default.htm`.

enumerated features in fact characterise the approach and the results of this paper.

Economic mental accounting theory, however, faces a significant challenge. Henderson and Peterson (1992) underlined that mental accounting theory had not adequately addressed the questions of why and how mental accounts are formed and what is included in them. They also noted that most discussions of mental accounting have focused on the consequences of framing decisions by forming psychological accounts of the advantages and disadvantages of an option or an event, rather than on the processes underlying mental accounting.

Mental accounting may result from processes described in categorization theories, which can be used to infer both the processes underlying mental accounting and its results (Henderson and Peterson 1992). However, the difficulty is that unlike categorisation, mental accounting necessitates that money contained in one mental account is nonfungible, i.e. it cannot be used in the remaining accounts; conversely, categorization theories allow fungibility. A straightforward way to overcome this problem is to assume that a person's funds are divided (through a behavioural process representing a person's needs and desires) into nonfungible categories. Categorisation theories give justification for the process of forming budgets or grouping expenditure for different purposes, contrary to mental accounting, which concerns either the outcomes of processing and grouping information, or the process of spending from current and permanent income (Henderson and Peterson 1992).

Nevertheless, no formal theoretical model of categorisation-enhanced mental-accounting theory has been developed. Moreover, it is demonstrated in this paper that representations of mental accounting by the means of intertemporal optimisation, including the behavioural life-cycle model of Shefrin and Thaler (1988), face several problems. The first is that they impose strict interrelations between categories of goods through optimality conditions, which can be viewed as a violation of the principle of nonfungibility. The second is that this cannot be justified by categorisation theories, as the elements within a category are context-dependent (Henderson and Peterson 1992) and thus the expenditures on various categories of goods are likely to depend also on other factors than prices and a utility function. Finally, the third and most important issue is that when the optimisation problem is formed with many goods and many separate budget constraints, then the function representing the problem is underidentified. These results, included in another paper, are summarised in appendix D.

Permanent income plays a crucial role in the existing formalisations of mental accounting and is still included in many empirical studies investigating this type of behaviour, despite of the numerous empirical literature refuting the permanent income hypothesis or any significant role of permanent income in consumer decision process (see the discussion in section 2). Additionally, the question of whether such representations are credible arises, especially that intertemporal optimisation approaches, including the behavioural life-cycle model, do not allow for infrequent expensive purchases necessitating using large amounts of debt, and sometimes also large payments using savings. But such expenditures are common in the contemporary economies: mortgages and consumer loans constitute a large amount of the assets of real-world banking sectors.

One of the main reasons for the use of permanent income model in the existing standard and mental-accounting economic theory is the familiarity of the concept and the tradition of using analytical methods for economic analyses. Another is that the behavioural life-cycle is one of the few formalisations of mental accounting. However, it is argued in this paper that credible representation of expenditure-oriented mental accounting necessitates an algorithmic approach. This is not a radical departure from the practice of economic modelling: computational economists and psychologists have argued for representing human decisions by the means of algorithms for a long time, in order to obtain a more transparent and realistic representation of people's behaviour and economic systems (Tesfatsion 2006).

Misconducted experiments may limit our ability to test different hypotheses. Similarly, incorrect theoretical models, which are later used as the bases of experiments or empirical estimation, can limit the conclusions drawn by the researchers (Wilson and Collins 2019; Donkin et al. 2014). In this paper, mental accounting theory is merged with categorisation theories, using two notions of nonfungibility: weak, satisfied by the behavioural life-cycle model and other intertemporal optimisation methods, and strong, in which all total and marginal propensities to consume out of category-specific mental budgets differ. A classification of possible consumer types is constructed. Moreover, an approach to model infrequent purchases of durable goods and debt-taking decisions is presented and analysed in a single-consumer working-life model. Finally, an overlapping-generations income-distribution agent-based model featuring agents using categorisation-enhanced mental-accounting rules is analysed. The results show not only outcomes consistent with micro- and macroeconomic evidence, but also point to consumer demand as a major force amplifying economic growth.

This paper is organised as follows. Section 2 contains the discussion of the documented unsuitability of the permanent income model, and provides the existing evidence for mental-accounting behaviour. The proposed theoretical solution to the problem of modelling consumer behaviour by merging categorisation and mental-accounting theory is presented in section 3. The features of the devised overlapping-generations income distribution agent-based model are presented in section 5. Section 6 contains the results of simulation and discussion. Finally, conclusions are contained in section 7. An example of single consumer's working-life cycle, analysed in a separate paper, is presented in appendix C, along with a short discussion of which features of consumption that are not accounted for by the permanent income or buffer-stock model are represented by the developed framework.

2. Consumption behaviour: what do we know?

2.1. The standard approach

There are two strands of the main approach to consumption modelling: the standard one, often termed the permanent income or life-cycle model (PIH/LCM), and the less frequently used buffer-stock/liquidity constraints model, proposed as a remedy for some of the shortcomings of the former. These interest-rate-based, intertemporal optimisation frameworks are still dominant in modelling consumption, despite ever-growing literature listing econometric, cognitive psychology and consumer research evidence against them and their consequences, such as the interest-rate-driven consumption Euler equation. Empirical research revealed that the reaction of consumption to variations in interest rates is weak or statistically irrelevant (Yogo 2004; Canzoneri et al. 2007; Boug et al. 2021). Conversely, consumers' expenditure was found to be highly responsive to changes in income (Campbell and Mankiw 1989; Parker 2017; Boug et al. 2021).

The history of the tests of the permanent income hypothesis can be described rather as a repeated rejection of the permanent income model. Flavin (1981) was one of the first to provide evidence against PIH/LCM. Mankiw's results indicate that expenditure on consumer durables is more sensitive to changes in the interest rate than spending on nondurables and services (Mankiw 1985). Furthermore, other studies that used aggregate time-series data from the United States of America rejected the restrictions on the data implied by the stochastic versions of the PIH/LCM (Hansen and Singleton 1983; Mankiw et al. 1985; Zeldes 1989).

Campbell and Deaton claimed that smoothness of consumption observed in the data cannot be explained by permanent income theory (Campbell and Deaton 1989). They argued that consumption is smooth because it responds with a lag to changes in income. Parker (2017) reached similar conclusions, namely that households' patterns of spending are highly predictable by past income.

While the buffer-stock theory has not been tested as much as the permanent income hypothesis, the existing empirical investigations performed by Ludvigson and Michaelides (2001) and Jappelli et al. (2008) did not find support for this version of the intertemporal optimisation approach. What is more, the buffer-stock theory shares the same weakness as any other framework based on intertemporal optimisation: in light of empirical studies, consumption is excessively related to the interest rate.

Both of the intertemporal-optimisation approaches – the PIH and the buffer-stock model – were found to be inconsistent with empirical data, yet neither was abandoned. Algorithmic methods offer an opportunity for developing more realistic representation of consumer behaviour. However, although they allow great flexibility in modelling choices, the question is: what valid theory can be used as a basis for a new, behavioural-computational approach?

There are psychology-based alternatives to intertemporal-optimisation framework. One of the main difficulties of incorporating psychological theories in microeconomic or macroeconomic models was the fact that optimisation has been the default way of economic modelling, while computational frameworks have traditionally been distrusted by most economists. The reasons have been various, from an attachment to equilibrium framework, to the 'black box' critique (Judd 2006; Caballero 2010). The latter has been undeserved: in fact, modelling the decision rules using algorithms makes them explicit.

2.2. Mental accounting

The majority of mental accounting research, both theoretical and empirical, including the behavioural life-cycle theory, focused on different sources of income and the nonfungibility of these funds. Nonetheless, for the description of consumer behaviour and spending, the key interest and the biggest potential of mental accounting theory lies in the categories-of-goods approach, i.e. the one that is focused on the objects of expenditure, not its sources. This is because merging it with categorisation theories will allow to model demand on various categories of goods with unequal total and marginal propensities to consume, and thus will enable modelling asymmetrical evolution of various markets, which is impossible under the assumption of consumers using constant-elasticity-of-substitution or any other utility function. This issue does not simply reduce to mental budgeting if the spending rates out of these budgets are time-variable.

That setting budgets in advance may simplify computational costs by reducing the number of alternatives when the available funds are limited is a point first raised by Simon (1947). It also facilitates comparison across possible choices (M. D. Johnson 1984). First mentioned by Tversky and Kahneman (1981) and Thaler (1985), this line of mental accounting research was undertaken also by Heath and Soll (1996). They have assumed that consumers set fixed budgets in advance of consumption, and because consumption opportunities change over time, the preset budgets are usually erroneous. However, they have only analysed a few specific case studies. A complete approach to modelling current consumer choices as well as allocation of resources over time has not yet been constructed.

Apart from to being formed as a psychological theory and applied in numerous experiments, mental accounting has been subject to a few tests against empirical data, which seem to corroborate the claim that households use mental accounting for making purchases. Antonides et al. (2011) have found empirical support for mental accounting consumer behaviour, basing on a large sample of Dutch population. Hastings and Shapiro (2013) have demonstrated that households treat money for different expenditure categories as nonfungible. More evidence supporting the mental accounting consumer behaviour was provided by Cheema and Soman (2006) in an experimental setting. They have shown that consumers flexibly classify expenses (construct accounts) to justify spending on various categories of goods, such as food, clothing and entertainment.

Mental accounting theory of economic behaviour has not been able to cope with the problem of representing consumer behaviour in a multi-market economic model. One of the reasons for this was the focus on the sources of income rather than on the categories of goods. Another reason is that pure mental accounting theory cannot account for individual differences.

Categorisation theories may be invoked to enhance the mental accounting approach. While most mental accounting theory focuses on the outcomes of actions, the process of forming type-of-good-related mental accounts as well as consumer decision making is almost never addressed within this line of research (with an exception of the work of Montgomery et al. (2019), discussed below).

It is demonstrated in this paper that any analytic optimisation-based representation of mental accounting is bound to either impose undesirable interdependencies between mental accounts for various categories of goods, or introduce ad hoc shocks to the optimality conditions to avoid such relations (as in (Montgomery et al. 2019)). This is contrary to the rationale of mental accounting and nonfungibility of funds. Therefore, it seems that credibly representing mental accounting consumer behaviour in a framework with many categories of goods is impossible in the standard, optimisation-based approach to economic modelling. The next subsection demonstrates that analytical models cannot represent category-of-goods mental accounting under assumptions on nonfungibility that rule out exact comovement of expenditure out of different mental budgets.

3. The theoretical solution

3.1. The overview of a consumer's mental-accounting system

3.2. Overview

The basic idea of the mental accounting consumer framework developed in this paper is to represent spending per each category of goods by the means of division-of-funds variables, each of which is constrained by the bounds of a respective mental budget.

The convention adopted in this paper is to use the notion of a 'mental account' in reference to the size of a category-related budget relative to the net disposable income $\tilde{\Omega}_t$ (see equation 1), while the word 'budget' refers to the maximum amount of money that can be spent on a given category. Thanks to this differentiation, the values of accounts are directly comparable with the total saving rates out of the corresponding budget.

The decision rule of a consumer consists of the following steps.

The mental-accounting system of a consumer

1) The consumer i mentally divides his/her current account (i.e. deposits bearing little or no interest, $d_t^{CA,i}$) into four purpose-related mental budgets:

- current expenditure ($d_t^{CA,curr,i}$),

- a mental budget devoted to the accumulation of funds for the purchase of a house or a flat ($d_t^{CA,H,i}$), growing over time

- a mental budget devoted to the accumulation of funds for the purchase of a vehicle ($d_t^{CA,veh,i}$), growing over time

- a mental budget devoted to the accumulation of funds for the purchase of other infrequently bought goods ($d_t^{CA,dur,i}$), growing over time.

2) First, the consumer pays – or reserves funds for the payment of – rents, bills and taxes, and makes debt payments, if there

is any outstanding debt, using the current expenditure budget. Of course, in reality, personal income taxes are paid only once a year, but it is assumed that people accumulate the required amount throughout that period. Quarterly tax payments are an approximation of this process.

3) The consumer spends on thirteen nondurable goods and frequently bought durable goods (see table **??**) from the current expenditure account, using net disposable income ($\tilde{\Omega}_t$, see equation 1) as a reference point for the expenditure. Its gross percentage changes drive the decision of how much to spend (equations 2 and 3). The current expenditure account net of payments of bills, rents and taxes, is divided into thirteen category-specific accounts (categories are labelled using the index s). Total spending rates out of each of the budgets (β_t^s) are time-variable, bounded functions of net disposable income (this framework may easily be extended to incorporate the effect of marketing, inflation and other stimuli). None of the category-specific spending rates can surpass or equate the size of the account (η^s). Therefore, each of the mental budgets consists of expenditure and savings. The saving rate out of an account is denoted $\sigma_{sr,t}^{s,i}$ (where i is an index identifying a particular consumer), while the resultant saving rate out of net disposable income is written as $\sigma_{sr,t}^{i}$.

4) The funds on the current expenditure account that are not spent (that are saved) are transferred to other mental budgets constituting the current account, $d_t^{CA,H,i}$, $d_t^{CA,veh,i}$, $d_t^{CA,dur,i}$, and to the saving account, $d_t^{SA,i}$.

5) The needs for a new house or a flat are present or not, and the needs for a new vehicle and other durable goods arise depending on whether the durability of the currently possessed product has been surpassed or not. The funds accumulated in the budgets $d_t^{CA,H,i}$, $d_t^{CA,veh,i}$, $d_t^{CA,dur,i}$ and available new debt are compared with the prices of these goods. For simplicity, it is assumed that credit is taken out only for houses and flats. If the available funds are sufficient and the need for a new product is present, then the purchase is made. Otherwise the consumer continues accumulating funds in these budgets.

A consumer i decides how much of a budget $\eta_s \cdot \tilde{\Omega}_t^i$ to spend in a given period using a time-varying variable governing the division of funds ascribed to a category s, $\beta_t^{s,i}$, with $\beta_t^{s,i} \in [\eta_{s,LB}, \eta_s]$, with $\eta_{s,LB}$ denoting the lower bound of the account. $\beta_t^{s,i}$ is the gross expenditure rate out of category-s-related account. Thus, the amount spent from a given budget in period t is equal to $\beta_t^{s,i} \cdot \tilde{\Omega}_t^i$.

The behavioural variables that govern the division of accounts into spending and savings are assumed to be functions of gross percentage changes of disposable income, $\tilde{\Omega}_t^i$, relative to the previous period. The disposable income is defined as the income earned in the previous period (quarter), Ω_{t-1}^i, net of rents, bills, taxes and debt payments, if the consumer i has any outstanding debt:

$$\tilde{\Omega}_t^i = \Omega_{t-1}^i - RBT_t^i - dps_t^i \tag{1}$$

Following preference reversal theory (Kahneman and Tversky 1984; Kahneman and Tversky 1979), it is assumed that the magnitude of changes of the behavioural division variables may vary for positive and negative disposable income net percentage changes. For the increases, we have

$$\beta_t^{s,i} = \beta_{s1}^i + \beta_{s5}^i \cdot \exp(\beta_{s3}^i \cdot (\frac{\tilde{\Omega}_t^i}{\tilde{\Omega}_{t-1}^i} - 1)), \tag{2}$$

while for the decreases

$$\beta_t^{s,i} = \beta_{s2}^i + \beta_{s6}^i \cdot \exp(\beta_{s4}^i \cdot (\frac{\tilde{\Omega}_t^i}{\tilde{\Omega}_{t-1}^i} - 1)), \tag{3}$$

and $\beta_{s1}^i, \beta_{s2}^i, \beta_{s3}^i, \beta_{s4}^i, \beta_{s5}^i, \beta_{s6}^i$ denote the parameters of the division-of-funds variables $\beta_t^{s,i}$. Due to the nonlinear character of these decision rules, only some of their parameters have an individual interpretation. The other may be treated as inseparable elements of the behavioural rule – as means of describing the patterns of behaviour. For more details, the reader is directed to section 5.

The resultant category-specific saving rates are defined as residuals $\sigma_t^{s,i} = \eta_s^i - \beta_t^{s,i}$, and therefore are time-variable. The aggregate saving rate equals the sum of category-specific saving rates and a special, minimum savings category: $\sigma_t^i = \sum_{s=0}^{S}(\sigma_t^{s,i}) + \sigma_{min}^i$. The limit range of possible variability of $\beta_t^{s,i}$ can be interpreted as a maximum possible saving rate out of the category s, $\sigma_{max}^{s,i}$. The category-specific saving rates at the same time define the space for demand variability: the larger the value of a saving rate corresponding to constant disposable income, i.e. to $\hat{\Omega}_t^i = 1$, the more can the demand of an individual i grow (section 5).

The sizes of transfers to mental accounts devoted to the accumulation of funds for infrequently bought durable goods and savings are determined by the residual saving rate,

$$\sigma_{sr,t}^{res,i} = 1 - \sum_{s}(\beta_t^s), \tag{4}$$

which is divided into transfers to the aforementioned mental accounts – but actually retained on the current account (i.e. a deposit bearing no interest) – and the transfer to the saving account:

$$tr_t^H = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^H \tag{5}$$

$$tr_t^{veh} = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^{veh} \tag{6}$$

$$tr_t^{dur} = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^{dur} \tag{7}$$

$$tr_t^{SA} = \sigma_{sr,t}^{SA} \cdot \tilde{\Omega}_t^i \cdot \beta^{dur} \tag{8}$$

where $\sigma_{sr,t}^{CA} = \omega^C A \cdot \sigma_{sr,t}^{res,i}$ and $\sigma_{sr,t}^{SA} = (1 - \omega^C A) \cdot \sigma_{sr,t}^{res,i}$.

When the net disposable income of an individual grows or declines, the resulting change of expenditure is affected also by an alteration of a consumer's behaviour. The latter is expressed as a new value of gross expenditure rate (β_t^s) out of net disposable income on a given category of frequently bought goods. Denote the values of β^s corresponding to the gross percentage increase and decrease of the net disposable income of δ_3 and δ_1 percent as β_{IN} and β_{DCR} respectively. We have that the percentage changes of category-s-related expenditure, y_{IN}^s and y_{DCR}^s, satisfy the following two equations:

$$y_{IN}^s \cdot \tilde{\Omega} = \beta_{IN}^s \cdot \tilde{\Omega} \cdot (1 + \delta_3) - \bar{\beta}^s \cdot \tilde{\Omega}, \tag{9}$$

$$y_{DCR}^s \cdot \tilde{\Omega} = \beta_{DCR}^s \cdot \tilde{\Omega} \cdot (1 + \delta_3) - \bar{\beta}^s \cdot \tilde{\Omega}, \tag{10}$$

where $\bar{\beta}^s$ is the value of β_t^s corresponding to constant net disposable income, $\tilde{\Omega}_t = \tilde{\Omega}_{t-1}$, while $\beta_{IN}^s = \bar{\beta}^s \cdot (1 + \Delta_{IN}^p)$, $\beta_{DCR}^s = \bar{\beta}^s \cdot (1 + \Delta_{DCR}^p)$, and the superscript p indicates that the changes of the β variables are taken to be the percentage changes (or 'proportional').

Consumers are reluctant to spend using savings (Thaler 1990). They are most likely to use them only if they have to, e.g. for the purchases of expensive durable goods. Thus, a continuous decision rule would be inappropriate for modelling such decisions. This, however, creates problems for the estimation or modelling of heterogeneous individuals using analytical tools. On the other hand, algorithmic methods can easily represent such behaviour. In the presented framework, only retired consumers use savings both for the purchases of nondurable and frequently bought durable goods, but all agents can spend them on infrequently bought durables.

4. Consumer types

4.1. Expansionary type

An expansionary consumer behaviour is defined as a spending pattern characterised by increases in expenditure, y^s_{IN}, surpassing the growth of disposable income $\tilde{\Omega}$, and decreases in spending, y^s_{DCR}, that are smaller in absolute value than the decline in disposable income. The second case can be interpreted as consumption habits behaviour when faced with a decrease of income. As for the growth of spending, this type of behaviour can be viewed as a demonstration of impatience or susceptibility to marketing.

Using the notation from the previous section, the following conditions must hold for an expansionary consumer type. First, the growth of expenditure caused by the combined amount of an increase of disposable income and the resultant additional (beyond one-to-one) increase of spending due to the change of β^s_t, cannot surpass the possible amount. This maximal amount is related to the size of possible variability within the mental account. I.e., relative to a situation without any income change, for which $\beta^s_t = \bar{\beta}^s$, the possible space for variability of the division-of-funds variable β^s_t is given by the 'average' saving rate $\bar{\sigma}^s_{sr} = \eta_s - \bar{\beta}^s$. We have

$$y^s_{IN} \leq \delta_3 + (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{11}$$

$$y^s_{IN} \geq \delta_3. \tag{12}$$

Similarly, for the change of expenditure after an income decrease – keeping in mind that an expansionary type tries to offset the effect of a decrease of net disposable income – we have

$$y^s_{DCR} \leq \delta_1 + (1 + \delta_1) \cdot \bar{\sigma}^s_{sr}, \tag{13}$$

$$y^s_{DCR} \geq \delta_1. \tag{14}$$

Thus, changes of expenditure can be described as

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 + \omega^{12}_s \cdot (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{15}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (1 + \delta_1) \cdot \bar{\sigma}^s_{sr}, \tag{16}$$

where $\omega^{11}_s, \omega^{12}_s, \omega^{21}_s, \omega^{22}_s \in [0, 1]$ and are such that all of the above conditions hold. In this paper it is assumed that $\forall_s\ \omega^{11}_s + \omega^{12}_s = 1$ and $\forall_s \omega^{21}_s + \omega^{22}_s = 1$, for all consumer types..

4.2. Volatile type

Volatile type of behaviour is characterised by both increases and decreases in expenditure surpassing (in absolute value) the growth and fall of disposable income income, respectively. Thus, it is a combination of prudent behaviour for the domain of disposable income income decreases and susceptibility to other stimuli such as marketing as well as impatience. Intuitively, volatile consumers ought to have the largest space for demand fluctuations of all other types. Note, however, that this also implies the largest maximum and average saving rates, which are determined by the differences between the sizes of accounts and the lower bounds of the accounts and the expenditure rates plus the saving account.

We have

$$y^s_{IN} \leq \delta_3 + (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{17}$$

$$y^s_{IN} \geq \delta_3. \tag{18}$$

The change of expenditure after an income decrease for a volatile type is limited by the lower bound of the account. We have $\eta_s - \beta_{s2} = \sigma^{s,max}_{sr}$, but the reference point is $\bar{\beta}^s$, associated with $\bar{\sigma}^s_{sr}$. Thus,

$$y^s_{DCR} \leq \delta_1, \tag{19}$$

$$y^s_{DCR} \geq \delta_1 - (1 + \delta_1) \cdot (\sigma^{s,max}_{sr} - \bar{\sigma}^s_{sr}). \tag{20}$$

Therefore, changes of expenditure for the volatile type can be described as

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 + \omega^{12}_s \cdot (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{21}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (\delta_1 - (1 + \delta_1) \cdot (\sigma^{s,max}_{sr} - \bar{\sigma}^s_{sr})), \tag{22}$$

4.3. Extreme saver type

An extreme saver type decreases expenditure after both a disposable income fall and increase. This means that a person exhibiting an extreme-saver consumer behaviour is one that increases savings when faced with any income change, relative to a situation in which his/her disposable income were constant.

We have

$$y^s_{IN} \geq \delta_3 - (1 + \delta_3) \cdot \sigma^{s,max}_{sr}, \tag{23}$$

$$y^s_{IN} \leq \delta_3. \tag{24}$$

The change of expenditure after an income decrease for a extreme saver type is limited by the lower bound of the account. We have $\eta_s - \beta_{s2} = \sigma^{s,max}_{sr}$, but the reference point is $\bar{\beta}^s$, associated with $\bar{\sigma}^s_{sr}$. Thus,

$$y^s_{DCR} \leq \delta_1, \tag{25}$$

$$y^s_{DCR} \geq \delta_1 - (1 + \delta_1) \cdot \sigma^{s,max}_{sr}. \tag{26}$$

Therefore, changes of expenditure for the extreme saver type can be described as

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 - \omega^{12}_s \cdot (1 + \delta_3) \cdot \sigma^{s,max}_{sr}, \tag{27}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (\delta_1 - (1 + \delta_1) \cdot \sigma^{s,max}_{sr}), \tag{28}$$

4.4. Prudent type

A prudent consumer type decreases expenditure more than proportionally after a disposable income fall. When faced with a growth of funds, this type increases spending by a factor smaller than one.

Therefore, a prudent consumer is a person who exhibits a saver's spending pattern given disposable income increases and precautionary savings behaviour when faced with decreases of $\tilde{\Omega}^i_t$.

$$y^s_{IN} \geq \delta_3 \cdot \bar{\beta}^s, \tag{29}$$

$$y^s_{IN} \leq \delta_3. \tag{30}$$

Just like for the for a extreme saver type, the change of expenditure after an income decrease for the prudent type is limited by the lower bound of the account. We have $\eta_s - \beta_{s2} = \sigma_{sr}^{s,max}$, but the reference point is $\bar{\beta}^s$, associated with $\bar{\sigma}_{sr}^s$. Thus,

$$y_{DCR}^s \leq \delta_1, \tag{31}$$

$$y_{DCR}^s \geq \delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}. \tag{32}$$

Therefore, changes of expenditure for the prudent type can be described as

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} \delta_3 \cdot \bar{\beta}^s, \tag{33}$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (\delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}), \tag{34}$$

but in this paper an alternative formulation for the increases was adopted, for the reason that the weights ω are kept constant across consumer types for the sake of comparison, but using the above rule yields very small changes of expenditure for income increases. Thus, the following alternative was adopted:

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12}(\delta_3 - \delta_3 \cdot \bar{\beta}^s). \tag{35}$$

The latter representation has the additional advantage that the size of $(\delta_3 - \delta_3 \cdot \bar{\beta}^s)$ can be interpreted as the degree of prudence.

4.5. Reversed type

A reversed consumer type decreases expenditure in response to a disposable income growth, but increases it when faced with the decline of disposable income, at least for small absolute values of changes of $\tilde{\Omega}_t^i$.

We have

$$y_{IN}^s \geq \delta_3 - (1 + \delta_3) \cdot \sigma_{sr}^{s,max}, \tag{36}$$

$$y_{IN}^s \leq \delta_3, \tag{37}$$

while for the decreases of income:

$$y_{DCR}^s \geq \delta_1, \tag{38}$$

$$y^s_{DCR} \leq \delta_1 + (1+\delta_1) \cdot \sigma^{s,max}_{sr}. \tag{39}$$

Thus, following the same reasoning as for the prudent type in the case of income increases:

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 + \omega^{12}_s (1+\delta_3) \cdot (\sigma^{s,max}_{sr} - \bar{\sigma}^s_{sr}), \tag{40}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (1+\delta_1) \cdot \bar{\sigma}^s_{sr}. \tag{41}$$

4.6. Consumption-habits type

A person displaying consumption habits is reluctant to decrease or increase expenditure by factors more than one. In other words, such a consumer will spend less than the excess income, but also will counteract the fall in income.

For increases of net disposable income, a consumption-habits type is characterised by:

$$y^s_{IN} \geq \delta_3 - \bar{\beta}^s \cdot \delta_3. \tag{42}$$

$$y^s_{IN} \leq \delta_3. \tag{43}$$

For the decreases of net disposable income we have:

$$y^s_{DCR} \geq \delta_1, \tag{44}$$

$$y^s_{DCR} \leq \delta_1 + (1+\delta_1) \cdot \sigma^{s,max}_{sr}. \tag{45}$$

Thus,

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 + \omega^{12}_s (\delta_3 - \delta_3 \cdot \bar{\beta}^s), \tag{46}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (1+\delta_1) \cdot \bar{\sigma}^s_{sr}. \tag{47}$$

5. An agent-based, multimarket, overlapping generations model with income heterogeneity of categorisation-enhanced mental-accounting consumption in the life-cycle

All agents in the model are assumed to display expansionary consumer behaviour, defined as a spending pattern characterised by increases in expenditure, y^s_{IN}, surpassing the growth of disposable income $\tilde{\Omega}$, and decreases in spending, y^s_{DCR}, that are smaller in absolute value than the decline in disposable income. The second case can be interpreted as consumption habits behaviour when faced with a decrease of income. As for the growth of spending, this type of behaviour can be viewed as a demonstration of impatience or susceptibility to marketing.

Using the notation from the previous section, the following conditions must hold for an expansionary consumer type. First, the growth of expenditure caused by the combined amount of an increase of disposable income and the resultant additional (beyond one-to-one) increase of spending due to the change of β^s_t, cannot surpass the possible amount. This maximal amount is related to the size of possible variability within the mental account. I.e., relative to a situation without any income change, for which $\beta^s_t = \bar{\beta}^s$, the possible space for variability of the division-of-funds variable β^s_t is given by the 'average' saving rate $\bar{\sigma}^s_{sr} = \eta_s - \bar{\beta}^s$. We have

$$y^s_{IN} \leq \delta_3 + (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{48}$$

$$y^s_{IN} \geq \delta_3. \tag{49}$$

Similarly, for the change of expenditure after an income decrease – keeping in mind that an expansionary type tries to offset the effect of a decrease of net disposable income – we have

$$y^s_{DCR} \leq \delta_1 + (1 + \delta_1) \cdot \bar{\sigma}^s_{sr}, \tag{50}$$

$$y^s_{DCR} \geq \delta_1. \tag{51}$$

Thus, changes of expenditure can be described as

$$y^s_{IN} = \omega^{11}_s \cdot \delta_3 + \omega^{12}_s \cdot (1 + \delta_3) \cdot \bar{\sigma}^s_{sr}, \tag{52}$$

$$y^s_{DCR} = \omega^{21}_s \cdot \delta_1 + \omega^{22}_s \cdot (1 + \delta_1) \cdot \bar{\sigma}^s_{sr}, \tag{53}$$

where $\omega^{11}_s, \omega^{12}_s, \omega^{21}_s, \omega^{22}_s \in [0, 1]$ and are such that all of the above conditions hold. In this paper it is assumed that $\forall_s\ \omega^{11}_s + \omega^{12}_s = 1$ and $\omega^{21}_s + \omega^{22}_s = 1$.

The modelled consumer is assumed to spend on frequently bought goods using the behavioural decision rule described in section 3. As for houses (H), flats (F), vehicles (veh) and other durable goods (dur), the consumer follows the behavioural rules described in procedure 1.

Although in this section only one consumer's working-life-cycle is modelled, the agent has assigned the superscripts i, a to keep the notation consistent with the model featuring many consumers. The index i denotes the family that a given agent i belongs to as well as places the consumer within the income distribution (see subsubsection 5.1), while the superscript a is equal to the agent's age. Moreover, it is assumed that the consumer tries to use only the funds on the housing good account, $d^{CA,H,i,a}_t$ to buy a flat or a house. Only if this amount is insufficient the decision-maker takes into account spending money accumulated on the savings account. If these combined funds are still too small to buy a flat or a house, the agent considers taking out a multiperiod loan. The consumer is subject to credit constraints: the first installment cannot be higher than the fraction $crc = 0.3$ of the agent's net disposable income.

Possible states of a decision-maker's funds relative to the prices of flats and houses are can be described by establishing whether the following six equations are true or not. Denote the index of a given agent by i, the agent's age by a, mental account devoted to the housing good (a house or a flat) by $d^{CA,H,i,a}_t$, net income by $NI^{i,a}_t$, saving account by $d^{SA,i,a}_t$ and the maximum admissible amount of new debt (only if there is no outstanding debt) by $HD^{adm,i,a}_t = (r + \frac{1}{CD})^{-1} \cdot \tilde{\Omega}^{i,a}_t \cdot crc$. The prices of a house and a flat are equal to P^H_t and P^F_t.

$$d^{CA,H,i,a}_t + NI^{i,a}_t \cdot \sigma^{CA,i,a}_{sr,t} \cdot \beta^H + d^{SA,i,a}_t \cdot (1+r) + HD^{adm,i,a}_t \geq P^F_t, (DRF_1) \tag{54}$$

$$d^{CA,H,i,a}_t + NI^{i,a}_t \cdot \sigma^{CA,i,a}_{sr,t} \cdot \beta^H + d^{SA,i,a}_t \cdot (1 + r) \geq P^F_t, (DRF_2) \tag{55}$$

$$d^{CA,H,i,a}_t + NI^{i,a}_t \cdot \sigma^{CA,i,a}_{sr,t} \cdot \beta^H \geq P^F_t, (DRF_3) \tag{56}$$

$$d^{CA,H,i,a}_t + NI^{i,a}_t \cdot \sigma^{CA,i,a}_{sr,t} \cdot \beta^H + d^{SA,i,a}_t \cdot (1+r) + HD^{adm,i,a}_t \geq P^H_t (DRH_1) \tag{57}$$

$$d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H + d_t^{SA,i,a} \cdot (1+r) \geq P_t^H (DRH_2) \tag{58}$$

$$d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H \geq P_t^H (DRH_3) \tag{59}$$

A consumer spends on frequently bought nondurable and durable goods, and then considers whether to buy a house or a flat, and whether to spend on a new vehicle (according to its depreciation value and price) and other durable goods (similarly to vehicles, depreciation and price are considered). This hierarchy of needs is the same for the single- and many agents models presented in this paper. Some of the cases are possible only in the model featuring many consumers, because only that version includes inheritance of flats and houses from the oldest cohorts. Given the presence of inheritance, an agent may possess more houses than flats; therefore, the hierarchy of needs concerning the housing is as follows:

1) First priority: obtain a flat.
2) If at least one flat but no house is possessed, attempt to buy a house.
3) If one flat and one house are in a consumer's posession, buy a second flat.
4) If two houses but only one flat are owned by a consumer, buy a second flat
5) If two flats and two houses are possessed by a consumer, do not buy any more flats or houses.

As for infrequently bought durables, the behaviour governing their purchases is described in Procedures 1, 2, 3, 4, 5, 5, 6 where $H_t^{i,a}$ and $F_t^{i,a}$ denote the number of houses and flats in consumer's i, a possession, while $dm_t^{i,a}$ is the indicator of debt maturity, where $dm_t^{i,a} = 0$ means that no debt is held by the consumer at the beginning of a period. Debt are multiperiod, last for 60 periods, and can be taken out only for the purpose of buying a house or a flat. Periods are interpreted as quarters, which translates into mortgages having 15-years maturity. The cases of the decision-maker having two or more flats but no houses and the reversed situation can occur only in the multi-agent model introduced in section 7, but are included in this section to avoid repetition.

Consumers are reluctant to spend using savings (Thaler 1990). They are most likely to use them only if they have to, e.g. for the purchases of expensive durable goods. Thus, a continuous decision rule would be inappropriate

Procedure 1 A consumer's decision rule for durable goods: part 1

if $F_t^{i,a} = 0 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:

  if $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = True)$:

    $F_{t+1}^{i,a} = 1$

    $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^F + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

    $HD_t^{i,a} = 0$

    $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

    $dm_{t+1} * i, a = 0$

  else if $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False)$:

    $F_{t+1}^{i,a} = 1$

    $d_{t+1}^{CA,H,i,a} = 0$

    $HD_t^{i,a} = 0$

    $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a} + d_t^{CA,H,i,a} - P_t^F + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

    $dm_{t+1} * i, a = 0$

  else if $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False)$:

    $F_{t+1}^{i,a} = 1$

    $d_{t+1}^{CA,H,i,a} = 0$

    $HD_t^{i,a} = P_t^F - d_t^{CA,H,i,a} - NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$

    $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

    $dm_{t+1} * i, a = 1$

  else

    $F_{t+1}^{i,a} = 0$

    $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

    $HD_t^{i,a} = 0$

    $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

    $dm_{t+1} * i, a = 0$

  $H_{t+1}^{i,a} = 0$

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:

  ...

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:

  ...

else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:

  ...

else if $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$:

  ...

else

  ...

Procedure 2 A consumer's decision rule for durable goods: part 2

if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$

...

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$

if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True):$

$H_{t+1}^{i,a} = 1$

$d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^H + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 0$

else if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False):$

$H_{t+1}^{i,a} = 1$

$d_{t+1}^{CA,H,i,a} = 0$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a} + d_t^{CA,H,i,a} - P_t^H + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$dm_{t+1} * i, a = 0$

else if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False):$

$H_{t+1}^{i,a} = 1$

$d_{t+1}^{CA,H,i,a} = 0$

$HD_t^{i,a} = P_t^H - d_t^{CA,H,i,a} - NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$

$d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 1$

else

$H_{t+1}^{i,a} = 0$

$d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 0$

$F_{t+1}^{i,a} = 1$

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$

...

else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$

...

else if $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2):$

...

else

...

Procedure 3 A consumer's decision rule for durable goods: part 3

if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$
    ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$
    ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$
    if $(DRF_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True):$
        $F_{t+1}^{i,a} = 2$
        $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^F + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$
        $HD_t^{i,a} = 0$
        $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$
        $dm_{t+1} * i, a = 0$
    else if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False):$
        $F_{t+1}^{i,a} = 2$
        $d_{t+1}^{CA,H,i,a} = 0$
        $HD_t^{i,a} = 0$
        $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a} + d_t^{CA,H,i,a} - P_t^F + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$
        $dm_{t+1} * i, a = 0$
    else if $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False):$
        $F_{t+1}^{i,a} = 2$
        $d_{t+1}^{CA,H,i,a} = 0$
        $HD_t^{i,a} = P_t^F - d_t^{CA,H,i,a} - NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$
        $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$
        $dm_{t+1} * i, a = 1$
    else
        $F_{t+1}^{i,a} = 1$
        $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$
        $HD_t^{i,a} = 0$
        $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$
        $dm_{t+1} * i, a = 0$
    $H_{t+1}^{i,a} = 1$
else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$
    ...
else if $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2):$
    ...
else
    ...

Procedure 4 A consumer's decision rule for durable goods: part 4

if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$

...

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0:$

...

else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$

...

else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0:$

if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True):$

$H_{t+1}^{i,a} = 2$

$d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^H + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 0$

else if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False):$

$H_{t+1}^{i,a} = 2$

$d_{t+1}^{CA,H,i,a} = 0$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a} + d_t^{CA,H,i,a} - P_t^H + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$dm_{t+1} * i, a = 0$

else if $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False):$

$H_{t+1}^{i,a} = 2$

$d_{t+1}^{CA,H,i,a} = 0$

$HD_t^{i,a} = P_t^H - d_t^{CA,H,i,a} - NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$

$d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 1$

else

$H_{t+1}^{i,a} = 1$

$d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$

$HD_t^{i,a} = 0$

$d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot NI_t^{i,a}$

$dm_{t+1} * i, a = 0$

$F_{t+1}^{i,a} = 2$

else if $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2):$

...

else

...

Procedure 5 A consumer's decision rule for durable goods: part 5

if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:
  ...
else if $dm_t^{i,a} > 0 \wedge \neg(F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$:
  $F_{t+1}^{i,a} = F_t^{i,a}$
  $H_{t+1}^{i,a} = H_t^{i,a}$
  $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$
  if $dm_t^{i,a} < CD$:
    $dm_{t+1}^{i,a} = dm_t^{i,a} + 1$
  else
    $dm_{t+1}^{i,a} = 0$
else
  ...

Procedure 6 A consumer's decision rule for durable goods: part 5

if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:
  ...
else if $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$:
  ...
else if $dm_t^{i,a} > 0 \wedge \neg(F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$:
  ...
else
  $F_{t+1}^{i,a} = F_t^{i,a}$
  $H_{t+1}^{i,a} = H_t^{i,a}$
  $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$
  if $dm_t^{i,a} < CD$:
    $dm_{t+1}^{i,a} = dm_t^{i,a} + 1$
  else
    $dm_{t+1}^{i,a} = 0$

for modelling such decisions. On the other hand, algorithmic methods can easily represent such behaviour.

In the presented framework, only retired consumers use savings also for the purchases of nondurable and frequently bought durable goods. These expenditures are subject to the same rules as spending from retirement pensions that the old receive, but the expenditure rates are much lower (see appendix A)

5.1. Income distribution and the demographic structure

The aim is to investigate what are the characteristics of the demand of consumers, exhibiting strongly nonfungible mental-accounting behaviour, for various categories of frequently bought goods as well as on the housing market. Agents are modelled in their life-cycle, treated as fourty years in the working age and twenty years during retirement. One period of the simulation of the model is interpreted as a quarter. Agents are divided into three large age groups, further differentiated into quarter-and-year-specific age cohorts: the young, the middle-aged, and the old. The agents are assumed to belong to families so that when the old finally die (after surpassing the maximum age), the new young and the youngest middle-aged (who have been the oldest young in the preceding period) inherit the flats and houses that the old agent who belonged to their family possessed. Thus, the new old do not inherit any housing good within a given family.

All consumers belong to a specific income percentile; most of the young are promoted – and change their income percentile, albeit in a predetermined way – after they become middle-aged. They receive the same places in the income distribution as the new old from their family have held in the previous period. Apart from this mechanism, all wages are assumed to grow at the rate determined by one of the four income processes used in the single-worker model, with the difference that they have been prolonged to 480 periods.

The initial income distribution of working-age agents (i.e., the young and middle-aged of all ages) is constructed from a truncated lognormal distribution with a shape parameter $\sigma_{LN} = 1.25$, location $\mu_{LN} = 0.003$, scale $\eta_{LN} = 0.25$ and a cut-off point of $\bar{w} = 5$. The support of the truncated distribution is rescaled by multiplying each value by 10^6. Define ID^{GRID} as the grid of the income distribution's percentiles. Tables 1 and 2 show the initial allocation of middle-aged and young workers to working-age income distribution quantiles.

Gini coefficient of the truncated lognormal distribution that was used to

Agent's index	Income assignment
$\forall a, i \in \{80, ..., 99\}$	linspace($ID_{90}^{GRID}, ID_{99}^{GRID}$)
$\forall a, i \in \{60, ..., 79\}$	linspace($ID_{70}^{GRID}, ID_{89}^{GRID}$)
$\forall a, i \in \{40, ..., 59\}$	linspace($ID_{50}^{GRID}, ID_{69}^{GRID}$)
$\forall a, i \in \{20, ..., 39\}$	linspace($ID_{30}^{GRID}, ID_{49}^{GRID}$)
$\forall a, i \in \{0, ..., 19\}$	linspace($ID_{0}^{GRID}, ID_{29}^{GRID}$)

Table 1 – The initial allocation of middle-aged workers to income distribution quantiles; 'linspace' denotes an equally spaced grid.

Agent's index	Income assignment
$\forall a, i \in \{80, ..., 99\}$	linspace($ID_{70}^{GRID}, ID_{89}^{GRID}$)
$\forall a, i \in \{60, ..., 79\}$	linspace($ID_{50}^{GRID}, ID_{69}^{GRID}$)
$\forall a, i \in \{40, ..., 59\}$	linspace($ID_{30}^{GRID}, ID_{49}^{GRID}$)
$\forall a, i \in \{0, ..., 39\}$	linspace($ID_{0}^{GRID}, ID_{29}^{GRID}$)

Table 2 – The initial allocation of young workers to income distribution quantiles; 'linspace' denotes an equally spaced grid.

create the income distribution used in the model has the value of 0.526154020[2], while the sample coefficient of the percentile points is 0.5807722935428327. The actual Gini coefficient, resulting from the procedure outlined in table 20, equals 0.6319106979211564. This way of creating an income distribution of workers can be easily extended to cases with more agents, or to models in which income earners are initially allocated to a specific percentile. Initial values of

Agents are assumed to form four-generations families, with twenty years separating each generation, and each generation is equally numerous. The youngest generation enters the model after finishing 80 quarters, becoming young agents of age $a = 0$). This approach is a simplification of the real demographic structure and serves the purpose of a benchmark model.

The old are assumed to spend using their received retirement pensions and savings accumulated in the life-cycle. The benefits received by the old play the role of their income, and their basic spending follows the same rules as for the young and the middle-aged. However, the old also spend some amount of their accumulated savings on nondurable and frequently bought durable goods. The behavioural decision rule is similar to spending using their net income, albeit the sizes of accounts are much smaller, as the old are assumed to attempt not to dissave too quicklt. Moreover, they do not save for a housing good or vehicles anymore, and the transfer of $NI_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a}$

[2]The calculation of this Gini coefficient was performed by numerical integration of the truncated lognormal distribution described above.

increases only the mental account for other durable goods, $d_t^{CA,dur,i,a}$.

It is assumed that the prices of houses, flats, vehicles and other infrequently-bought durable goods are differentiated for each agent type (i). This market segmentation, however, is not ideal, since it does not clear the market.

Income is assumed to grow at the same quarterly rateS of income as in the single-worker model. Two versions of prices' growth are considered: in the first one, prices of houses, flats, vehicles and other infrequently bought durable goods grow at the exogenous rate of

Only the first two generations of a family inherit flats, houses, saving and current accounts of the deceased old of their family. This means that each cohort of the young of age $a = 0$ and middle-aged of age $a = 0$ (an agent's age is counted separately, in quarters from 0 to $\hat{a} = 79$, for each part of life - youth, middle-age and senility) inherits the housing goods of the old who were of age $\hat{a}$ in the previous period.

6. Results

The model is simulated for 480 periods, which amounts to 3 full working-life and 2 adult-life generations.

6.1. Expenses on nondurable and frequently bought durable goods

Unlike the single-consumer model, in a multi-agent framework the expenditures on nondurable and frequently bought durable goods constantly grow except for the second income process, which causes some fluctuations of the growth rate. However, even in this case the aggregate consumption is much smoother than that of an individual. This is because debt-taking and purchases of durable goods are distributed in time and across income distribution percentiles; while some agents reduce their expenses due to the necessity of making debt payments, other expand their expenses according to the behavioural rule and income increases. These individual behaviours level balance to some extent when the aggregate values are calculated. This points to heterogeneous labour market fluctuations and marketing activities of firms as the causes of aggregate fluctuations of demand for these types of goods.

As for the aggregate expenditure on nondurable and frequently bought durable goods, there are very few differences between the two scenarios of housing goods inflation, therefore only the results for the first one are shown.

Figures of growth rates of expenditures on various categories of infrequently bought goods (Fig. 1,2,3,4) demonstrate how strongly aggregate demand can be enhanced within the presented framework. The initial jump of the growth rate of consumption for income versions 1,3 and 4 shows the effect of demand variability and the impact of the behavioural decision rules. The results for the second income process give support to the claim that the presented model of categorisation-enhanced mental-accounting consumer behaviour is characterised by great amplification of income changes: depending on the category, maximal growth rate of expenditures ranges from five to nine percent per quarter, depending on the category of goods, while the quarterly growth rate of income is approximately equal to 0.0099. This augmentation is not an effect of quarterly raises of wages when the oldest young agents become the youngest middle-aged workers, because then a similar increase would be observed for the other three income processes. The emergent property of aggregate category-specific demand has substantial consequences: it indicates that consumer demand has a much more significant role in economic fluctuations than in models in which consumer behaviour can be easily integrated or described via a representative agent.

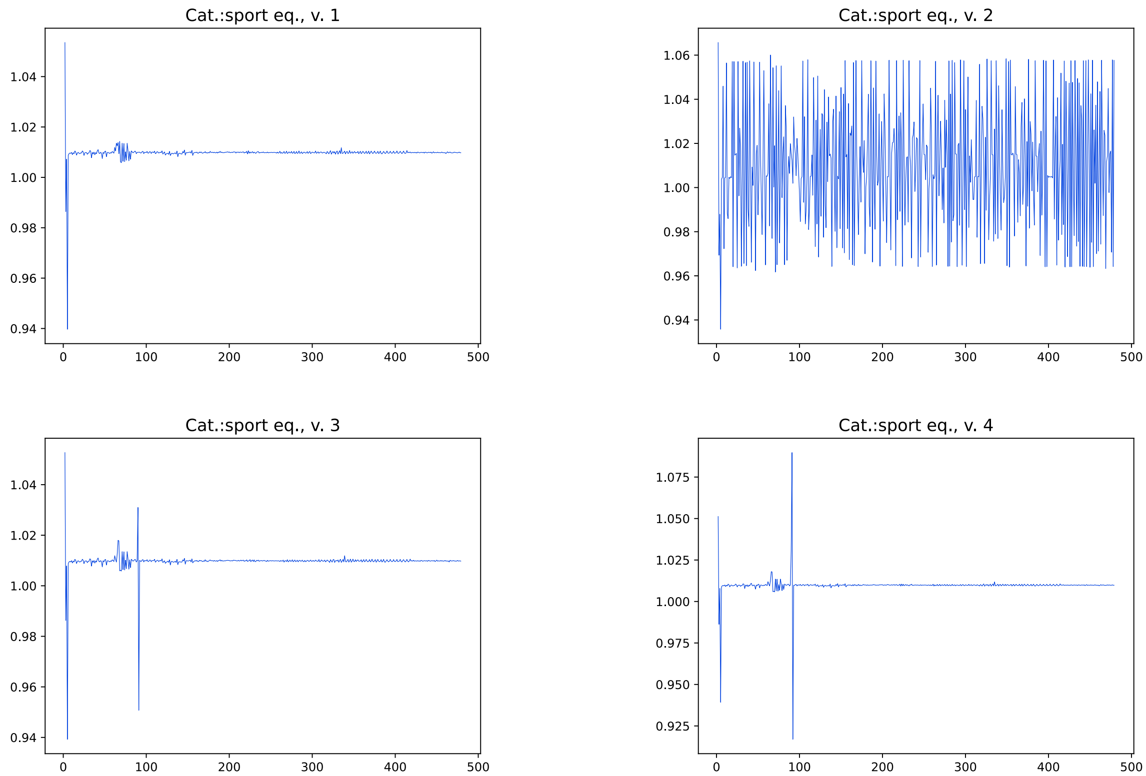

Fig. 1 – Gross percentage change of the aggregate expenditure, category: sports equipment. All agents are assumed to be of the same, expansionary consumer type. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures.

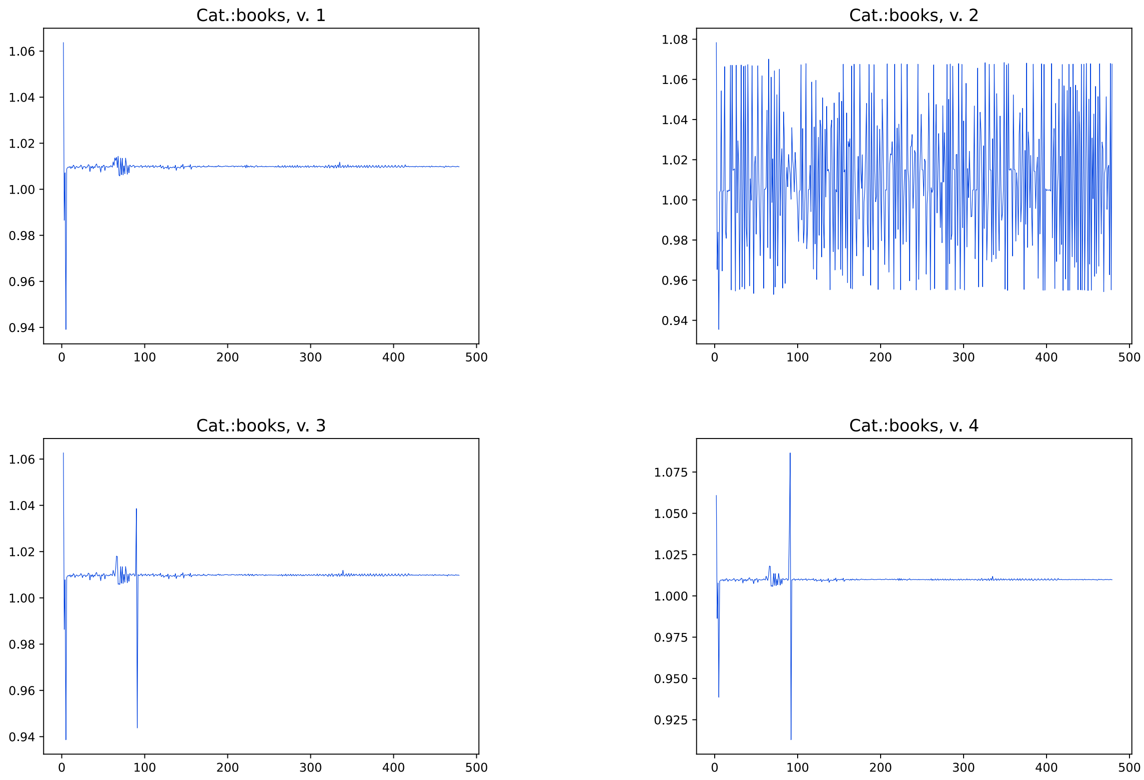

Fig. 2 – Gross percentage change of the aggregate expenditure, category: books. All agents are assumed to be of the same, expansionary consumer type. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures.

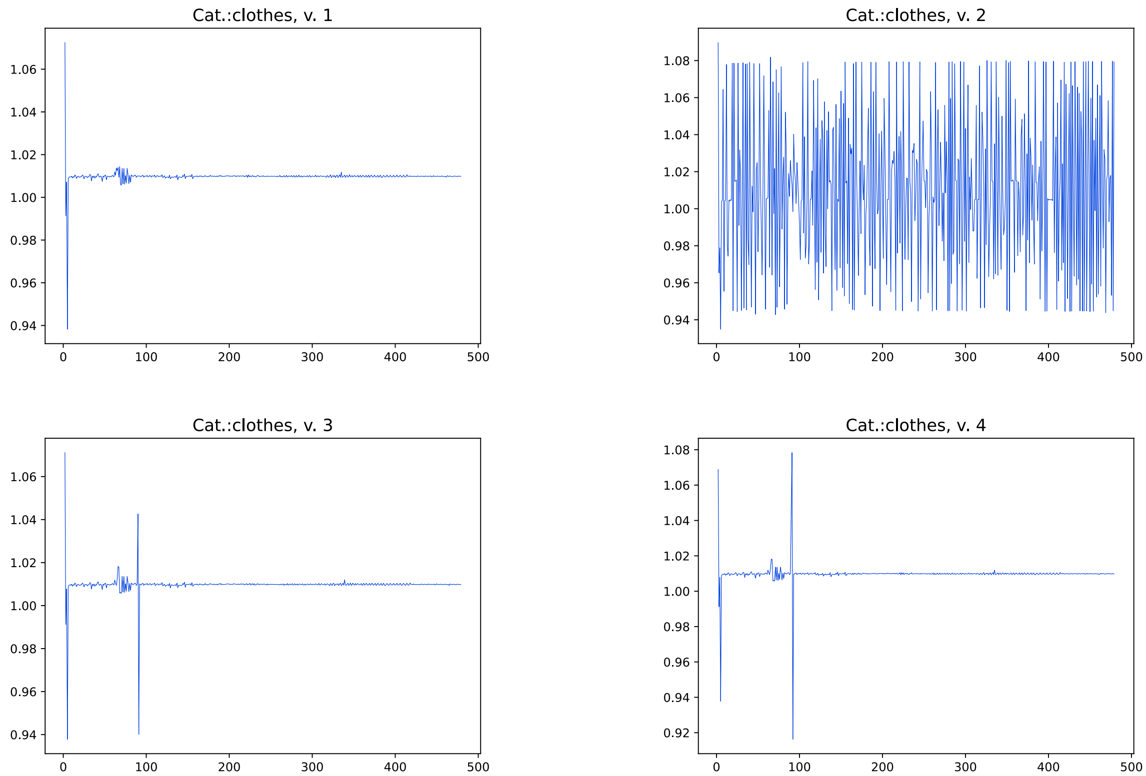

Fig. 3 – Gross percentage change of the aggregate expenditure, category: clothes. All agents are assumed to be of the same, expansionary consumer type. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures.

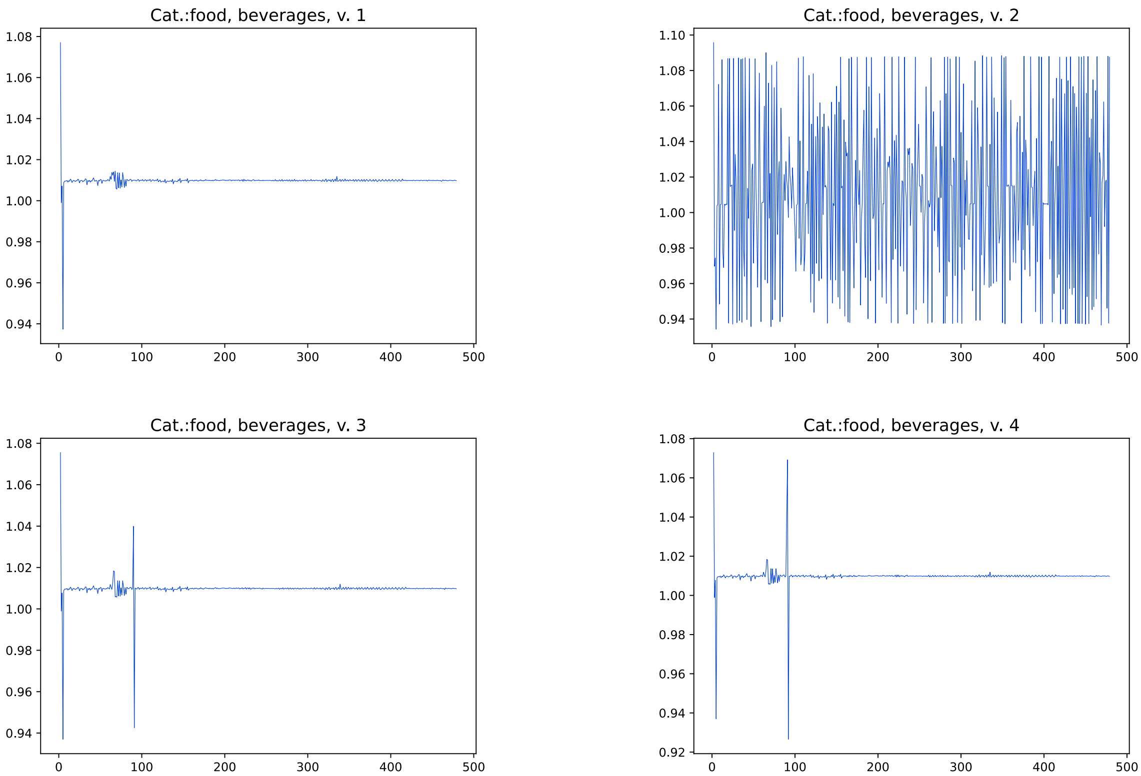

Fig. 4 – Gross percentage change of the aggregate expenditure, category: food and beverages. All agents are assumed to be of the same, expansionary consumer type. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures.

6.2. Expenditure on durable goods: flats

The effect of quicker increase of prices on the growth of the number of flats is clearly visible in figure 5: faster housing goods inflation imply much smaller ownership at the end of simulation, despite the assumption that percentiles from the thirtieth to the ninety-ninth face different prices due to market segmentation.

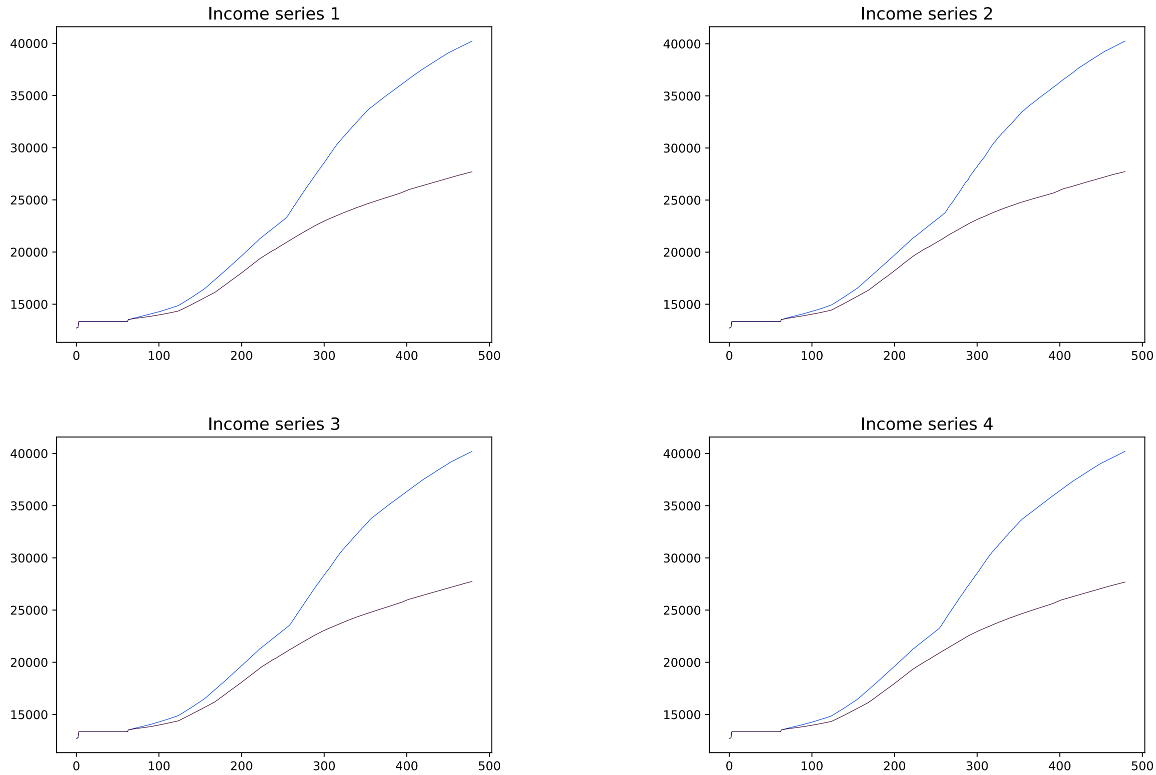

Fig. 5 – The number of owned flats for all periods. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

After an initial jump of over four percent of the number of flats owned by workers, its rate of growth decreases, remaining positive but usually below one percent per quarter (Fig. 6).

Figures 7 and 8 show the gross rates of change of flat ownership among the young (where the rate of 0 indicates that in a given period no flats were owned by the young, while the first period in which some of them buy new flats is taken to have the gross rate of growth equal to 1). It is worth noting

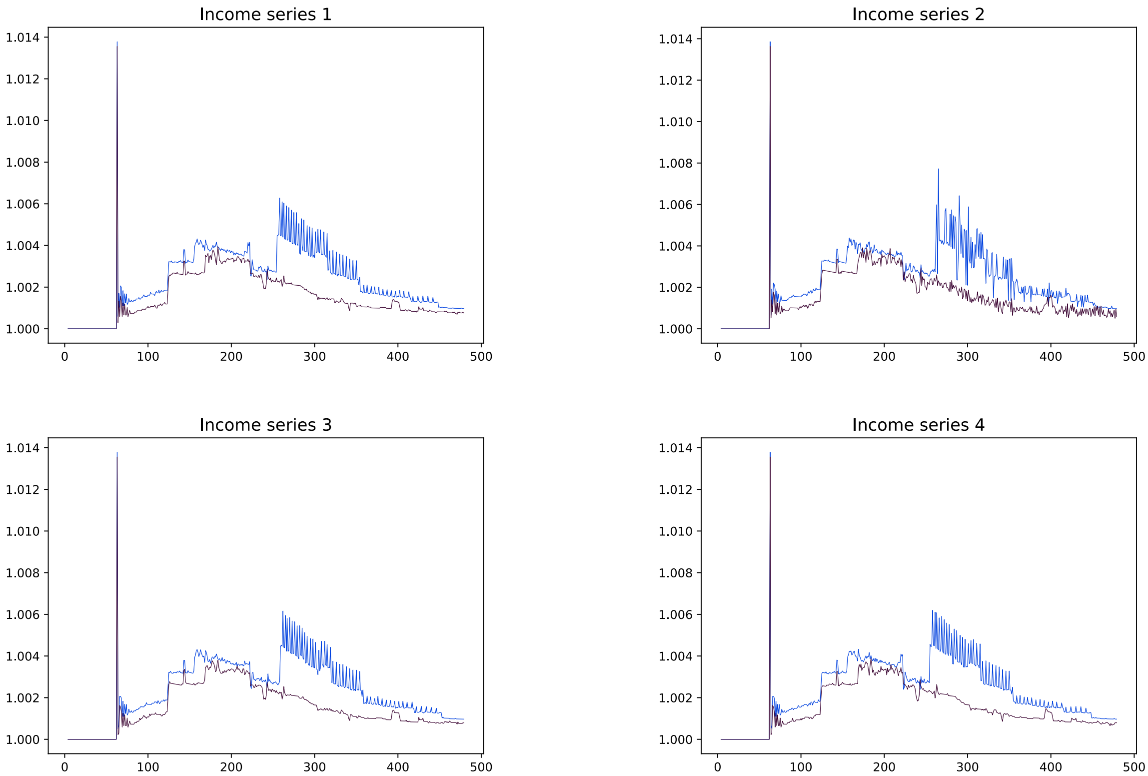

Fig. 6 – Gross percentage change of aggregate ownership, category: flats; periods from 4 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

that the income structure is such that for both price growth scenarios the income of the young is insufficient to buy new flats until period 82, since which young agents with age $a = 0$ start to inherit flats from the deceased old. Thus, the modelled market is too tight for young agents, and they cannot afford a flat or a house.

In the first price growth scenario, this situation changes when the growth of income finally increases the purchasing power of the young enough to allow them to make purchases of flats before they become middle-aged (and, for some families, move up the income distribution). In the second scenario flats' and houses' inflation grows at the same pace as income, and credit constraints remain unchanged relative to agents' net incomes. But it is not just the inheritance of flats and houses that causes the observed increase in the ownership of housing goods by young cohorts: in many cases the inheritance increases a young agent's assets enough to make him/her eligible for credit. Debt of the young (after a few ephemeral quarters featuring debt-taking by the young; see fig. 9 and 10) appears and grows since $t = 155$ for versions $1, 3, 4$ of the income process or $t = 154$ for variant 2, for the first version of price growth, and, for quicker price growth variant, since $t = 168$ for versions $1, 2, 3$ of the income process or $t = 169$ for variant 3. New young agents inherit also one third of the current and saving accounts of the deceased old of their family, which increases the funds of some of these agents enough to be eligible for a credit. However, the period of the fastest increase of the number of flats owned by young cohorts occurs before peaks in new debt growth rate and is the same for all versions of simulations: it is the quarter $t = 83$.

A few first periods (fig. 11) are characterised by a surge in the number of flats owned by middle-aged agents, mostly due to the initial ownership needs that could be satisfied with the help of debt, and because of the fact that initial prices were differentiated for 'market segments', i.e. income distribution percentiles (from the thirtieth).

A closer investigation of the subsequent periods shows that the model generates long- and short-term stochastic cyclicality and outlier, shock-like behaviour (fig. 12). This is even more evident on figure 13, which shows the gross rates of change of the number of flats owned by the old, who do not buy new housing goods, only hold what they have bought or inherited before retirement.

Contrary to young cohorts, the group of middle-aged agents take out debt since the second period. In the second part of simulations the growth rates of the nominal values of credit display sizeable cyclicality – fig. 14. Debts of old agents are non-zero only if the loans they have taken out in the past have not been fully paid before their retirement. It can be seen on figure 15

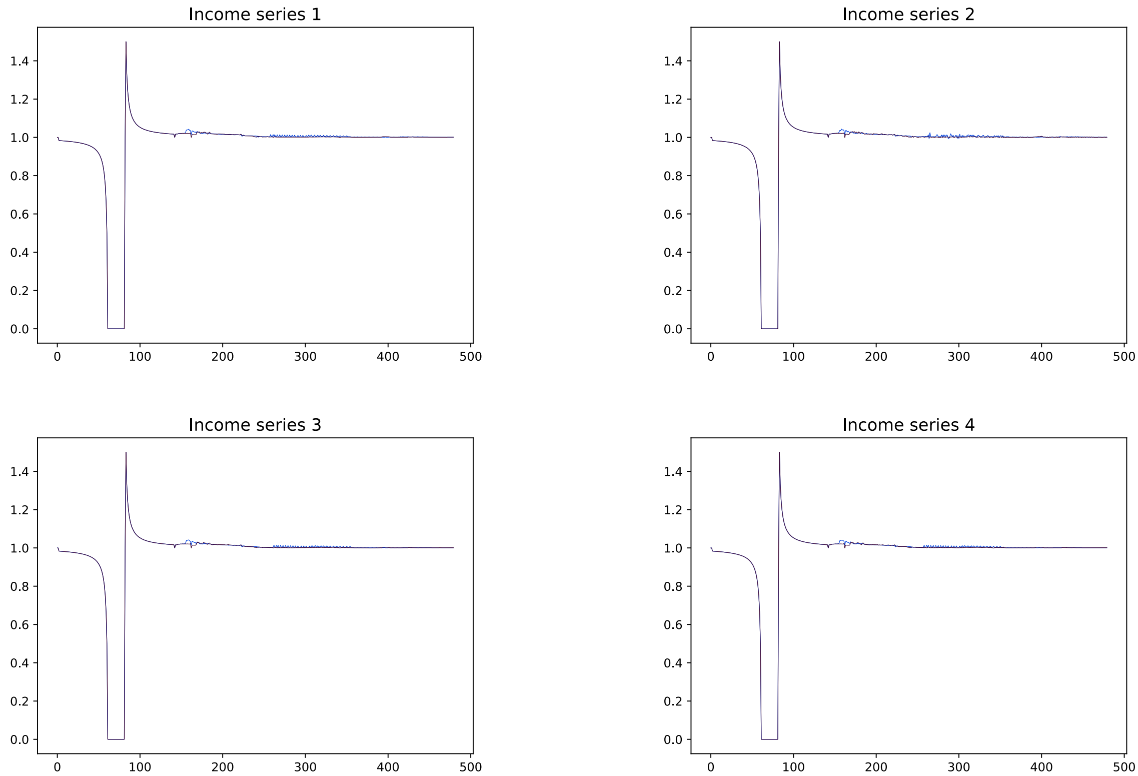

Fig. 7 – The level of change of the aggregate ownership of all age cohorts of the young gross percentage change, category: flats; periods from 1 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

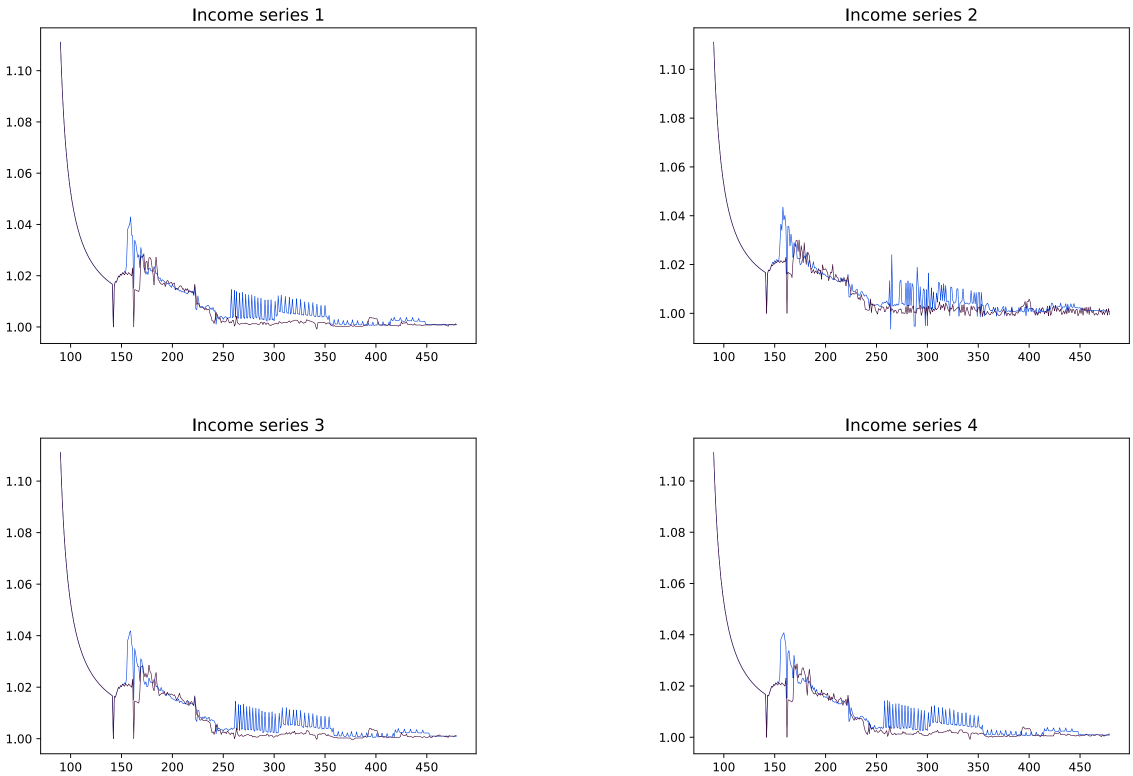

Fig. 8 – Gross percentage change of the aggregate ownership of all age cohorts of the young gross percentage change, category: flats; periods from 90 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

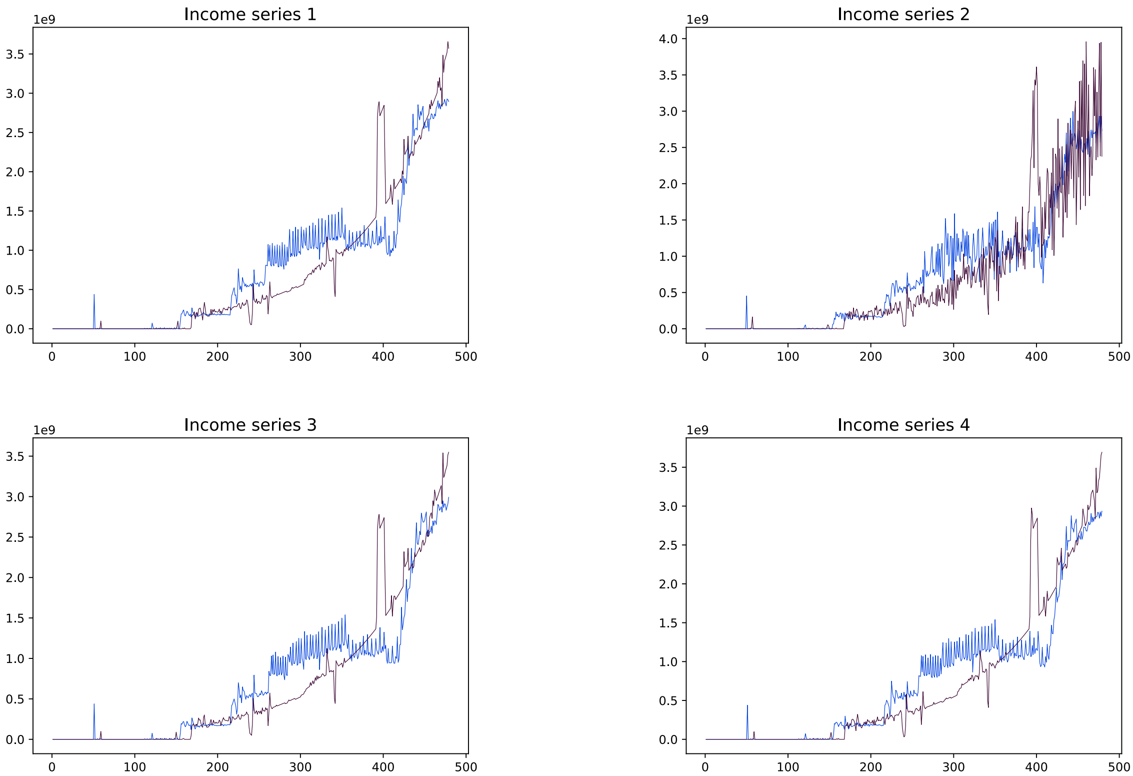

Fig. 9 – The stock of debt held by all age cohorts of the young; periods from 1 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

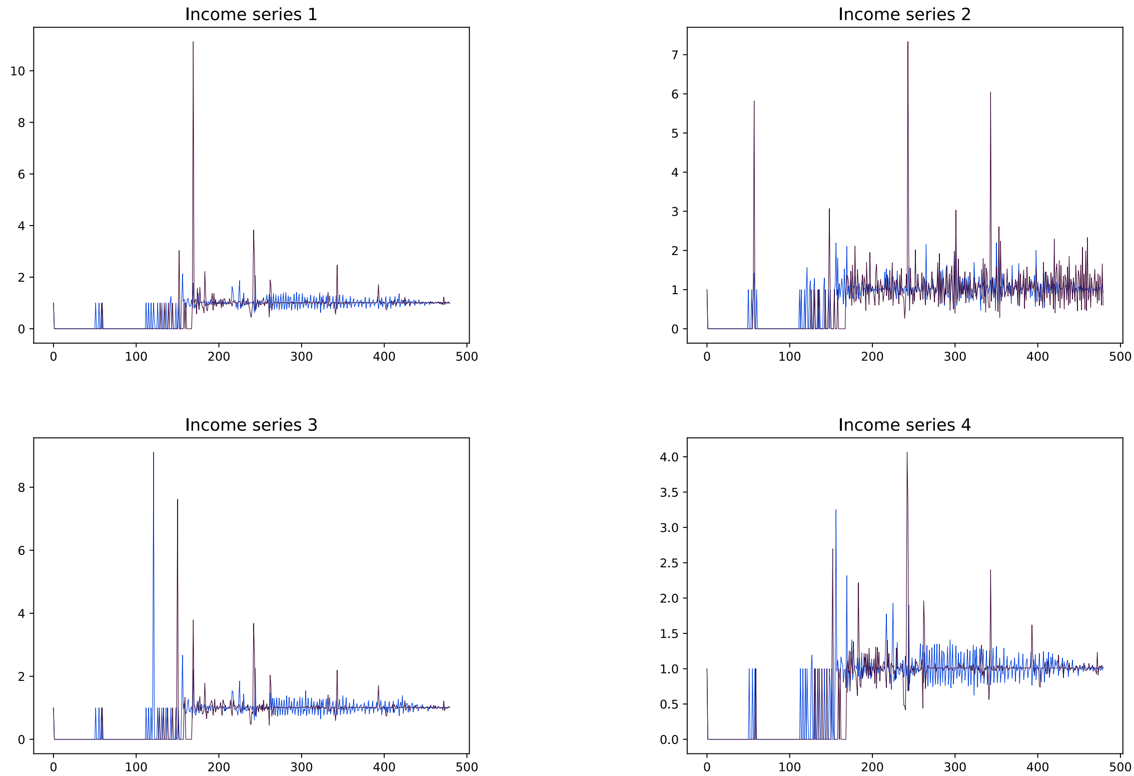

Fig. 10 – Gross percentage change of the stock of debt held by all age cohorts of the young; periods from 1 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

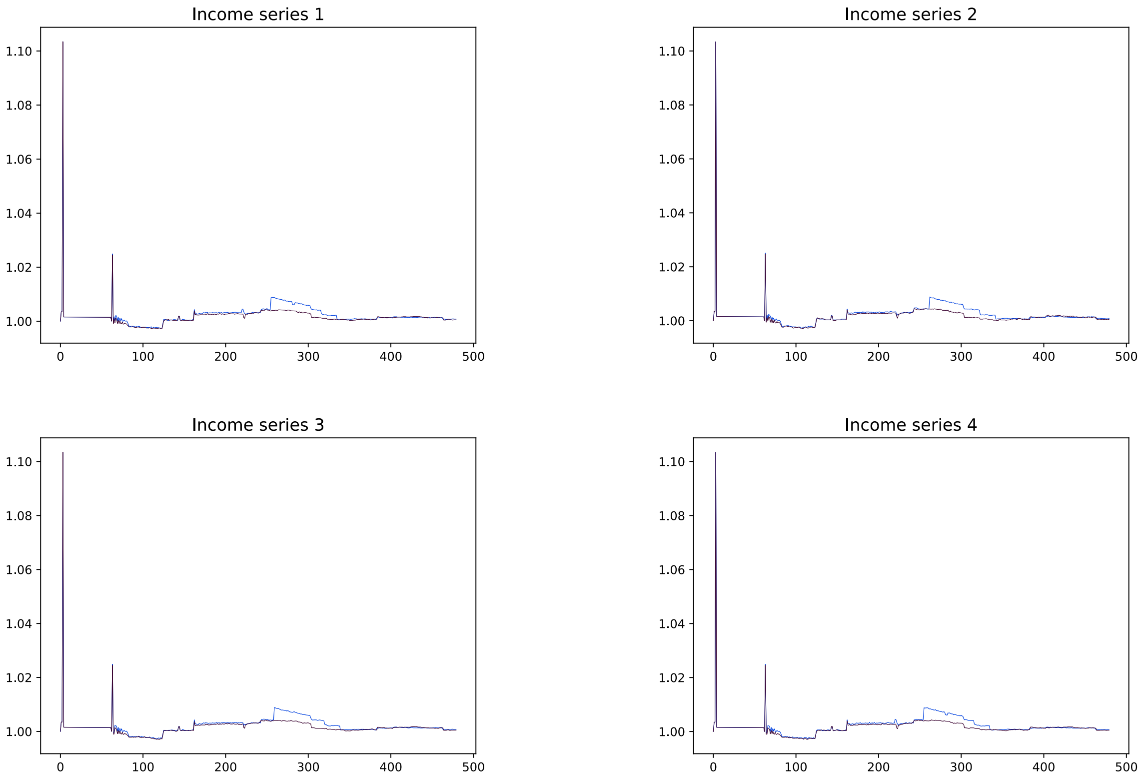

Fig. 11 – Gross percentage change of the aggregate ownership of all age cohorts of the middle-aged gross percentage change, category: flats; all periods are shown. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

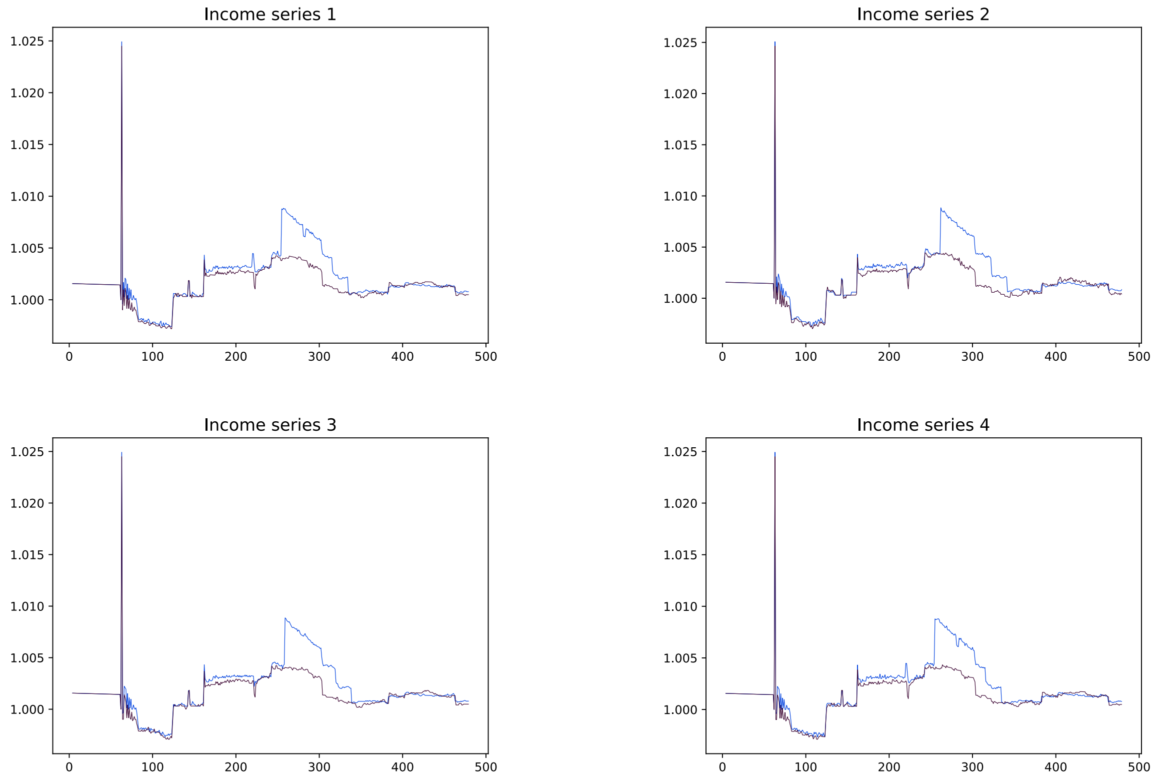

Fig. 12 – Gross percentage change of the aggregate ownership of all age cohorts of the middle-aged gross percentage change, category: flats; periods from 4 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

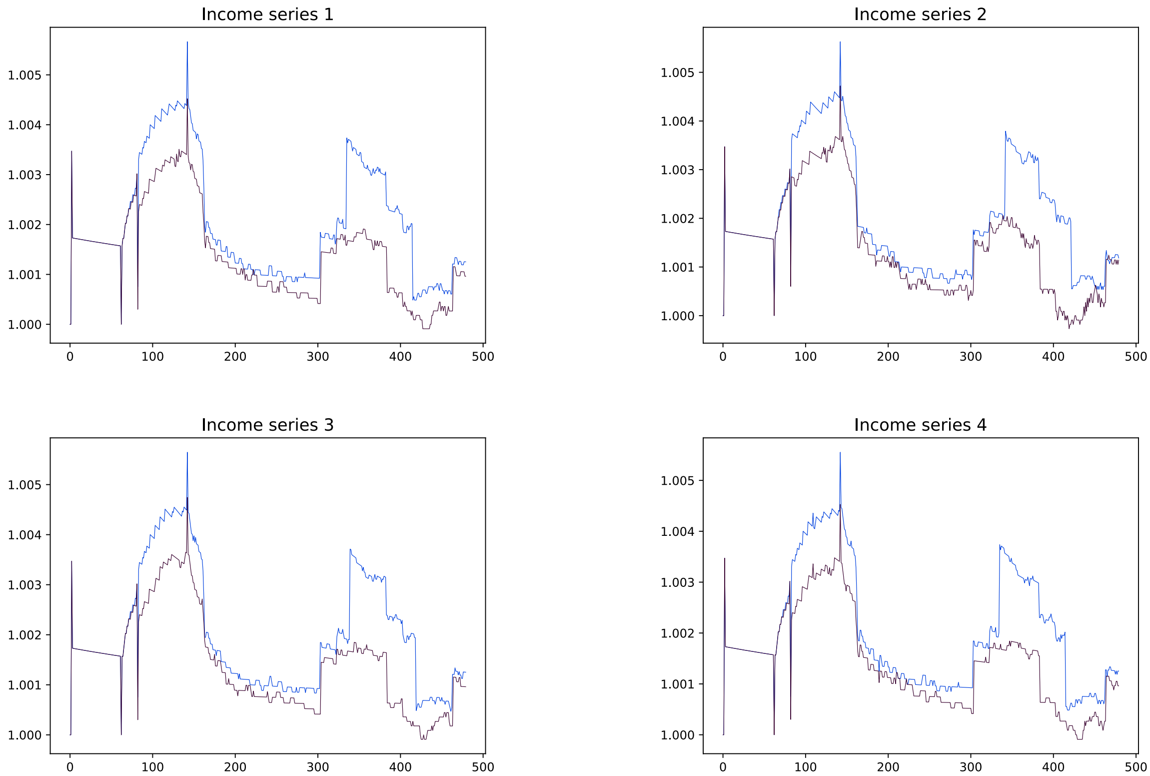

Fig. 13 – Gross percentage change of the aggregate ownership of all age cohorts of the old gross percentage change, category: flats; periods from 1 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

that such a situation occurs rarely and debts carried into the retirement age are usually almost repaid.

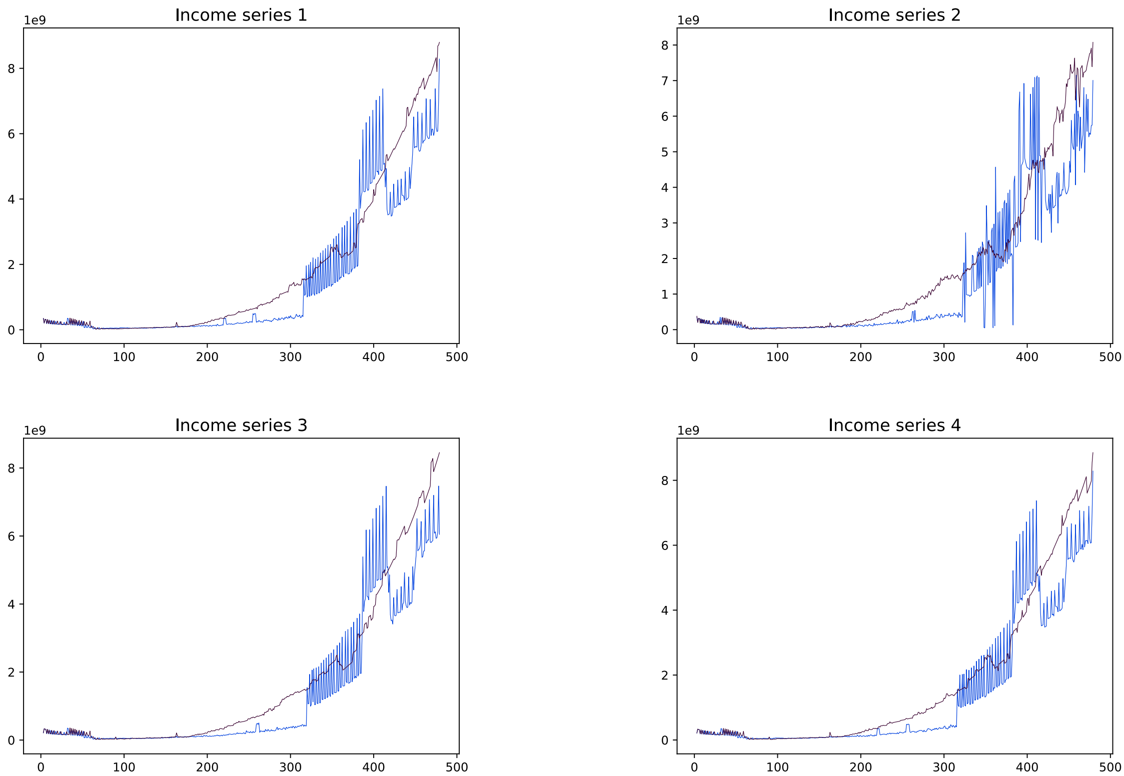

Fig. 14 – The stock of debt held by all age cohorts of the middle-aged; periods from 3 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one – the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

6.3. Expenditure on durable goods: houses

The number of all houses owned by agents in the model increases steadily, but at a lower pace than the number of flats; moreover, in the first 100 quarters, after the initial surge, the quantity of houses in possession of all middle-aged cohorts decreases, which indicates both that the remaining agents did not satisfy the conditions for obtaining new debt and that the savings of the families have not been accumulated sufficiently (through saving and inheritance) to allow taking out credit for the housing good (figures 16, 17, 18).

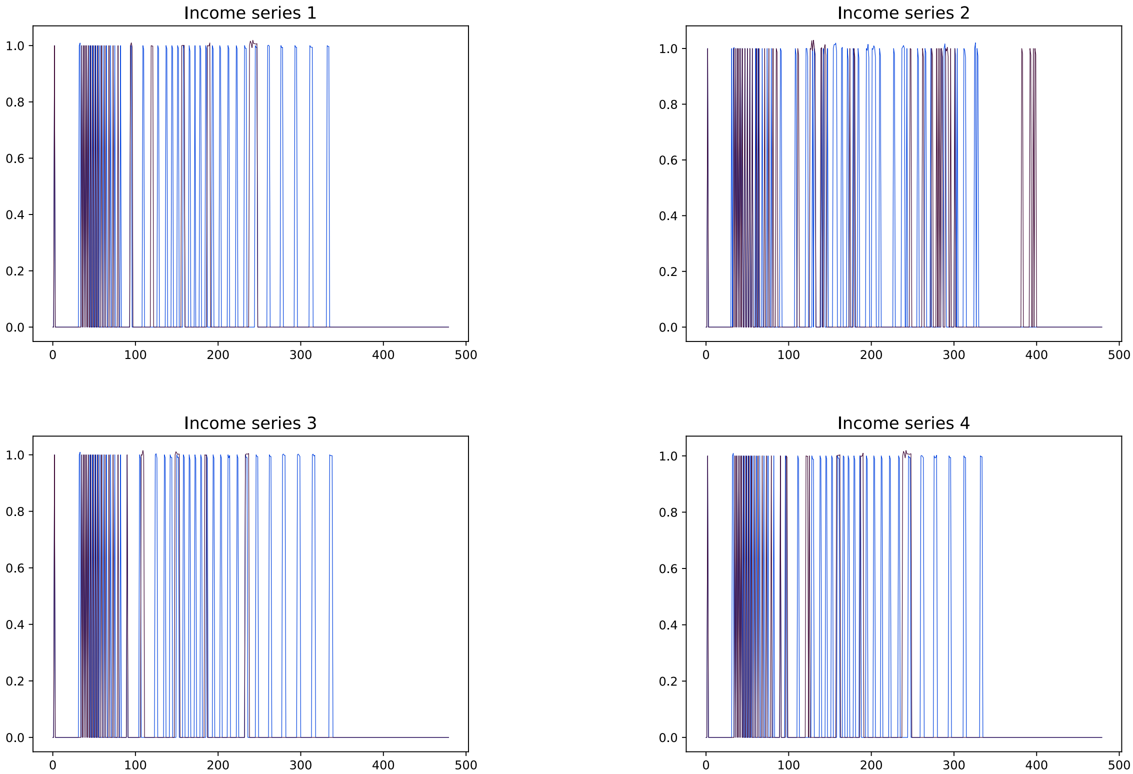

Fig. 15 – Gross percentage change of debt of all age cohorts of the old, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second. A single vertical line indicates that the debts carried into retirement age were only one period short of repayment.

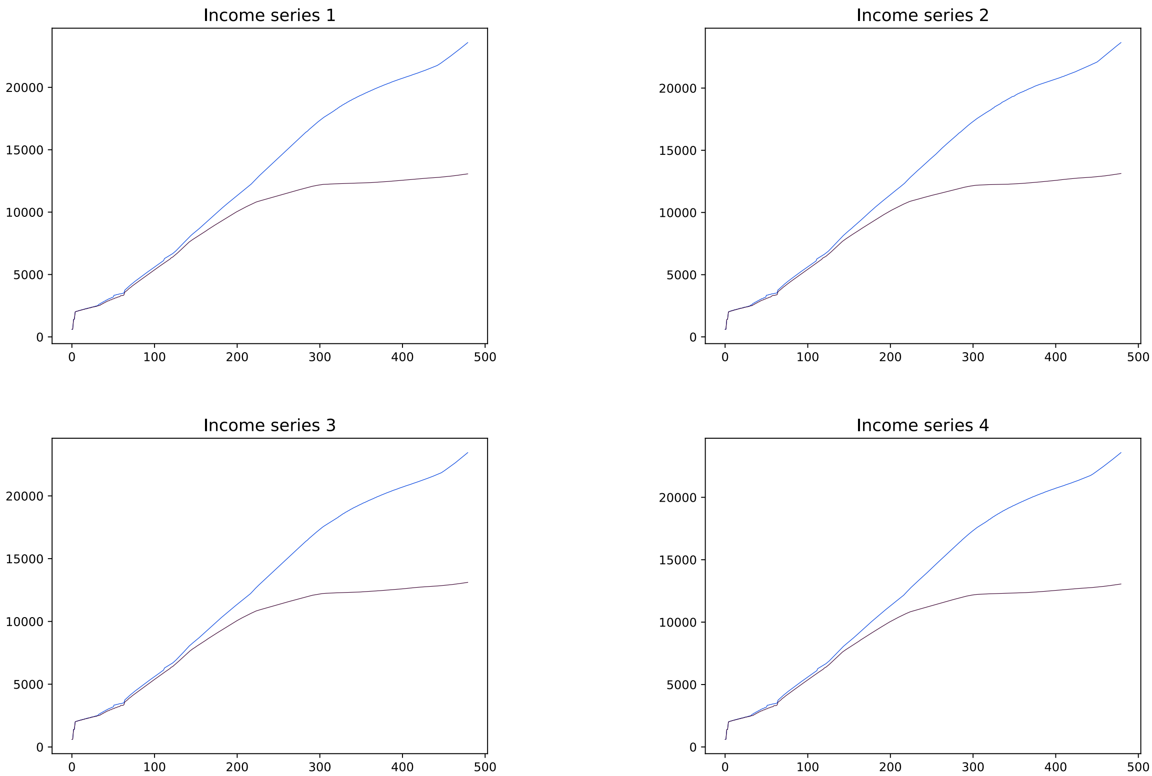

Fig. 16 – Aggregate ownership, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

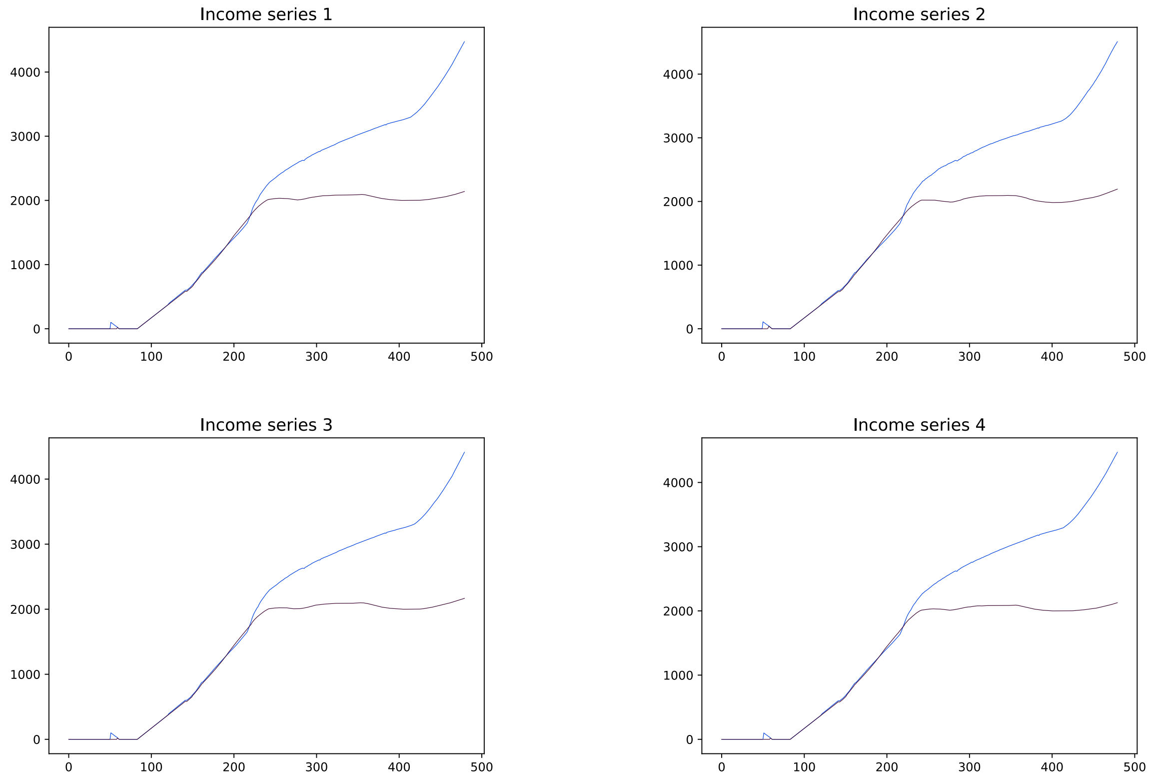

Fig. 17 – Aggregate ownership of all age cohorts of the young, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

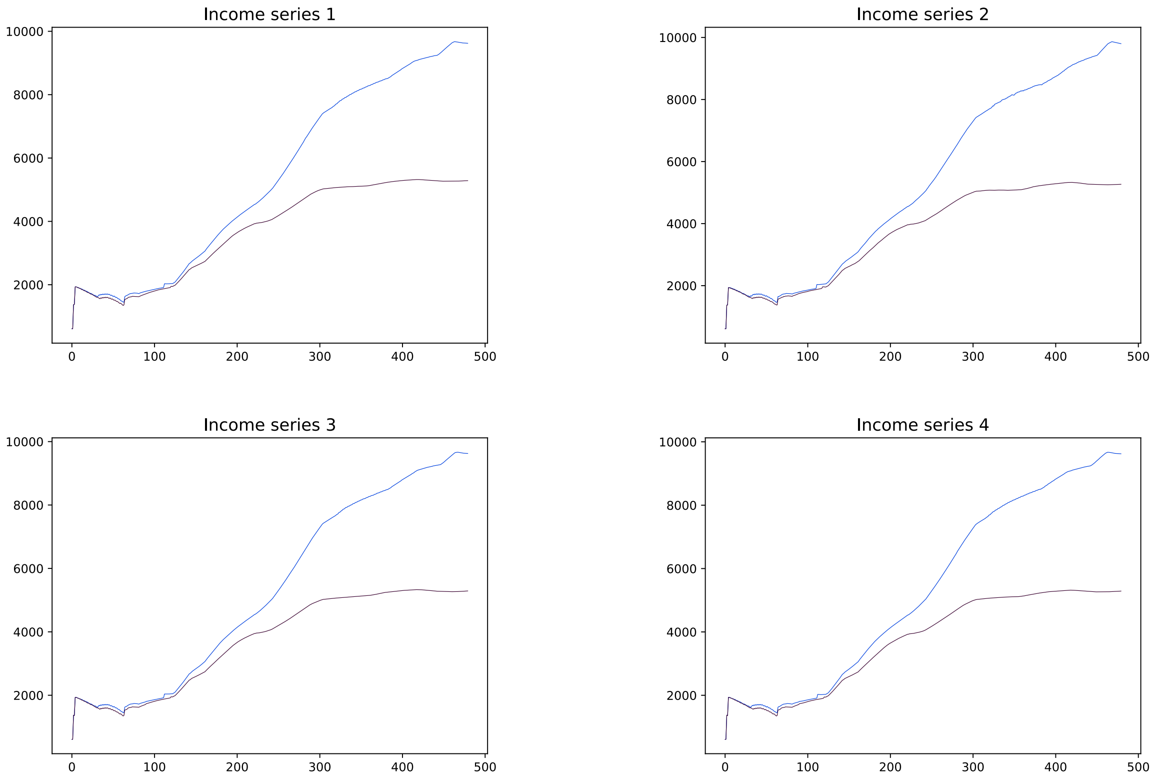

Fig. 18 – Aggregate ownership of all age cohorts of the middle-aged, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

Contrary to the aggregate number of flats series, the series of the number of all houses owned by the agents in the model is more volatile than the former in the first part of time periods' span (Fig. ??). It is also less volatile in the second part of the simulation than the counterpart series for flats.

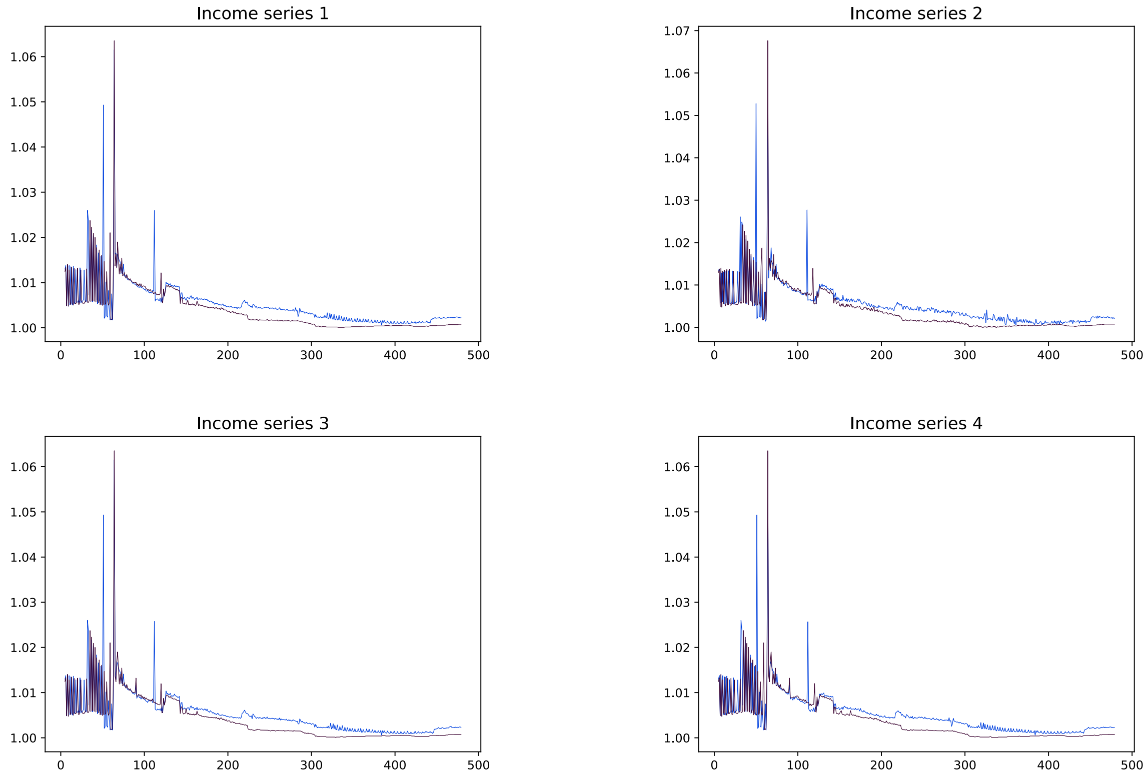

Fig. 19 – Gross percentage changes of the aggregate ownership of all agents, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

For the rate of growth of the number of houses owned by the young, we can observe a pattern that bears some resemblance to the one that characterises the growth rate of the number of flats owned by these agents. This group starts with no houses in possession and, for the first version of price growth, until period $t = 84$ no agents from this group buy houses, except for the period $t = 51$, when a hundred flats is bought (which can be observed as the ownership of a hundred flats in period $t = 51$). All of these flats are bought by the ten oldest cohorts of the young (i.e. $a \in \{70, ..., 79\}$) of the top ten income distribution percentiles. The fact that these flats were bought was corroborated by the fact that the corresponding oldest cohorts of the old in the previous period (i.e., $t = 50$) did not possess any houses.

The growth of the number of houses owned by the young starting in $t = 84$ (see fig. 20) is due to inheritance, which can be also partly observed by noticing that the young start to take out debt much later (fig. 10). The highest gross rate of growth of the number of houses in possession of the young is the same (equals 2 and occurs in $t = 85$) for all versions of income in the two variants of price processes. The only exception is the second income series version for the quicker increase of prices variant, for which highest gross growth rate of houses owned by all young cohorts is $6\frac{2}{3}$ for $t = 57$.

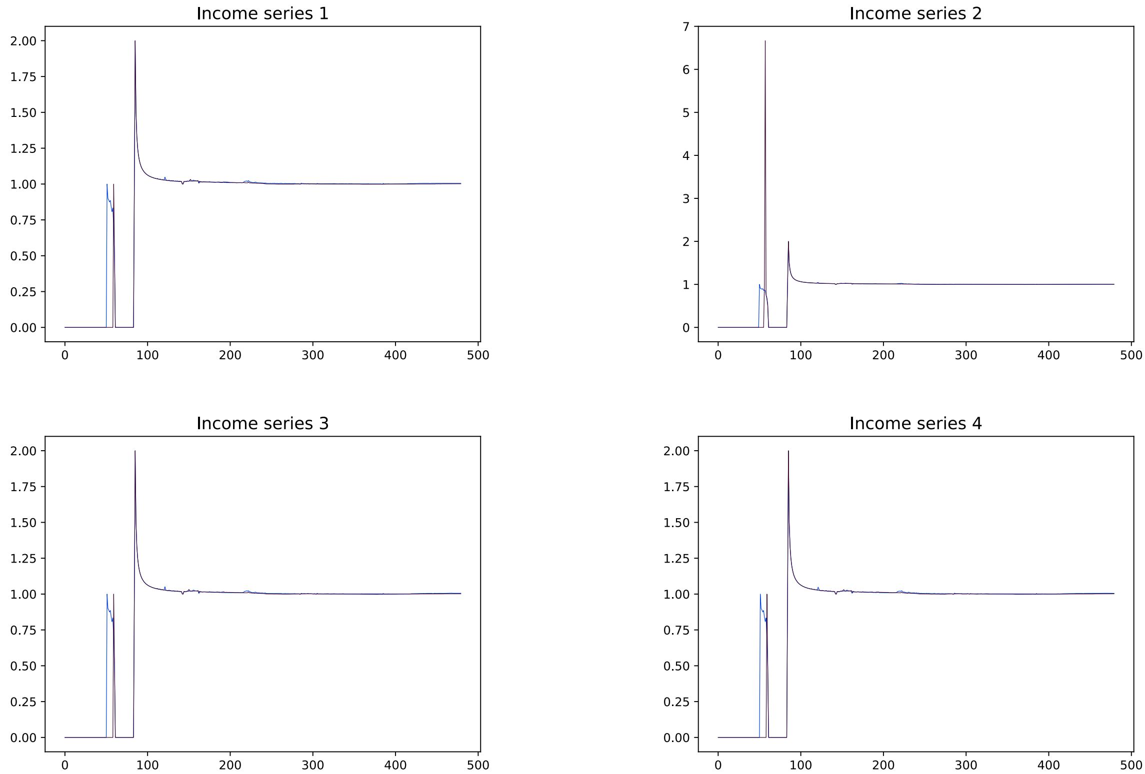

Fig. 20 – Gross percentage change of the aggregate ownership of all age cohorts of the young, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

In the case of young agents, differences in volatility of house number growth for the two price growth scenarios are less pronounced than for the analogous series for flats (figures 20, 21).

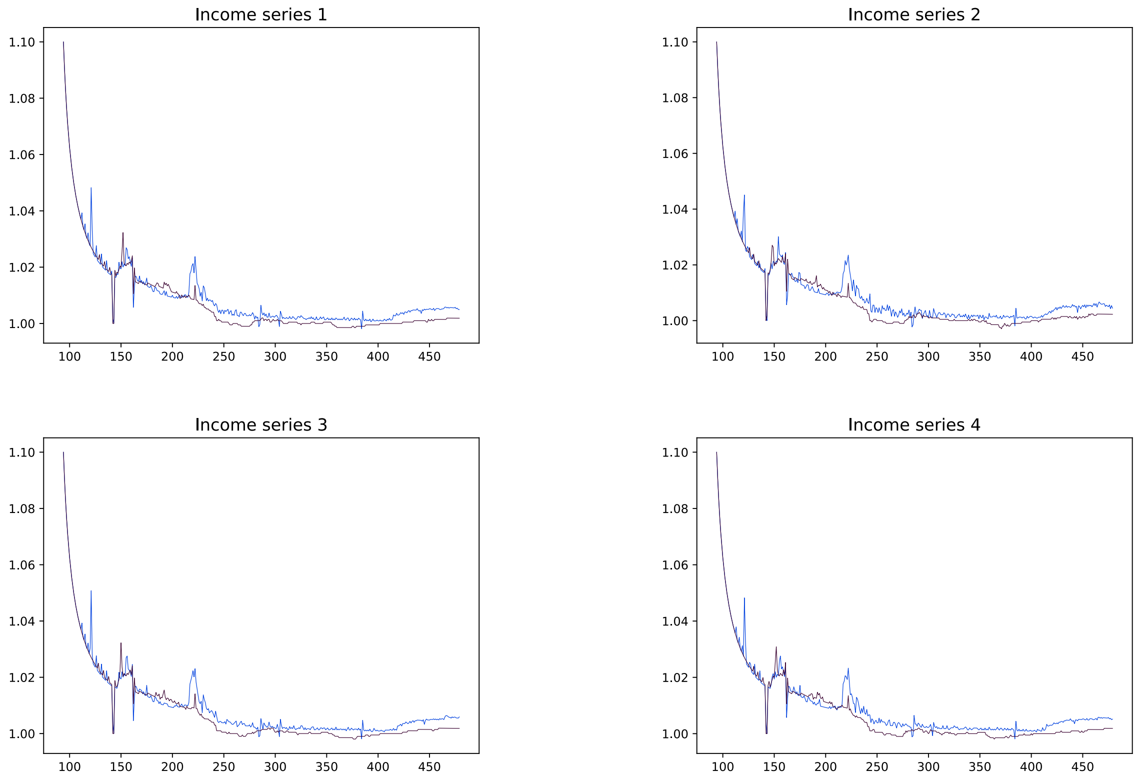

Fig. 21 – Gross percentage change of the aggregate ownership of all age cohorts of the young, category: houses; periods from 94 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

The largest one-period increase (after $t = 4$) in the amount of houses owned by middle-aged cohorts occurs in $t = 64$ for all income series and both price increases versions, with slightly larger value for the second one (fig. 22). This rise is not associated with the largest increase of aggregate debt of the middle-aged, which occurs in $t = 316$ for the first and fourth versions of the income process and the first version of inflation, while for the second and third series the peak of one-period debt increases occurs in $t = 360$ and $t = 320$ respectively (figure 14). Thus, these peaks in growth rates of houses owned by the middle-aged result from inheritance of deposits and houses of the deceased old.

The number of houses in possession of the old cohorts shows less cyclicality than the ownership of flats by this group (fig. 23, 24). This is another

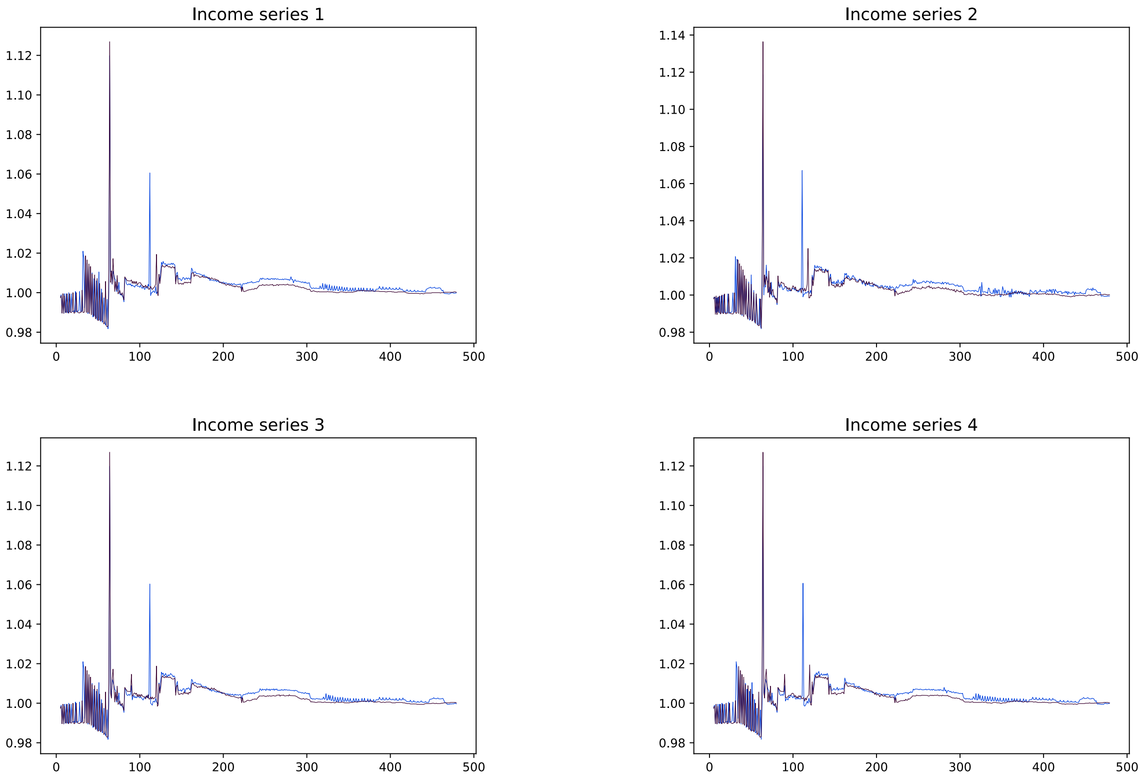

Fig. 22 – Gross percentage change of the aggregate ownership of all age cohorts of the middle-aged, category: houses; periods from 5 to 480 are displayed. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

indication that changes of the aggregate owned houses stock are less volatile than those of the aggregate owned flats stock.

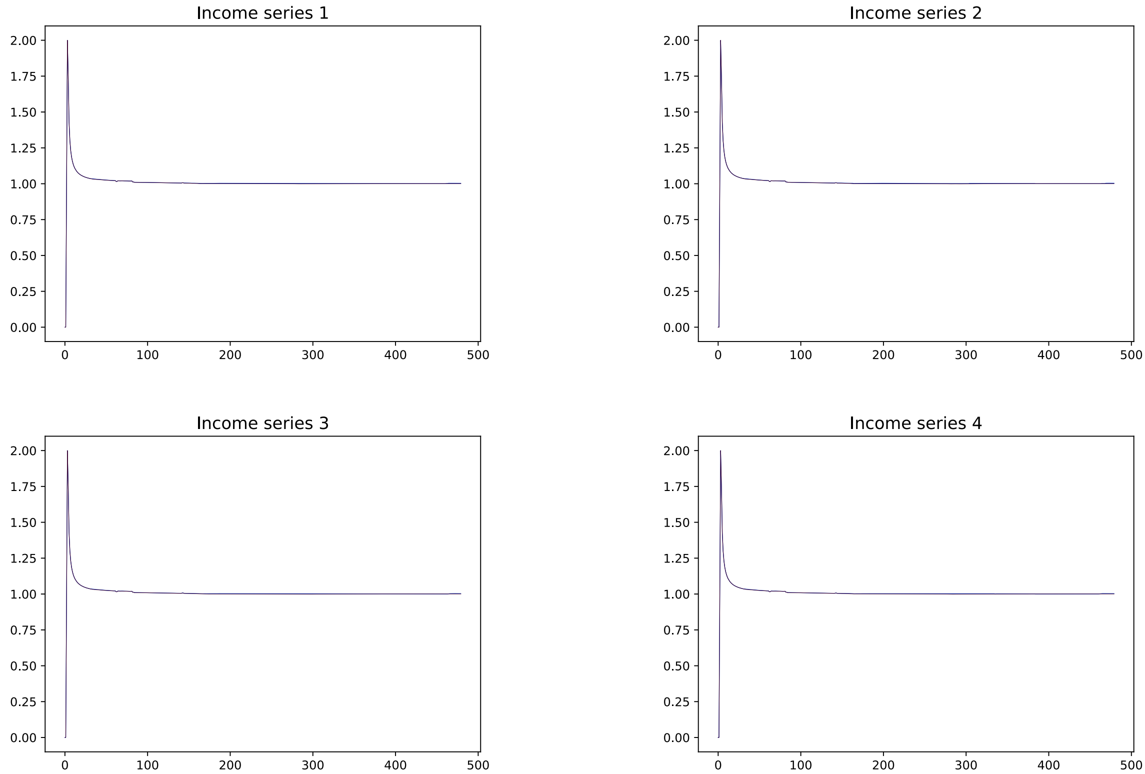

Fig. 23 – Gross percentage change of the aggregate ownership of all age cohorts of the old gross percentage change, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

6.4. Expenditure on durable goods: vehicles and other infrequently-bought durables

Differences in the housing goods inflation have no effect on the aggregate sales of new vehicles. The two considered versions of the model produce exactly the same results, characterised by a radically different pattern from those characteristic of the time series of purchases of houses and flats. This shows that categorisation-enhanced mental accounting consumer behaviour leads to differentiation of the evolution of markets. Moreover, absent of marketing

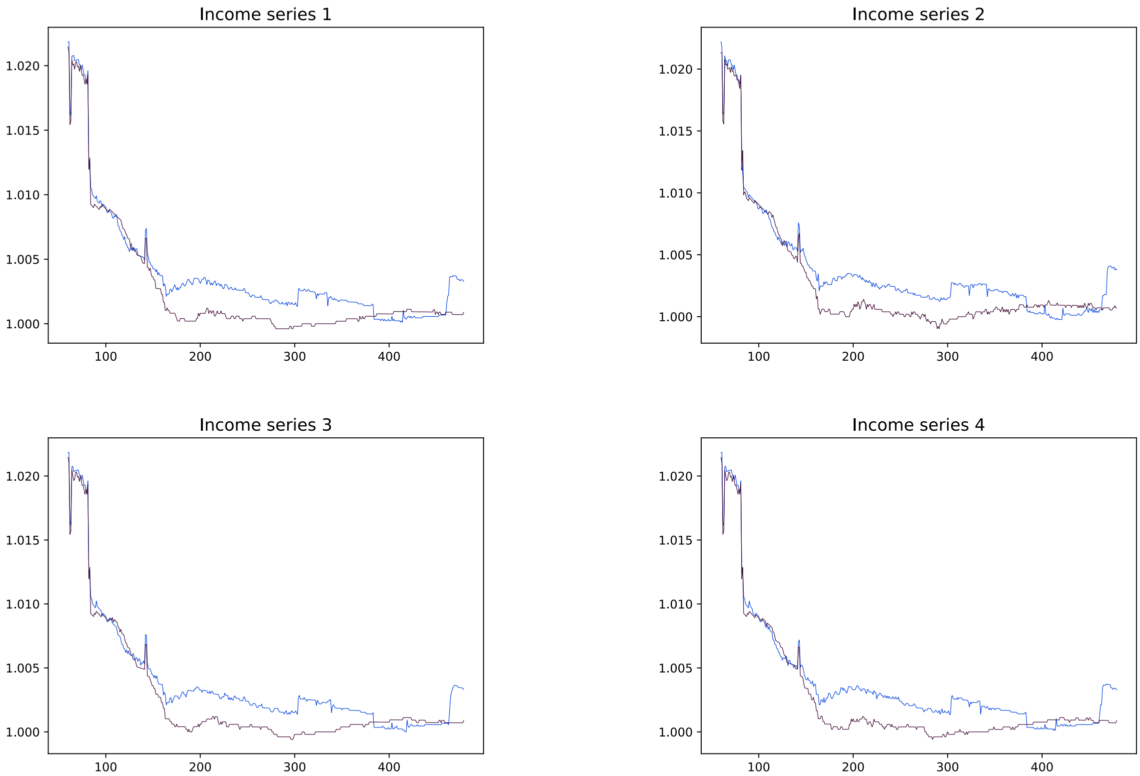

Fig. 24 – Gross percentage change of the aggregate ownership of all age cohorts of the old, category: houses. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

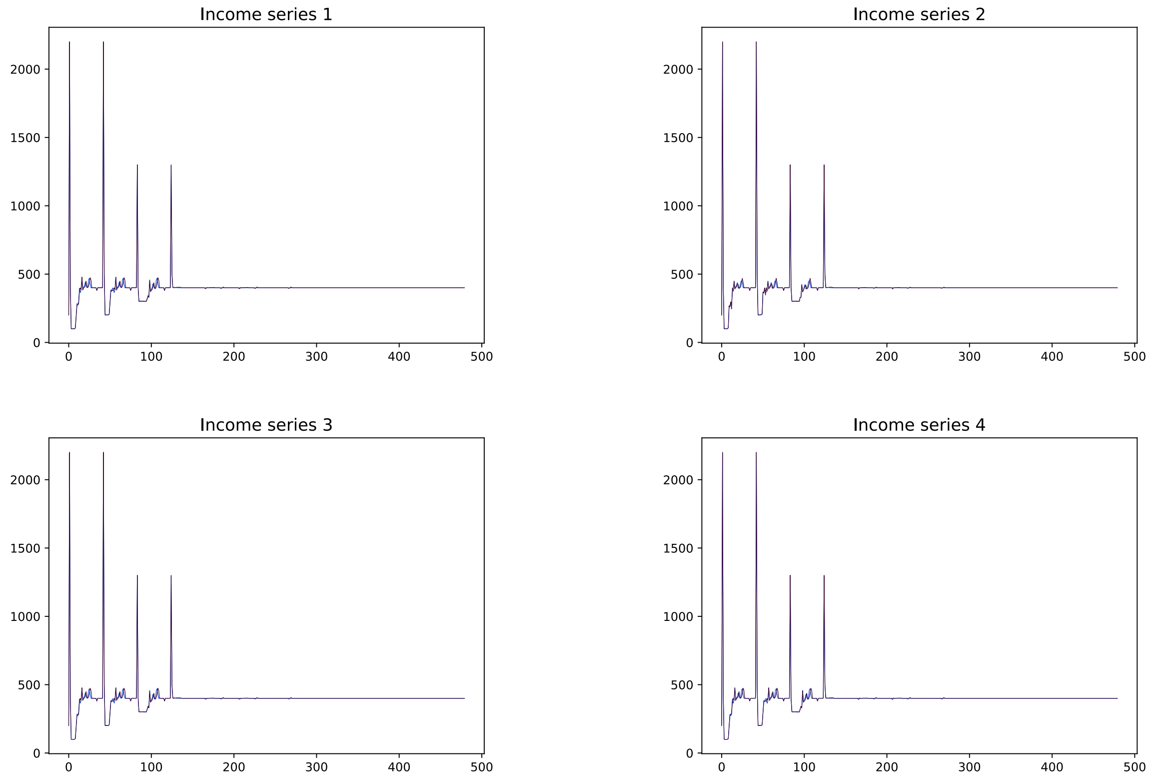

Fig. 25 – The number of new purchases by young and middle-aged agents, category: vehicles. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

shocks or more elaborate labour market fluctuations, the market for vehicles stabilises after the period $t = 125$.

Similarly, differences in the housing goods inflation have no effect on the aggregate sales of other infrequently bought durable goods. Without marketing shocks or more elaborate labour market fluctuations, the market for other durable goods stabilises after the period $t = 137$ (with the exception of $t = 145$ and $t = 154$).

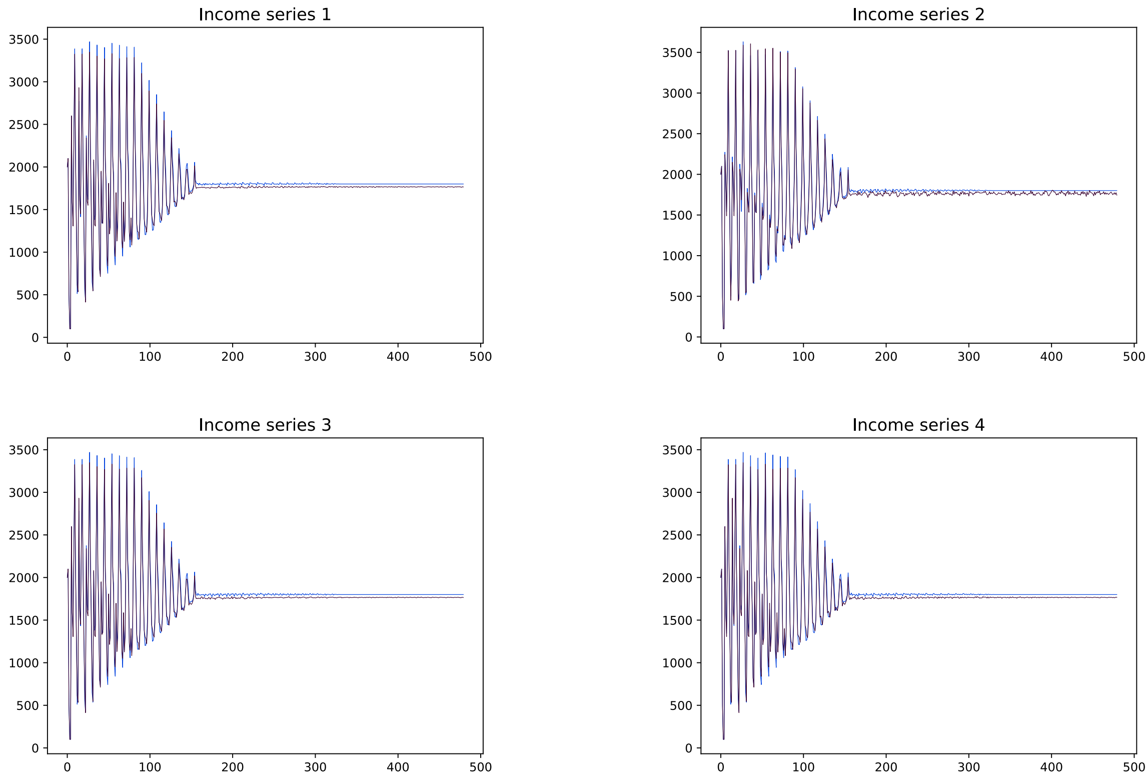

Fig. 26 – The number of new purchases by all young and middle-aged agents, category: other durables. Starting from the upper left corner, the results for the first, second, third and fourth version of the income process are displayed. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Blue lines show the results for the first price process, while the aubergine lines display the outcomes for the second.

7. Conclusions

It was demonstrated in this paper that mental accounting oriented at objects and categories of expenditure cannot be credibly represented using any intertemporal optimisation approach.The notions of strong and weak nonfungibility were introduced, and the behavioural life-cycle model was shown to satisfy the principle of nonfungibility only in the weak sense. Intertemporal optimisation either leads to underdetermination of the variables constituting the consumer problem or to breaking the principle of nonfungibility (in the strong or weak sense), or both.

For the purpose of developing a theoretical model of category-of-goods mental accounting consumer expenditure, mental-accounting theory was merged with categorisation theories, following the call of Henderson and Peterson (1992). The resultant framework is characterised by procedural and limited rationality; objective computational, informational and cognitive limitations of consumers are mitigated by the categorisation and mental accounting behaviour. This facilitates the decision process, and thus is beneficial for a decision-maker.

The crucial features of the devised framework are disposable income, category-related mental budgets and division-of-funds variables, which determine how much of each budget is spent. A classification of six basic consumer types was devised, basing on the differences of changes of expenditure in response to variations of net disposable income and other possible stimuli. Individual spending rates out of each mental budget are time-variable, but the distinct frequency of purchases of durable and nondurable goods observed in real economies necessitate different decision processes for products that are bought infrequently, especially if such purchases are often supported by large amounts of consumer credit or housing loans.

In reality, some of the categories are not (or not always) spent on in every quarter. The framework presented in this paper is an approximation of the actual process. Extensions, for example featuring infrequent purchases for more categories of goods are possible, but some degree of parsimony was sought in this paper.

The presented single- and multi-agent models of consumer behaviour are consistent both with microeconomic and macroeconomic evidence on consumption. A single consumer's working-life-cycle expenditure on nondurable and frequently bought durable goods tracks income, but is disrupted by debt taking and repayments. Moreover, if a consumer's income is subject to changes of various magnitude, his/her category-related expenditure may be amplified beyond income variations, or smoothed out, depending on a consumer's type. The aggregate series of category-specific consumption, however, are much smoother than the individual series because falls in net disposable incomes resulting from debt repayments are not widespread across consumers. Moreover, various age cohorts and income percentiles take out loans in different time periods. Nonetheless, if a large part of the working-age population suffered from a net disposable income decline, the impact on consumption would be noticeable.

The devised agent-based overlapping-generations model is the first of its kind, not only because of its multi-cohort and income-distribution structure, but also due to the introduction of the behavioural-procedural consumer behaviour, characterised by purposeful infrequent purchases of flats and houses and intentional debt-taking. The analysis of this model allowed to uncover that if people's behaviour can be approximated by categorisation-enhanced mental accounting (or if this actually is the real-world human behaviour), then income changes are greatly enhanced by behavioural responses of consumers, thereby causing high aggregate demand growth. This is evident from the results of the agent-based overlapping-generations model, characterised by quarterly growth rates of demand on various frequently-bought categories strongly surpassing the growth rates of income. Therefore, the results of this paper support the hypothesis that in reality consumer demand plays a much more important role in business cycle fluctuations and economic growth than the analytical, general-equilibrium-based models suggest.

This framework offers vast possibilities of theoretical investigations of issues that cannot be addressed using analytical methods due to the curse of dimensionality and the limitations of representations of behaviour based on intertemporal optimisation. Labour market mobility and fluctuations within the age-income distribution is the most obvious one. The effects of marketing efforts and firms' competition on various category-related markets, or contagion of downturns on separate markets are another. Moreover, decision rules of consumers may be further modified, for instance by investigating the possibility that a consumer's type changes throughout the life-cycle.

The devised models also prompt questions about the real-world behaviour of consumers: what is the distribution of consumer types within a population? What are the magnitudes of consumer responses to changes of net disposable income and other stimuli, such as firms' marketing or inflation? These questions open vast possibilities for future empirical and experimental studies.

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A. The sizes of category-related mental accounts and reactions of expenditure rates to stimulus

Given the minimum values of the saving rates $\sigma^{CA,i}_{sr,min} = 0.1, \sigma^{SA,i}_{sr,min} = 0.05$, we have that

$$\sum_s \eta_s = 1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min} \tag{60}$$

In this paper, for exposition purposes, the following assumptions were made:

$$\eta_b = \frac{2}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{61}$$

$$\eta_{seq} = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{62}$$

$$\eta_t = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{63}$$

$$\eta_{cl} = \frac{3}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{64}$$

$$\eta_{mds} = \frac{2}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{65}$$

$$\eta_{fb} = \frac{5}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{66}$$

$$\eta_{csm} = \frac{2}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{67}$$

$$\eta_{cs:ent} = \frac{3}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{68}$$

$$\eta_{cs:sp} = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{69}$$

$$\eta_{cs:hc} = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{70}$$

$$\eta_{cs:tr} = \frac{3}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{71}$$

$$\eta_{cs:sti} = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}), \tag{72}$$

$$\eta_{cs:edu} = \frac{1}{26}(1 - \sigma^{CA,i}_{sr,min} - \sigma^{SA,i}_{sr,min}). \tag{73}$$

Furthermore, it was assumed that the sum of account-specific saving rates, corresponding to no change in net disposable income and no other stimuli, equals $\sigma^{CA,i}_{sr,min} + \sigma^{SA,i}_{sr,min}$, and that the proportions of these saving rates are equal to the ratios of the corresponding mental accounts, i.e.

$$\sigma^{b}_{sr,0} = \frac{2}{26}(\sigma^{CA,i}_{sr,min} + \sigma^{SA,i}_{sr,min}), \tag{74}$$

etc.

For the weak fungibility demonstrations and simulations it was assumed that for all categories s the coefficients ω_s, described in the presentation of different possible consumer types in section 4, are all equal to 0.5. As for the strong nonfungibility versions, the values of ω_s coefficients are given in table 3 (recall that $\omega_s^{12} = 1 - \omega_s^{11}$, $\omega_s^{22} = 1 - \omega_s^{21}$.).

Spending of the old on frequently bought goods follows the same rules as expenditure from net disposable income, with the exception that the respective mental accounts η_s^{SA} are much smaller, since they are defined using $\sigma^{SA,SA,i}_{sr,min} = 0.55, \sigma^{CA,SA,i}_{sr,min} = 0.45$.

Account	Value of ω_s^{11} and ω_s^{21}
b	0.5
seq	0.1
t	0.51
cl	0.3
mds	0.4
fb	0.6
csm	0.7
$cs : ent$	0.15
$cs : sp$	0.8
$cs : hc$	0.05
$cs : tr$	0.25
$cs : sti$	0.35
$cs : edu$	0.75

Table 3 – Values of ω_s^{11} and ω_s^{21} for all categories of frequently-bought non-durable and durable goods, for young, middle-aged and old consumers of all age cohorts and income percentiles.

B. Initial values

B.1. Income distribution

The initial income distribution is constructed on the basis of a truncated log-normal distribution with shape parameter $\sigma_{LN} = 1.25$, location $\mu_{LN} = 0.003$, scale $\eta_{LN} = 0.25$ and a cut-off point of $\bar{w} = 5$. The support of the truncated distribution is rescaled by multiplying each value by 10^6; the construction of the income distribution used in the model is described in the main body of the paper. Table 4 shows the Gini coefficient for the original truncated log-normal distribution (G_{tLN}), calculated using numerical integration, the sample Gini coefficient calculated for that distribution's percentile points (G_{StLN}), and the Gini coefficient for the population of agents used in the model G_{INC}.

$Version$	$Value$
G_{tLN}	0.526154020
G_{StLN}	0.5807722935428327
G_{INC}	0.6319106979211564

Table 4 – Values of Gini indices for the original truncated log-normal distribution, its percentiles and cut-off points, and the actual income distribution of agents in the agent-based model.

B.2. Prices

Initial prices of housing goods are assumed to reflect a form of market segmentation; there are 70 different prices of houses and the same number of flat prices, reflecting different quality of the product, e.g. size and unobserved location and urban facilities. Initial prices are set as follows. The top 10 percent of house prices satisfy the following equations in $t = 0$:

$$\forall_{n\in\{90,\ldots,99\}} P_0^{H,n} = HD_0^{adm,Y,n} \cdot (crc)^{-1} + \beta^H \cdot \tilde{\Omega}_0^{Y,n,20} \cdot 20, \tag{75}$$

$$\forall_{n\in\{0,\ldots,89\}} P_0^{H,n} = P_0^{H,90}. \tag{76}$$

where $HD_0^{adm,Y,n}$, crc is as in subsection **??** and $\tilde{\Omega}_0^{Y,n,20}$ indicate that initial prices are set on the basis of net incomes and admissible debts of young agents.

For flats we have

$$\forall_{n\in\{30,\ldots,99\}} P_0^{F,n} = HD_0^{adm,Y,n} \cdot (crc)^{-1} + \beta^H \cdot \tilde{\Omega}_0^{Y,n,20} \cdot 20, \tag{77}$$

$$\forall_{n\in\{0,\ldots,29\}} P_0^{F,n} = P_0^{F,30}. \tag{78}$$

C. An example of a single agent's working-life cycle expenditure

These results were obtained in a separate paper; they serve as a demonstration of consistency of the framework with facts on consumption expenditure established by microeconometric research.

The consumer expenditure model's dynamics are consistent with many facts that are unaccounted for by the permanent income model but were documented by microeconometric studies of consumption. Consistently with the findings of Zeldes (1989), debt-taking and repayment of loans affect the paths of consumption of goods from all categories of frequently bought goods. The presented model is also consistent with the implications of the study of Stephens (2003), i.e. the arrival of new income, higher than the past one, increases the level of consumption of frequently purchased products. Consistently with evidence provided by Stephens (2008), the modelled consumer reacts to the increase in discretionary income caused by paying off a loan and the disappearance of debt repayments. In line with the results of the study of D. S. Johnson et al. (2006), category-enhanced mental-accounting consumer framework does not feature forecasting future income or any anticipatory response to its variation. The finding of Parker (2017), that predictable changes in income have a significant impact on consumption at the time of their realisation, is a feature characterising the categorisation-enhanced mental-accounting consumer framework. It is manifested through the effect the non-stochastic parts of income series have on consumer's decisions (through the division-of-funds variables β_t^s) and behaviour, i.e. expenditure changes that are not a result of smoothing, but a compound of behavioural reactions and income changes.

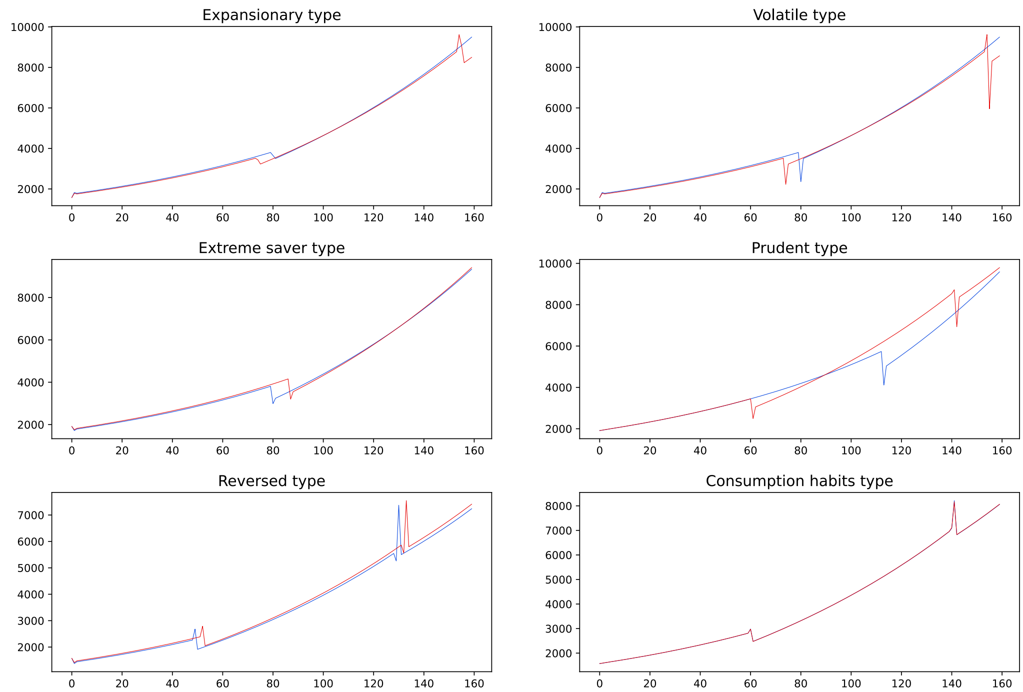

Fig. 27 – Expenditure in the life-cycle for various types of consumers, for the first version of the income process, with growing durable goods' prices, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

D. The problems with various intertemporal-optimisation representations of expenditure-oriented mental accounting consumer behaviour

All of intertemporal optimisation approaches to modelling mental-accounting behaviour share one of two problems: either they imply a form of fungibility of funds devoted for separate categories of goods (i.e. between money in different mental budgets) or their parameters are underidentified, or both.

The concept of nonfungibility of funds has not been analytically defined in mental accounting theory. While keeping separate accounts for various categories of goods may seem to be a defining characteristic of nonfungibility, if the consumer problem is modelled using intertemporal optimisation, the optimality conditions will impose strict interrelations between spending on each of the categories. Such a behaviour is not much different from a behaviour of an optimising individual with a constant elasticity of substitution utility function. However, maintaining the same ratios of consumed goods would require significant cognitive effort. Moreover, such an assumption precludes the possibility that the dynamics of people's needs for various categories of products differ, or that the effectiveness of marketing in different branches of the economy varies. Therefore, two notions of nonfungibility are used in this paper.

Definition D.1 (Weak nonfungibility). *A consumer's decision rule exhibits* ***weak nonfungibility*** *if spending on each of categories of goods is confined to separate mental budgets, but category-related expenditures co-move and their ratios are proportional to the ratios of the budgets, and the ratios of goods' prices.*

Definition D.2 (Strong nonfungibility). *A consumer's decision rule exhibits* ***strong nonfungibility*** *if spending on each of categories of goods is confined to separate mental budgets and the spending rates out of each of the accounts are not proportional and their ratios are time-variable and not proportional to the ratios of the corresponding budgets or the goods' prices.*

Thus, strong nonfungibility of funds stresses the notion that goods from various categories satisfy different needs and that the dynamics of these needs differ. This implies, among else, that elasticities of substitution are time-variable and do not depend only on price ratios, but also on intrinsic needs of consumers.

D.0.1. Intertemporal optimisation with a single budget

In this simple case, consumer's problem may be represented as

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \lambda_t(w_t - \sum_{m=0}^{M}(P_t^m \cdot x_t^m) - P_t^k \cdot k_{t+1} + (1 - \delta + r_t) \cdot k_t). \tag{79}$$

where the function $\mathcal{U}()$ is an agent's utility, m is a category-specific index of a good x_t^m, P_t^m is its price in period t, w_t represents the individual's available funds, k_t is a saving asset and P_t^k the corresponding price. λ_t is the Lagrange multiplier on the budget constraint, while r_t is the interest rate on the saving asset k_t, while δ is its the depreciation rate.

First-order conditions imply

$$\frac{P_t^m}{P_t^n} = \frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}}{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^n}}. \tag{80}$$

This condition violates the strong notion of nonfungibility of funds from different accounts.

D.0.2. Intertemporal optimisation with multiple budgets and no saving asset

Here, the consumer's lagrangian is

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^{M}(\lambda_t^m(w_t^m - P_t^m \cdot x_t^m)). \tag{81}$$

where λ_t^n is the category-specific Lagrange multiplier. Transforming first-order conditions, we obtain equation 80 and

$$\frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_{t+1}^m}}{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}} = \frac{\lambda_t^m}{\lambda_t^n} \cdot \frac{P_t^m}{P_t^n} = \frac{P_t^k}{1 - \delta_k - r_{t+1}} \cdot \frac{P_t^m}{P_t^n}. \tag{82}$$

But again, equation 80 violates the strong notion of nonfungibility of funds from different accounts.

D.0.3. Intertemporal optimisation with multiple budgets and saving assets

If we want to use a saving asset to introduce differences between budgets, then the asset must appear in only one period. Otherwise the relation between budgets will not be identified. Suppose that $f^m(k_{t+1})$ is the amount of an account m devoted to the saving asset. Note that all budgets but one ought to have nonnegative $f^m(k_{t+1})$ in order to assure the differences between expenditures on each of categories.

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^{M}(\lambda_t^m(w_t^m - P_t^m \cdot x_t^m - P_t^k \cdot f^m(k_{t+1})) \tag{83}$$

We have, again, equation 80, so if there exists a way to make consumer expenditure for different types of goods strongly nonfungible then it must result from the first-order condition with respect to capital. However, we have

$$0 = \sum_{m=0}^{M}(\lambda_t^m \cdot (-P_t^k \cdot \frac{\partial f^m(k_{t+1})}{\partial k_{t+1}})) \tag{84}$$

This implies that $\{\lambda_t^m\}_{m=0}^M$ are undefined unless $\{\frac{\partial f^m(k_{t+1})}{\partial k_{t+1}}\}_{m=0}^M$ are such that the hyperplane defined by $\{\lambda_t^m\}_{m=0}^M$ in equation 84 collapses to a single solution for all time periods t.

D.0.4. The behavioural life-cycle without a saving asset

In the work of Shefrin and Thaler (1988) the formula for the consumer problem in intertemporal optimisation framework is not given. Instead, the properties of the assumed utility function are analysed, and it is postulated that consumers spend differently from the current and permanent incomes. However, when the optimisation problem is constructed, then the mental-accounting rules are broken. Defining y_t^m to be the category-m related income, y_t – the whole income of a consumer while F_t – the consumer's future income and s – the saving rate (as in (Shefrin and Thaler 1988)), the function to be optimised may be written as

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^{M} \lambda_t^{CIA,m}((1-s)\cdot y_t^m - x_t^m \cdot P_t^m)$$
$$+ \sum_{m=1}^{M} \mu_t^{CWA,m}(\alpha^m \cdot (\sum_{\tau=1}^{t-1}((1-s)\cdot y_\tau^m - c_\tau)) - z_t^m \cdot P_t^m) \tag{85}$$
$$+ \sum_{m=1}^{M} \zeta_t^{FI,m} \cdot (\gamma^m \cdot F_t - v_t^m \cdot P_t^m)$$

where z_t^m is the amount of category-m good bought using permanent income and s is the saving rate (savings build the permanent income). If there is no saving asset, then the multipliers are not defined if their values differ. If, however, $\forall_m \lambda_t^{CIA,m} = \lambda_t^{CIA}$, then transforming first-order conditions yields

$$\frac{P_t^m}{P_t^n} = \frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}}{\frac{\partial \mathcal{U}(\{x_t^n\}_{m=0}^M)}{\partial x_t^n}} \tag{86}$$

which is the same as equation 80.

D.0.5. The behavioural life-cycle with a saving asset

In this subsection, it will be verified whether the introduction of a saving asset k_t into the behavioural life-cycle model would break the interrelations between the first-order conditions of various types of goods and allow the identification of lagrange multipliers. Using the notation and definitions from Shefrin and Thaler (1988), we need to disaggregate a consumer's current income y_t into current wage w_t and the stock of assets k_t: $y_t = w_t + (1 - \delta_k + r_t^k) \cdot k_t$. Then, $k_{t+1} = s \cdot (w_t + (1 - \delta_k + r_t^k) \cdot k_t)$. Thus, substituting this into 85 and taking first-order condition with respect to k_t, we get

$$
\begin{aligned}
0 = \sum_{m=1}^{M}(\lambda_t^{CIA,m} \cdot (1-s) \cdot ((1-\delta_k + r_t^k)))+ \\
\sum_{m=1}^{M}(\zeta_t^{FI,m} \cdot \gamma^m \cdot \frac{\partial FI_t}{\partial k_t})+ \\
\beta \cdot \sum_{m=0}^{M}(\lambda_{t+1}^{CIA,m}) \cdot ((1-s) \cdot ((1-\delta_k + r_{t+1}^k) \cdot s \cdot ((1-\delta_k + r_t^k)))+ \\
+\beta \cdot \sum_{m=0}^{M} \mu_{t+1}^{CWA,m} \cdot (\alpha_m \cdot (1-s) \cdot (1-\delta_k + r_t^k))+ \\
\beta \cdot \sum_{m=1}^{M}(\zeta_{t+1}^{FI,m} \cdot \gamma^m \cdot \frac{\partial F_{t+1}}{\partial k_t})
\end{aligned}
\tag{87}
$$

Again, this means that Lagrange multipliers $\{\lambda_t^{CIA,m}\}, \{\zeta_t^{FI,m}\}, \{\lambda_{t+1}^{CIA,m}\}$, $\{\mu_{t+1}^{CWA,m}\}, \{\zeta_{t+1}^{FI,m}\}$ define a hyperplane and are underidentified. In the case $\forall_m \lambda_t^{CIA,m}$

D.0.6. What does this tell us?

The analysis of the above cases shows that credible representation of objects-oriented mental-accounting consumer behaviour satisfying strong nonfungibility in an intertemporal optimisation framework is impossible. Nevertheless, algorithmic approaches are suitable for this purpose, as they provide much more flexibility and do not impose interrelations between first-order conditions, as they do not require any such calculations.