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## Abstract

Credibly representing category-of-goods mental accounting in an intertemporal optimisation framework is notoriously difficult, as this modelling approach imposes interrelations between the demand for different categories through first-order conditions. This breaks the principle of nonfungibility, contrary to the rationale of mental-accounting theory. Proofs that using intertemporal optimisation is futile in modelling this kind of behaviour are provided, and an alternative is developed: a procedural-behavioural merger of mental accounting and categorisation theories. The merger is necessary to enhance mental-accounting theory, which by itself does not inform about how mental budgets are formed, what they include and how money is spent from various accounts. A classification of six basic consumer types was devised, basing on the differences between their mental-accounting systems and variations of changes of expenditure in response to variations of net disposable income and other possible stimuli. Representing the consumer problem as a behavioural procedure including spending on nondurable and frequently-bought durable goods and decisions whether or not to purchase very expensive durable goods, such as houses and flats, allows to model real-world features such as infrequent purchases and rare debt-taking. The devised working-life cycle models of consumer behaviour are consistent with microeconomic evidence on consumption, including those features that are not accounted for by various versions of the permanent income or buffer-stock models.

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# 1. Introduction

Is mental-accounting or any other psychology-based theory of consumer behaviour necessary for better understanding of economic systems? If credibility and realism of microfoundations of economic models and the theoretical foundations of econometric studies of consumption matter, then the answer to this question is positive. After all, as Canova (2009) argues, if an estimated theoretical model does not reflect the structure of the real process, then the estimation will be biased. The same reasoning applies to drawing conclusions from theoretical models themselves. Mental-accounting behaviour – the set of cognitive rules used by investors or consumers to make financial decisions or choose the desired amount of consumption expenditure – has been documented in experimental and empirical studies, therefore it constitutes a good candidate for behavioural microfoundations of economic theory (Antonides et al. 2011; Thaler 1999; Thaler 1994b; Thaler 1990; Thaler 1985; Thaler and Shefrin 1981).

However, economic mental accounting theory faces a significant challenge. Henderson and Peterson (1992) underlined that mental accounting theory had not adequately addressed the questions of why and how mental accounts are formed and what is included in them. They also noted that most discussions of mental accounting have focused on the consequences of framing decisions by forming psychological accounts of the advantages and disadvantages of an option or an event, rather than on the processes underlying mental accounting. Mental accounting may result from processes described in categorization theories, which can be used to infer both the processes underlying mental accounting and its results (Henderson and Peterson 1992). A merger of these two concepts – categorisation and mental accounting – into one theory of consumer behaviour is devised in this paper. The resultant framework not only merges these theories, but also allows for modelling infrequent purchases of various durable goods, which is impossible under the usual continuity assumptions characterising the standard approach to modelling consumer choice. It also enables representing expenditure on various categories of goods satisfying the rule of nonfungibility; this paper contains proofs that expenditure-oriented mental accounting consumer behaviour cannot be credibly represented using solely analytical methods due to the assumptions of nonfungibility of funds from different category-related accounts. Conversely, an algorithmic (decision-tree) representation allows to model such consumer behaviour, as well as infrequent purchases of expensive durable goods, often combined with debt taking.

The difficulty is that unlike categorisation, mental accounting necessi-

tates that money contained in one mental account is nonfungible, i.e. it cannot be used in the remaining accounts; conversely, categorization theories allow fungibility. A straightforward way to overcome this problem is to assume that a person's funds are divided (through a behavioural process representing a person's needs and desires) into nonfungible categories. Categorisation theories give justification for the process of forming budgets or grouping expenditure for different purposes, contrary to mental accounting, which concerns either the outcomes of processing and grouping information, or the process of spending from current and permanent income (Henderson and Peterson 1992).

Nevertheless, no formal theoretical model of categorisation-enhanced mental-accounting theory has been developed. Moreover, it is demonstrated in this paper that representations of mental accounting by the means of intertemporal optimisation, including the behavioural life-cycle model of Shefrin and Thaler (1988), face several problems. The first is that they impose strict interrelations between categories of goods through optimality conditions, which can be viewed as a violation of the principle of nonfungibility. The second is that this cannot be justified by categorisation theories, as the elements within a category are context-dependent (Henderson and Peterson 1992) and thus the expenditures on various categories of goods are likely to depend also on other factors than prices and a utility function. Finally, the third issue is that when the optimisation problem is formed with many goods and many separate budget constraints, then the function representing the problem is underidentified.

Permanent income plays a crucial role in the existing formalisations of mental accounting and is still included in many empirical studies investigating this type of behaviour, despite numerous empirical literature refuting the permanent income hypothesis or any significant role of permanent income in consumer decision process (see the discussion in section 2). Additionally, the question of whether such representations are credible arises, especially that intertemporal optimisation approaches, including the behavioural life-cycle model, do not allow for infrequent expensive purchases necessitating using large amounts of debt, and sometimes also large payments using savings. Nonetheless, such expenditures are common in contemporary economies: mortgages and consumer loans constitute a large amount of assets of the real-world banking sectors.

One of the main reasons for the use of permanent income model in the existing standard and mental-accounting economic theory is the familiarity of the concept and the tradition of using analytical methods for economic analyses. Another is that the behavioural life-cycle is one of the few formalisations of mental accounting. However, it is argued in this paper that credible

representation of expenditure-oriented mental accounting necessitates an algorithmic approach. This is not a radical departure from the practice of economic modelling: computational economists and psychologists have argued for representing human decisions by the means of algorithms for a long time, in order to obtain a more transparent and realistic representation of people's behaviour and economic systems (Tsfatsion 2006).

In this paper, insights from behavioural economics, consumer and psychological research are used to create behavioural, evidence-based modelling assumptions in the form of a merger of categorisation and mental accounting theories. In order to preserve a form of continuity between this and the previously existing mental-accounting frameworks, two notions of nonfungibility are used. The first, weak, is satisfied by the behavioural life-cycle model and other intertemporal optimisation methods, and the second, strong, in which all total and marginal propensities to consume out of category-specific mental budgets differ. A classification of possible consumer types is constructed, on the basis of technical characteristics of the approach and economic intuition behind the behaviour they characterise. Moreover, an approach to represent infrequent purchases of durable goods and debt-taking decisions is presented and analysed in a single-consumer working-life model. The methodological motivation for the framework is provided by behavioural and agent-based computational economics, as well as psychological and consumer research.

Additionally to the model incorporating infrequent purchases and debt-taking that cannot be included in an intertemporal optimisation model, the results of the simulation of a working-life cycle for a single consumer are consistent with various, empirically documented, facts that the permanent income model cannot account for. These characteristics are discussed together with the results in section 5.

This conceptual work constitutes a methodological contribution to mental-accounting and consumer theory from a computational and microeconomic perspective. While differences in expenditure between consumers exhibiting weak and strong nonfungibility of funds from different, category-specific, accounts may seem small for certain consumer types, these discrepancies will accumulate in a macroeconomic context. Moreover, the differences depend on the assumptions on the variability of expenditure rates out of an account for weakly and strongly nonfungible spending behaviour (see section 3). The author has also devised and analysed the framework in a multi-market income distribution overlapping generations model with multiple age cohorts, in which seemingly inconsistent microeconomic and macroeconomic evidence on consumption are reconciled, but it is beyond the scope of this paper.

This paper is organised as follows. Section 2 contains the discussion of the documented unsuitability of the permanent income model, and provides

the existing evidence for mental-accounting behaviour; in subsection 2.3 it is demonstrated why it is infeasible to reliably represent category-of-goods mental accounting using intertemporal-optimisation approach. The proposed theoretical solution to the problem of modelling consumer behaviour by merging categorisation and mental-accounting theory is presented in section 3. Section 4 contains the categorisation and characterisation of consumer types as well as illustrations of basic behavioural dynamics. A single consumer's working-life cycle is presented in section 5. Finally, conclusions are contained in section 6. Appendix A contains information on the used sizes of category-related mental accounts.

## **2. Consumption behaviour: what do we know?**

### **2.1. The standard approach**

There are two strands of the main approach to consumption modelling: the standard one, often termed the permanent income or life-cycle model (PIH/LCM), and the less frequently used buffer-stock/liquidity constraints model, proposed as a remedy for some of the shortcomings of the former. These interest-rate-based, intertemporal optimisation frameworks are still dominant in modelling consumption, despite ever-growing literature listing econometric, cognitive psychology and consumer research evidence against them and their consequences, such as the interest-rate-driven consumption Euler equation. Empirical research revealed that the reaction of consumption to variations in interest rates is weak or statistically irrelevant (Yogo 2004; Canzoneri et al. 2007; Boug et al. 2021). Conversely, consumers' expenditure was found to be highly responsive to changes in income (Campbell and Mankiw 1989; Parker 2017; Boug et al. 2021).

The history of the tests of the permanent income hypothesis can be described rather as a repeated rejection of the permanent income model. Flavin (1981) was one of the first to provide evidence against PIH/LCM. Mankiw's results indicate that expenditure on consumer durables is more sensitive to changes in the interest rate than spending on nondurables and services (Mankiw 1985). Furthermore, other studies that used aggregate time-series data from the United States of America rejected the restrictions on the data implied by the stochastic versions of the PIH/LCM (Hansen and Singleton 1983; Mankiw et al. 1985; Zeldes 1989).

Campbell and Deaton claimed that smoothness of consumption observed in the data cannot be explained by permanent income theory (Campbell and Deaton 1989). They argued that consumption is smooth because it responds

with a lag to changes in income. Parker (2017) reached similar conclusions, namely that households' patterns of spending are highly predictable by past income.

While the buffer-stock theory has not been tested as much as the permanent income hypothesis, the existing empirical investigations performed by Ludvigson and Michaelides (2001) and Jappelli et al. (2008) did not find support for this version of the intertemporal optimisation approach. What is more, the buffer-stock theory shares the same weakness as any other framework based on intertemporal optimisation: in light of empirical studies, consumption is excessively related to the interest rate.

Both of the intertemporal-optimisation approaches – the PIH and the buffer-stock model – were found to be inconsistent with empirical data. Algorithmic methods offer an opportunity for developing more realistic representation of consumer behaviour. However, although they allow great flexibility in modelling choices, the question is: what valid theory can be used as a basis for a new, behavioural-computational approach?

There are psychology-based alternatives to intertemporal-optimisation framework. One of the main difficulties of incorporating psychological theories in microeconomic or macroeconomic models was the fact that optimisation has been the default way of economic modelling, while computational frameworks have traditionally been distrusted by most economists. The reasons have been various, from an attachment to equilibrium framework, to the 'black box' critique (Judd 2006; Ricardo J Caballero 2010). The latter has been undeserved: in fact, modelling the decision rules using algorithms makes them explicit.

## 2.2. Mental accounting

The majority of mental accounting research, both theoretical and empirical, including the behavioural life-cycle theory, focused on different sources of income and the nonfungibility of these funds. Nonetheless, for the description of consumer behaviour and spending, the key interest and the biggest potential of mental accounting theory lies in the categories-of-goods approach, i.e. the one that is focused on the objects of expenditure, not its sources. This is because merging it with categorisation theories will allow to model demand on various categories of goods with unequal total and marginal propensities to consume, and thus will enable modelling asymmetrical evolution of various markets, which is impossible under the assumption of consumers using constant-elasticity-of-substitution or any other utility function. This issue does not simply reduce to mental budgeting if the spending rates out of these budgets are time-variable.

That setting budgets in advance may simplify computational costs by reducing the number of alternatives when the available funds are limited is a point first raised by Simon (1947). It also facilitates comparison across possible choices (M. D. Johnson 1984). First mentioned by Tversky and Kahneman (1981) and Thaler (1985), this line of mental accounting research was undertaken also by Heath and Soll (1996). They have assumed that consumers set fixed budgets in advance of consumption, and because consumption opportunities change over time, the preset budgets are usually erroneous. However, they have only analysed a few specific case studies. A complete approach to modelling current consumer choices as well as allocation of resources over time has not yet been constructed.

Apart from to being formed as a psychological theory and applied in numerous experiments, mental accounting has been subject to a few tests against empirical data, which seem to corroborate the claim that households use mental accounting for making purchases. Antonides et al. (2011) have found empirical support for mental accounting consumer behaviour, basing on a large sample of Dutch population. Hastings and Shapiro (2013) have demonstrated that households treat money for different expenditure categories as nonfungible. More evidence supporting the mental accounting consumer behaviour was provided by Cheema and Soman (2006) in an experimental setting. They have shown that consumers flexibly classify expenses (construct accounts) to justify spending on various categories of goods, such as food, clothing and entertainment.

Mental accounting theory of economic behaviour has not been able to cope with the problem of representing consumer behaviour in a multi-market economic model. One of the reasons for this was the focus on the sources of income rather than on the categories of goods. Another reason is that pure mental accounting theory cannot account for individual differences.

Categorisation theories may be invoked to enhance the mental accounting approach. While most mental accounting theory focuses on the outcomes of actions, the process of forming type-of-good-related mental accounts as well as consumer decision making is almost never addressed within this line of research (with an exception of the work of Montgomery et al. (2019), discussed below).

It is demonstrated in this paper that any analytic optimisation-based representation of mental accounting is bound to either impose undesirable interdependencies between mental accounts for various categories of goods, or introduce ad hoc shocks to the optimality conditions to avoid such relations (as in (Montgomery et al. 2019)). This is contrary to the rationale of mental accounting and nonfungibility of funds. Therefore, it seems that credibly representing mental accounting consumer behaviour in a framework



with many categories of goods is impossible in the standard, optimisation-based approach to economic modelling. The next subsection demonstrates that analytical models cannot represent category-of-goods mental accounting under assumptions on nonfungibility that rule out exact comovement of expenditure out of different mental budgets.

### 2.3. The problems with various intertemporal-optimisation representations of expenditure-oriented mental accounting consumer behaviour

This section presents various forms of intertemporal optimisation approaches to modelling mental-accounting behaviour. All of them share one of two problems: either they imply a form of fungibility of funds devoted for separate categories of goods (i.e. between money in different mental budgets) or their parameters are underidentified, or both.

The concept of nonfungibility of funds has not been analytically defined in mental accounting theory. While keeping separate accounts for various categories of goods may seem to be a defining characteristic of nonfungibility, if the consumer problem is modelled using intertemporal optimisation, the optimality conditions will impose strict interrelations between spending on each of the categories. Such a behaviour is not much different from a behaviour of an optimising individual with a constant elasticity of substitution utility function. However, maintaining the same ratios of consumed goods would require significant cognitive effort. Moreover, such an assumption precludes the possibility that the dynamics of people's needs for various categories of products differ, or that the effectiveness of marketing in different branches of the economy varies. Therefore, two notions of nonfungibility are used in this paper.

**Definition 2.1** (Weak nonfungibility). *A consumer's decision rule exhibits **weak nonfungibility** if spending on each of categories of goods is confined to separate mental budgets, but at least one ratio of two category-specific total or marginal propensities to consume out of category-related budgets are always equal to another such ratio, or constant.*

**Definition 2.2** (Strong nonfungibility). *A consumer's decision rule exhibits **strong nonfungibility** if spending on each of categories of goods is confined to separate mental budgets and none of the ratios of total or marginal propensities to consume out of category-related budgets are equal to another ratio or constant.*

Thus, strong nonfungibility of funds stresses the notion that goods from various categories satisfy different needs and that the dynamics of these needs differ. This implies, among else, that elasticities of substitution are time-variable and do not depend only on price ratios, but also on intrinsic needs of consumers.

### 2.3.1. Intertemporal optimisation with a single budget

In this simple case, consumer's problem may be represented as

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \lambda_t(w_t - \sum_{m=0}^M (P_t^m \cdot x_t^m) - P_t^k \cdot k_{t+1} + (1 - \delta + r_t) \cdot k_t). \quad (1)$$

where the function  $\mathcal{U}()$  is an agent's utility,  $m$  is a category-specific index of a good  $x_t^m$ ,  $P_t^m$  is its price in period  $t$ ,  $w_t$  represents the individual's available funds,  $k_t$  is a saving asset and  $P_t^k$  the corresponding price.  $\lambda_t$  is the Lagrange multiplier on the budget constraint, while  $r_t$  is the interest rate on the saving asset  $k_t$ , while  $\delta$  is its the depreciation rate.

First-order conditions imply

$$\frac{P_t^m}{P_t^n} = \frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}}{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^n}}. \quad (2)$$

This condition violates the strong notion of nonfungibility of funds from different accounts.

### 2.3.2. Intertemporal optimisation with multiple budgets and no saving asset

Here, the consumer's lagrangian is

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^M (\lambda_t^m (w_t^m - P_t^m \cdot x_t^m)). \quad (3)$$

where  $\lambda_t^n$  is the category-specific Lagrange multiplier. Transforming first-order conditions, we obtain equation 2 and

$$\frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_{t+1}^m}}{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}} = \frac{\lambda_t^m}{\lambda_t^n} \cdot \frac{P_t^m}{P_t^n} = \frac{P_t^k}{1 - \delta_k - r_{t+1}} \cdot \frac{P_t^m}{P_t^n}. \quad (4)$$

But again, equation 2 violates the strong notion of nonfungibility of funds from different accounts.

### 2.3.3. Intertemporal optimisation with multiple budgets and saving assets

If we want to use a saving asset to introduce differences between budgets, then the asset must appear in only one period. Otherwise the relation between budgets will not be identified. Suppose that  $f^m(k_{t+1})$  is the amount of an account  $m$  devoted to the saving asset. Note that all budgets but one ought to have nonnegative  $f^m(k_{t+1})$  in order to assure the differences between expenditures on each of categories.

$$\mathcal{L} = \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^M (\lambda_t^m (w_t^m - P_t^m \cdot x_t^m - P_t^k \cdot f^m(k_{t+1}))) \quad (5)$$

We have, again, equation 2, so if there exists a way to make consumer expenditure for different types of goods strongly nonfungible then it must result from the first-order condition with respect to capital. However, we have

$$0 = \sum_{m=0}^M (\lambda_t^m \cdot (-P_t^k \cdot \frac{\partial f^m(k_{t+1})}{\partial k_{t+1}})) \quad (6)$$

This implies that  $\{\lambda_t^m\}_{m=0}^M$  are undefined unless  $\{\frac{\partial f^m(k_{t+1})}{\partial k_{t+1}}\}_{m=0}^M$  are such that the hyperplane defined by  $\{\lambda_t^m\}_{m=0}^M$  in equation 6 collapses to a single solution for all time periods  $t$ .

### 2.3.4. The behavioural life-cycle without a saving asset

In the work of Shefrin and Thaler (1988) the formula for the consumer problem in intertemporal optimisation framework is not given. Instead, the properties of the assumed utility function are analysed, and it is postulated that consumers spend differently from the current and permanent incomes. However, when the optimisation problem is constructed, then the mental-accounting rules are broken. Defining  $y_t^m$  to be the category- $m$  related income,  $y_t$  – the whole income of a consumer while  $F_t$  – the consumer’s future income and  $s$  – the saving rate (as in (Shefrin and Thaler 1988)), the function to be optimised may be written as

$$\begin{aligned}
\mathcal{L} = & \mathcal{U}(\{x_t^m\}_{m=0}^M) + \sum_{m=1}^M \lambda_t^{CIA,m} ((1-s) \cdot y_t^m - x_t^m \cdot P_t^m) \\
& + \sum_{m=1}^M \mu_t^{CWA,m} (\alpha^m \cdot (\sum_{\tau=1}^{t-1} ((1-s) \cdot y_\tau^m - c_\tau)) - z_t^m \cdot P_t^m) \\
& + \sum_{m=1}^M \zeta_t^{FI,m} \cdot (\gamma^m \cdot F_t - v_t^m \cdot P_t^m)
\end{aligned} \tag{7}$$

where  $z_t^m$  is the amount of category- $m$  good bought using permanent income and  $s$  is the saving rate (savings build the permanent income). If there is no saving asset, then the multipliers are not defined if their values differ. If, however,  $\forall_m \lambda_t^{CIA,m} = \lambda_t^{CIA}$ , then transforming first-order conditions yields

$$\frac{P_t^m}{P_t^n} = \frac{\frac{\partial \mathcal{U}(\{x_t^m\}_{m=0}^M)}{\partial x_t^m}}{\frac{\partial \mathcal{U}(\{x_t^n\}_{m=0}^M)}{\partial x_t^n}} \tag{8}$$

which is the same as equation 2.

### 2.3.5. The behavioural life-cycle with a saving asset

In this subsection, it will be verified whether the introduction of a saving asset  $k_t$  into the behavioural life-cycle model would break the interrelations between the first-order conditions of various types of goods and allow the identification of lagrange multipliers. Using the notation and definitions from Shefrin and Thaler (1988), we need to disaggregate a consumer's current income  $y_t$  into current wage  $w_t$  and the stock of assets  $k_t$ :  $y_t = w_t + (1 - \delta_k + r_t^k) \cdot k_t$ . Then,  $k_{t+1} = s \cdot (w_t + (1 - \delta_k + r_t^k) \cdot k_t)$ . Thus, substituting this into 7 and taking first-order condition with respect to  $k_t$ , we get

$$\begin{aligned}
0 = & \sum_{m=1}^M (\lambda_t^{CIA,m} \cdot (1-s) \cdot ((1-\delta_k + r_t^k))) + \\
& \sum_{m=1}^M (\zeta_t^{FI,m} \cdot \gamma^m \cdot \frac{\partial FI_t}{\partial k_t}) + \\
\beta \cdot & \sum_{m=0}^M (\lambda_{t+1}^{CIA,m}) \cdot ((1-s) \cdot ((1-\delta_k + r_{t+1}^k) \cdot s \cdot ((1-\delta_k + r_t^k)))) + \quad (9) \\
& + \beta \cdot \sum_{m=0}^M \mu_{t+1}^{CWA,m} \cdot (\alpha_m \cdot (1-s) \cdot (1-\delta_k + r_t^k)) + \\
& \beta \cdot \sum_{m=1}^M (\zeta_{t+1}^{FI,m} \cdot \gamma^m \cdot \frac{\partial FI_{t+1}}{\partial k_t})
\end{aligned}$$

Again, this means that Lagrange multipliers  $\{\lambda_t^{CIA,m}\}$ ,  $\{\zeta_t^{FI,m}\}$ ,  $\{\lambda_{t+1}^{CIA,m}\}$ ,  $\{\mu_{t+1}^{CWA,m}\}$ ,  $\{\zeta_{t+1}^{FI,m}\}$  define a hyperplane and are underidentified. In the case  $\forall_m \lambda_t^{CIA,m}$

### 2.3.6. What does this tell us?

The analysis of the above cases shows that credible representation of objects-oriented mental-accounting consumer behaviour satisfying strong nonfungibility in an intertemporal optimisation framework is impossible. Nevertheless, algorithmic approaches are suitable for this purpose, as they provide much more flexibility and do not impose interrelations between first-order conditions, as they do not require any such calculations.

## 2.4. Cognitive limitations of consumers, different spending patterns for various types of goods

The existing models of mental accounting are characterised by consumers using permanent or future income in their decision process. However, this would require exquisite forecasting skills of real-world consumers as well as considerable amount of time and information. If consumers were capable of performing such tasks as well as of acquiring and analysing the necessary information, there is little justification for the claim that they stop at this point and do not spend using optimisation rules, making individual saving plans, rather than behaving in accordance with mental accounting. Moreover, psychological and cognitive science literature suggests that people's cognitive

capabilities are limited and they often act irrationally from the point of view of standard economic theory (Lieder and Griffiths 2020).

Most people cannot and do not calculate the value of their permanent income and they spend using savings (Thaler 1994a). It is also well-known that durable goods are bought infrequently (Ricardo J. Caballero 1993). Note that the latter characteristic of behaviour is in contrast to the assumptions behind the behavioural life-cycle model or the buffer-stock model, which were proposed as remedies for the shortcomings of the permanent income framework.

There are, however, many types of durable goods (table 1), differing in the strength of consumer demand, durability, and possibly also the limits to which consumer spending can be enhanced by demand-generating activities of firms, such as marketing. Some of these types, such as books, household goods, sport equipment, tools, some consumer electronics and jewellery, consist of goods that are usually bought with cash, or with the use of current funds. Other types, e.g. houses, flats, and more expensive consumer electronics consist of products that are most often paid at least to some extent with borrowed funds. Moreover, these categories differ in terms of the frequency of making purchases by consumers, and prices, and inflation in each sector differs slightly from its counterparts.

<i>Category</i>
Books
Clothing
Houses and flats
Home appliances
Furniture
Tools
Cars and vehicles
Jewelry
Consumer electronics
Accessories
Long-term insurance

**Table 1** – Various categories of consumer durables

Additionally, there are many categories of consumer nondurables (table 2), each satisfying different needs and requiring various marketing techniques to affect consumer behaviour. At the level of an individual, the demand for nondurables is more steady than on durables (Ricardo J. Caballero 1993).

Durable goods can be divided into two groups: ones that are bought infrequently and those that are purchased frequently by some consumers (table 1). Quarters seem to be a good measure of frequency in this case. Most typically, it is the first type of goods that is associated with debt-taking, but not all infrequently bought durable goods require to be paid with new consumer loans.

Category of nondurable and low-price, frequently bought durable goods
Books
Sports equipment
Tools
Clothing
Medicines and dietary supplements
Food and beverages
Cosmetics
Consumer services: entertainment
Consumer services: Sports
Consumer services: Health care
Consumer services: Transportation
Consumer services: Short-term insurance
Consumer services: Education

**Table 2** – Various categories of consumer nondurables and frequently bought durable goods

### 3. The mental-accounting system of a consumer

#### 3.1. Overview

The basic idea of the mental accounting consumer framework developed in this paper is to represent spending per each category of goods by the means of division-of-funds variables, each of which is constrained by the bounds of a respective mental budget.

The convention adopted in this paper is to use the notion of a ‘mental account’ in reference to the size of a category-related budget relative to the net disposable income  $\tilde{\Omega}_t$  (see equation 10), while the word ‘budget’ refers to the maximum amount of money that can be spent on a given category.

Thanks to this differentiation, the values of accounts are directly comparable with the total saving rates out of the corresponding budget.

The decision rule of a consumer consists of the following steps.

### **The mental-accounting system of a consumer**

1) The consumer  $i$  mentally divides his/her current account (i.e. deposits bearing little or no interest,  $d_t^{CA,i}$ ) into four purpose-related mental budgets:

- current expenditure ( $d_t^{CA,curr,i}$ ),
- a mental budget devoted to the accumulation of funds for the purchase of a house or a flat ( $d_t^{CA,H,i}$ ), growing over time
- a mental budget devoted to the accumulation of funds for the purchase of a vehicle ( $d_t^{CA,veh,i}$ ), growing over time
- a mental budget devoted to the accumulation of funds for the purchase of other infrequently bought goods ( $d_t^{CA,dur,i}$ ), growing over time.

2) First, the consumer pays – or reserves funds for the payment of – rents, bills and taxes, and makes debt payments, if there is any outstanding debt, using the current expenditure budget. Of course, in reality, personal income taxes are paid only once a year, but it is assumed that people accumulate the required amount throughout that period. Quarterly tax payments are an approximation of this process.

3) The consumer spends on thirteen nondurable goods and frequently bought durable goods (see table 2) from the current expenditure account, using net disposable income ( $\tilde{\Omega}_t$ , see equation 10) as a reference point for the expenditure. Its gross percentage changes drive the decision of how much to spend (equations 11 and 12). The current expenditure account net of payments of bills, rents and taxes, is divided into thirteen category-specific accounts (categories are labelled using the index  $s$ ). Total spending rates out of each of the budgets ( $\beta_t^s$ ) are time-variable, bounded functions of net disposable income (this framework may easily be extended to incorporate the effect of marketing, inflation and other stimuli). None of the category-specific spending rates can surpass or equate the size of the account ( $\eta^s$ ). Therefore, each of the mental budgets consists of expenditure and savings. The saving rate out of an account is denoted  $\sigma_{sr,t}^{s,i}$  (where  $i$  is an index



identifying a particular consumer), while the resultant saving rate out of net disposable income is written as  $\sigma_{sr,t}^i$ .

4) The funds on the current expenditure account that are not spent (that are saved) are transferred to other mental budgets constituting the current account,  $d_t^{CA,H,i}$ ,  $d_t^{CA,veh,i}$ ,  $d_t^{CA,dur,i}$ , and to the saving account,  $d_t^{SA,i}$ .

5) The needs for a new house or a flat are present or not, and the needs for a new vehicle and other durable goods arise depending on whether the durability of the currently possessed product has been surpassed or not. The funds accumulated in the budgets  $d_t^{CA,H,i}$ ,  $d_t^{CA,veh,i}$ ,  $d_t^{CA,dur,i}$  and available new debt are compared with the prices of these goods. For simplicity, it is assumed that credit is taken out only for houses and flats. If the available funds are sufficient and the need for a new product is present, then the purchase is made. Otherwise the consumer continues accumulating funds in these budgets.

A consumer  $i$  decides how much of a budget  $\eta_s \cdot \tilde{\Omega}_t^i$  to spend in a given period using a time-varying variable governing the division of funds ascribed to a category  $s$ ,  $\beta_t^{s,i}$ , with  $\beta_t^{s,i} \in [\eta_{s,LB}, \eta_s]$ , with  $\eta_{s,LB}$  denoting the lower bound of the account.  $\beta_t^{s,i}$  is the gross expenditure rate out of category- $s$ -related account. Thus, the amount spent from a given budget in period  $t$  is equal to  $\beta_t^{s,i} \cdot \tilde{\Omega}_t^i$ .

The behavioural variables that govern the division of accounts into spending and savings are assumed to be functions of gross percentage changes of disposable income,  $\tilde{\Omega}_t^i$ , relative to the previous period. The disposable income is defined as the income earned in the previous period (quarter),  $\Omega_{t-1}^i$ , net of rents, bills, taxes and debt payments, if the consumer  $i$  has any outstanding debt:

$$\tilde{\Omega}_t^i = \Omega_{t-1}^i - RBT_t^i - dps_t^i \quad (10)$$

Following preference reversal theory (Kahneman and Tversky 1984; Kahneman and Tversky 1979), it is assumed that the magnitude of changes of the behavioural division variables may vary for positive and negative disposable income net percentage changes. For the increases, we have

$$\beta_t^{s,i} = \beta_{s1}^i + \beta_{s5}^i \cdot \exp(\beta_{s3}^i \cdot (\frac{\tilde{\Omega}_t^i}{\tilde{\Omega}_{t-1}^i} - 1)), \quad (11)$$

while for the decreases

$$\beta_t^{s,i} = \beta_{s2}^i + \beta_{s6}^i \cdot \exp(\beta_{s4}^i \cdot (\frac{\tilde{\Omega}_t^i}{\tilde{\Omega}_{t-1}^i} - 1)), \quad (12)$$

and  $\beta_{s1}^i, \beta_{s2}^i, \beta_{s3}^i, \beta_{s4}^i, \beta_{s5}^i, \beta_{s6}^i$  denote the parameters of the division-of-funds variables  $\beta_t^{s,i}$ . Due to the nonlinear character of these decision rules, only some of their parameters have an individual interpretation. The other may be treated as inseparable elements of the behavioural rule – as means of describing the patterns of behaviour. For more details, the reader is directed to section 4.

The resultant category-specific saving rates are defined as residuals  $\sigma_t^{s,i} = \eta_s^i - \beta_t^{s,i}$ , and therefore are time-variable. The aggregate saving rate equals the sum of category-specific saving rates and a special, minimum savings category:  $\sigma_t^i = \sum_{s=0}^S (\sigma_t^{s,i}) + \sigma_{min}^i$ . The limit range of possible variability of  $\beta_t^{s,i}$  can be interpreted as a maximum possible saving rate out of the category  $s$ ,  $\sigma_{max}^{s,i}$ . The category-specific saving rates at the same time define the space for demand variability: the larger the value of a saving rate corresponding to constant disposable income, i.e. to  $\hat{\Omega}_t^i = 1$ , the more can the demand of an individual  $i$  grow (section 4).

The sizes of transfers to mental accounts devoted to the accumulation of funds for infrequently bought durable goods and savings are determined by the residual saving rate,

$$\sigma_{sr,t}^{res,i} = 1 - \sum_s (\beta_t^s), \quad (13)$$

which is divided into transfers to the aforementioned mental accounts – but actually retained on the current account (i.e. a deposit bearing no interest) – and the transfer to the saving account:

$$tr_t^H = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^H \quad (14)$$

$$tr_t^{veh} = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^{veh} \quad (15)$$

$$tr_t^{dur} = \sigma_{sr,t}^{CA} \cdot \tilde{\Omega}_t^i \cdot \beta^{dur} \quad (16)$$

$$tr_t^{SA} = \sigma_{sr,t}^{SA} \cdot \tilde{\Omega}_t^i \cdot \beta^{dur} \quad (17)$$

where  $\sigma_{sr,t}^{CA} = \omega^C A \cdot \sigma_{sr,t}^{res,i}$  and  $\sigma_{sr,t}^{SA} = (1 - \omega^C A) \cdot \sigma_{sr,t}^{res,i}$ .

When the net disposable income of an individual grows or declines, the resulting change of expenditure is affected also by an alteration of a consumer's

behaviour. The latter is expressed as a new value of gross expenditure rate ( $\beta_t^s$ ) out of net disposable income on a given category of frequently bought goods. Denote the values of  $\beta^s$  corresponding to the gross percentage increase and decrease of the net disposable income of  $\delta_3$  and  $\delta_1$  percent as  $\beta_{IN}$  and  $\beta_{DCR}$  respectively. We have that the percentage changes of category- $s$ -related expenditure,  $y_{IN}^s$  and  $y_{DCR}^s$ , satisfy the following two equations:

$$y_{IN}^s \cdot \tilde{\Omega} = \beta_{IN}^s \cdot \tilde{\Omega} \cdot (1 + \delta_3) - \bar{\beta}^s \cdot \tilde{\Omega}, \quad (18)$$

$$y_{DCR}^s \cdot \tilde{\Omega} = \beta_{DCR}^s \cdot \tilde{\Omega} \cdot (1 + \delta_3) - \bar{\beta}^s \cdot \tilde{\Omega}, \quad (19)$$

where  $\bar{\beta}^s$  is the value of  $\beta_t^s$  corresponding to constant net disposable income,  $\tilde{\Omega}_t = \tilde{\Omega}_{t-1}$ , while  $\beta_{IN}^s = \bar{\beta}^s \cdot (1 + \Delta_{IN}^p)$ ,  $\beta_{DCR}^s = \bar{\beta}^s \cdot (1 + \Delta_{DCR}^p)$ , and the superscript  $p$  indicates that the changes of the  $\beta$  variables are taken to be the percentage changes (or ‘proportional’).

Consumers are reluctant to spend using savings (Thaler 1990). They are most likely to use them only if they have to, e.g. for the purchases of expensive durable goods. Thus, a continuous decision rule would be inappropriate for modelling such decisions. This, however, creates problems for the estimation or modelling of heterogeneous individuals using analytical tools. On the other hand, algorithmic methods can easily represent such behaviour. In the presented framework, only retired consumers use savings both for the purchases of nondurable and frequently bought durable goods, but all agents can spend them on infrequently bought durables.

## 4. Consumer types

### 4.1. Expansionary type

An expansionary consumer behaviour is defined as a spending pattern characterised by increases in expenditure,  $y_{IN}^s$ , surpassing the growth of disposable income  $\tilde{\Omega}$ , and decreases in spending,  $y_{DCR}^s$ , that are smaller in absolute value than the decline in disposable income. The second case can be interpreted as consumption habits behaviour when faced with a decrease of income. As for the growth of spending, this type of behaviour can be viewed as a demonstration of impatience or susceptibility to marketing.

Using the notation from the previous section, the following conditions must hold for an expansionary consumer type. First, the growth of expenditure caused by the combined amount of an increase of disposable income and the resultant additional (beyond one-to-one) increase of spending due to

the change of  $\beta_t^s$ , cannot surpass the possible amount. This maximal amount is related to the size of possible variability within the mental account. I.e., relative to a situation without any income change, for which  $\beta_t^s = \bar{\beta}^s$ , the possible space for variability of the division-of-funds variable  $\beta_t^s$  is given by the ‘average’ saving rate  $\bar{\sigma}_{sr}^s = \eta_s - \bar{\beta}^s$ . We have

$$y_{IN}^s \leq \delta_3 + (1 + \delta_3) \cdot \bar{\sigma}_{sr}^s, \quad (20)$$

$$y_{IN}^s \geq \delta_3. \quad (21)$$

Similarly, for the change of expenditure after an income decrease – keeping in mind that an expansionary type tries to offset the effect of a decrease of net disposable income – we have

$$y_{DCR}^s \leq \delta_1 + (1 + \delta_1) \cdot \bar{\sigma}_{sr}^s, \quad (22)$$

$$y_{DCR}^s \geq \delta_1. \quad (23)$$

Thus, changes of expenditure can be described as

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} \cdot (1 + \delta_3) \cdot \bar{\sigma}_{sr}^s, \quad (24)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (1 + \delta_1) \cdot \bar{\sigma}_{sr}^s, \quad (25)$$

where  $\omega_s^{11}, \omega_s^{12}, \omega_s^{21}, \omega_s^{22} \in [0, 1]$  and are such that all of the above conditions hold. In this paper it is assumed that  $\forall_s \omega_s^{11} + \omega_s^{12} = 1$  and  $\forall_s \omega_s^{21} + \omega_s^{22} = 1$ , for all consumer types.

## 4.2. Volatile type

Volatile type of behaviour is characterised by both increases and decreases in expenditure surpassing (in absolute value) the growth and fall of disposable income, respectively. Thus, it is a combination of prudent behaviour for the domain of disposable income income decreases and susceptibility to its increases or to other positive stimuli such as marketing as well as impatience. Intuitively, volatile consumers ought to have the largest space for demand fluctuations of all other types. Note, however, that this also implies the largest maximum and average saving rates, which are determined by the differences between the sizes of accounts and their lower bounds.

We have

$$y_{IN}^s \leq \delta_3 + (1 + \delta_3) \cdot \bar{\sigma}_{sr}^s, \quad (26)$$

$$y_{IN}^s \geq \delta_3. \quad (27)$$

The change of expenditure after an income decrease for a volatile type is limited by the lower bound of the account. We have  $\eta_s - \beta_{s2} = \sigma_{sr}^{s,max}$ , but the reference point is  $\bar{\beta}^s$ , associated with  $\bar{\sigma}_{sr}^s$ , which is zero for this type. Thus,

$$y_{DCR}^s \leq \delta_1, \quad (28)$$

$$y_{DCR}^s \geq \delta_1 - (1 + \delta_1) \cdot (\sigma_{sr}^{s,max} - \bar{\sigma}_{sr}^s). \quad (29)$$

Therefore, changes of expenditure for the volatile type can be described as

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} \cdot (1 + \delta_3) \cdot \bar{\sigma}_{sr}^s, \quad (30)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (\delta_1 - (1 + \delta_1) \cdot (\sigma_{sr}^{s,max} - \bar{\sigma}_{sr}^s)), \quad (31)$$

### 4.3. Extreme saver type

An extreme saver type decreases expenditure after both a disposable income fall and increase. This means that a person exhibiting an extreme-saver consumer behaviour is one that increases savings when faced with any income change, relative to a situation in which his/her disposable income were constant.

We have

$$y_{IN}^s \geq \delta_3 - (1 + \delta_3) \cdot \sigma_{sr}^{s,max}, \quad (32)$$

$$y_{IN}^s \leq \delta_3. \quad (33)$$

The change of expenditure after an income decrease for a extreme saver type is limited by the lower bound of the account. We have  $\eta_s - \beta_{s2} = \sigma_{sr}^{s,max}$ , but the reference point is  $\bar{\beta}^s$ , associated with  $\bar{\sigma}_{sr}^s$ . Thus,

$$y_{DCR}^s \leq \delta_1, \quad (34)$$

$$y_{DCR}^s \geq \delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}. \quad (35)$$

Therefore, changes of expenditure for the extreme saver type can be described as

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 - \omega_s^{12} \cdot (\delta_3 - (1 + \delta_3) \cdot \sigma_{sr}^{s,max}), \quad (36)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (\delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}), \quad (37)$$

#### 4.4. Prudent type

A prudent consumer type decreases expenditure more than proportionally after a disposable income fall. When faced with a growth of funds, this type increases spending by a factor smaller than one.

Therefore, a prudent consumer is a person who exhibits a saver's spending pattern given disposable income increases and precautionary savings behaviour when faced with decreases of  $\tilde{\Omega}_t^i$ .

$$y_{IN}^s \geq \delta_3 \cdot \bar{\beta}^s, \quad (38)$$

$$y_{IN}^s \leq \delta_3. \quad (39)$$

Just like for the extreme saver type, the change of expenditure after an income decrease for the prudent type is limited by the lower bound of the account. We have  $\eta_s - \beta_{s2} = \sigma_{sr}^{s,max}$ , but the reference point is  $\bar{\beta}^s$ , associated with  $\bar{\sigma}_{sr}^s$ . Thus,

$$y_{DCR}^s \leq \delta_1, \quad (40)$$

$$y_{DCR}^s \geq \delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}. \quad (41)$$

Therefore, changes of expenditure for the prudent type can be described as

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} \delta_3 \cdot \bar{\beta}^s, \quad (42)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (\delta_1 - (1 + \delta_1) \cdot \sigma_{sr}^{s,max}), \quad (43)$$

but in this paper an alternative formulation for the increases was adopted, for the reason that the weights  $\omega$  are kept constant across consumer types for the sake of comparison, but using the above rule yields very small changes of expenditure for income increases. Thus, the following alternative was adopted:

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} (\delta_3 - \delta_3 \cdot \bar{\beta}^s). \quad (44)$$

The latter representation has the additional advantage that the size of  $(\delta_3 - \delta_3 \cdot \bar{\beta}^s)$  can be interpreted as the degree of prudence.

#### 4.5. Reversed type

A reversed consumer type decreases expenditure in response to a disposable income growth, but increases it when faced with the decline of disposable income, at least for small absolute values of changes of  $\tilde{\Omega}_t^i$ .

We have

$$y_{IN}^s \geq \delta_3 - (1 + \delta_3) \cdot \sigma_{sr}^{s,max}, \quad (45)$$

$$y_{IN}^s \leq \delta_3, \quad (46)$$

while for the decreases of income:

$$y_{DCR}^s \geq \delta_1, \quad (47)$$

$$y_{DCR}^s \leq \delta_1 + (1 + \delta_1) \cdot \sigma_{sr}^{s,max}. \quad (48)$$

Thus, following the same reasoning as for the prudent type in the case of income increases, we have:

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} (1 + \delta_3) \cdot (\sigma_{sr}^{s,max} - \bar{\sigma}_{sr}^s), \quad (49)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (1 + \delta_1) \cdot \bar{\sigma}_{sr}^s. \quad (50)$$

#### 4.6. Consumption-habits type

A person displaying consumption habits is reluctant to decrease or increase expenditure by factors more than one. In other words, such a consumer will spend less than the excess income, but also will counteract the fall in income.

For increases of net disposable income, a consumption-habits type is characterised by:

$$y_{IN}^s \geq \delta_3 - \bar{\beta}^s \cdot \delta_3. \quad (51)$$

$$y_{IN}^s \leq \delta_3. \quad (52)$$

For the decreases of net disposable income we have:

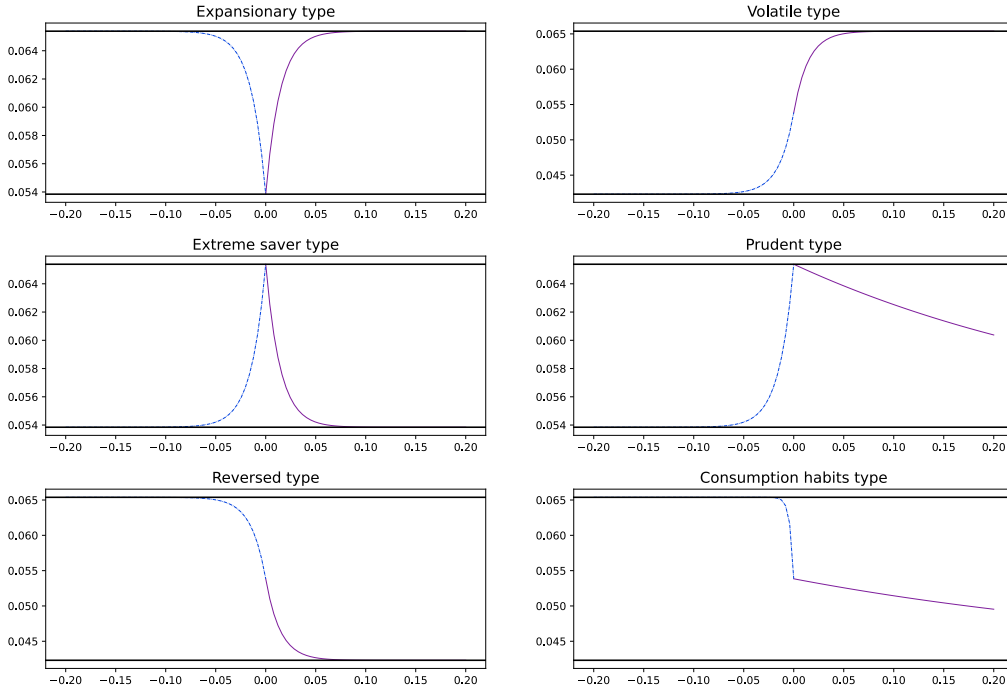
$$y_{DCR}^s \geq \delta_1, \quad (53)$$

$$y_{DCR}^s \leq \delta_1 + (1 + \delta_1) \cdot \sigma_{sr}^{s,max}. \quad (54)$$

Thus,

$$y_{IN}^s = \omega_s^{11} \cdot \delta_3 + \omega_s^{12} (\delta_3 - \delta_3 \cdot \bar{\beta}^s), \quad (55)$$

$$y_{DCR}^s = \omega_s^{21} \cdot \delta_1 + \omega_s^{22} \cdot (1 + \delta_1) \cdot \bar{\sigma}_{sr}^s. \quad (56)$$

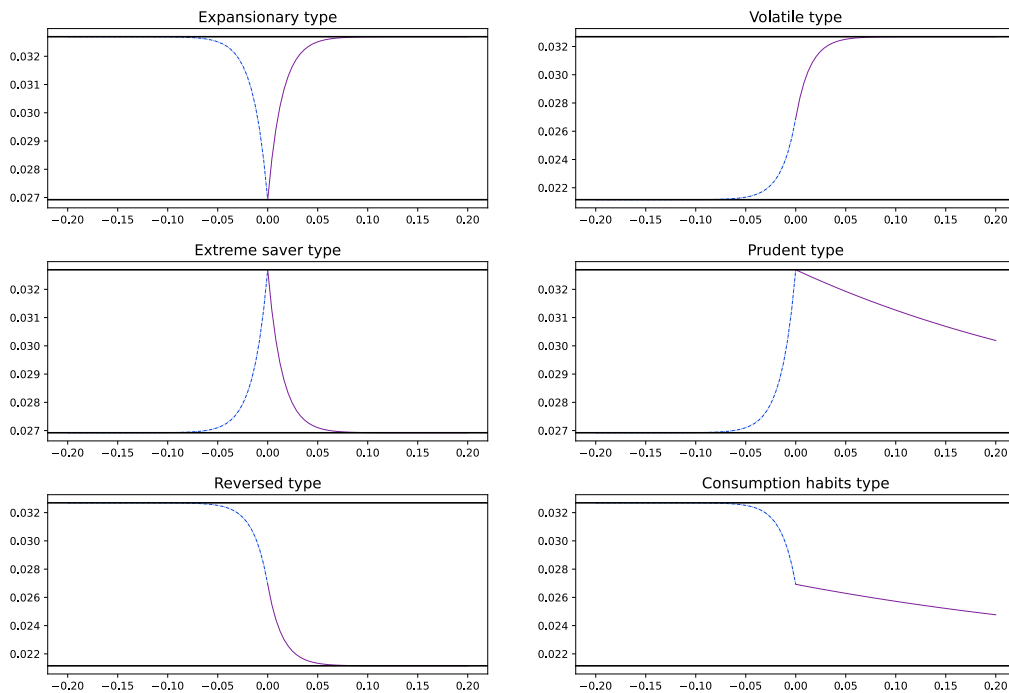


**Fig. 1** – The division-of-funds variables for various types of consumers, under weak non-fungibility, category: books. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\tilde{\Omega}$ , while the vertical one displays the values of the division-of-funds variable,  $\beta^b$ .



## 4.7. The range of variability of the division-of-funds variables

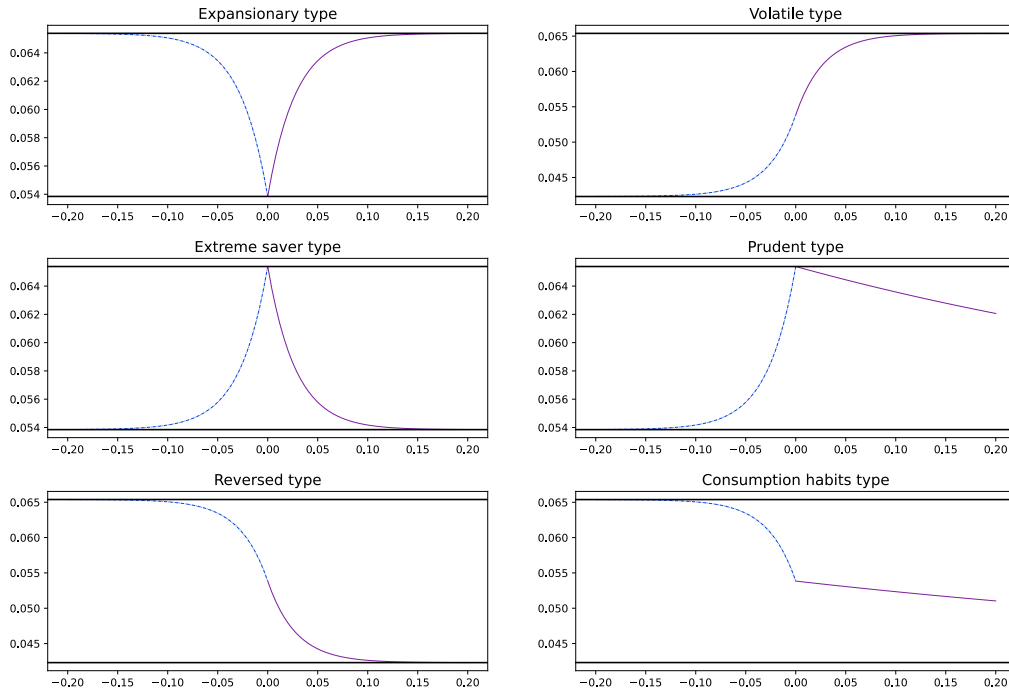
Figures 1, 2 show mental accounts and the range of variability of the division-of-funds variables,  $\beta^s$ , satisfying only weak nonfungibility, for the six types of consumers described above and two frequently-bought goods categories: books and sports equipment. These figures display the crucial feature of weak nonfungibility: the responses of category-related expenditure for various categories are proportional.



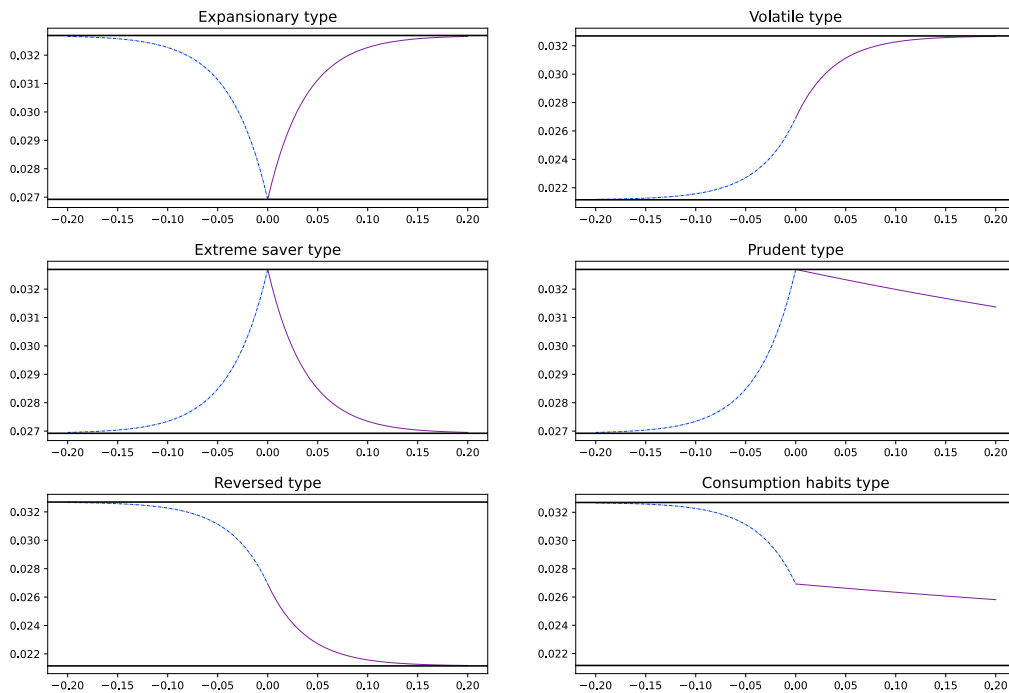
**Fig. 2** – The division-of-funds variables for various types of consumers, under weak nonfungibility, category: sports equipment. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\Omega$ , while the vertical one displays the values of the division-of-funds variable,  $\beta^b$ .

Figures 3, 4 display mental accounts and the range of variability of the division-of-funds variables,  $\beta^s$ , satisfying strong nonfungibility, for the six types of consumers described above and two frequently-bought goods categories: books and sports equipment. Comparing figures depicting expenditure for consumers whose behaviour satisfies weak and strong nonfungibility, we may observe that the reactions of consumer spending are not proportional in the second case, contrary to the responses of a consumer whose decision

rule satisfies only weak nonfungibility.



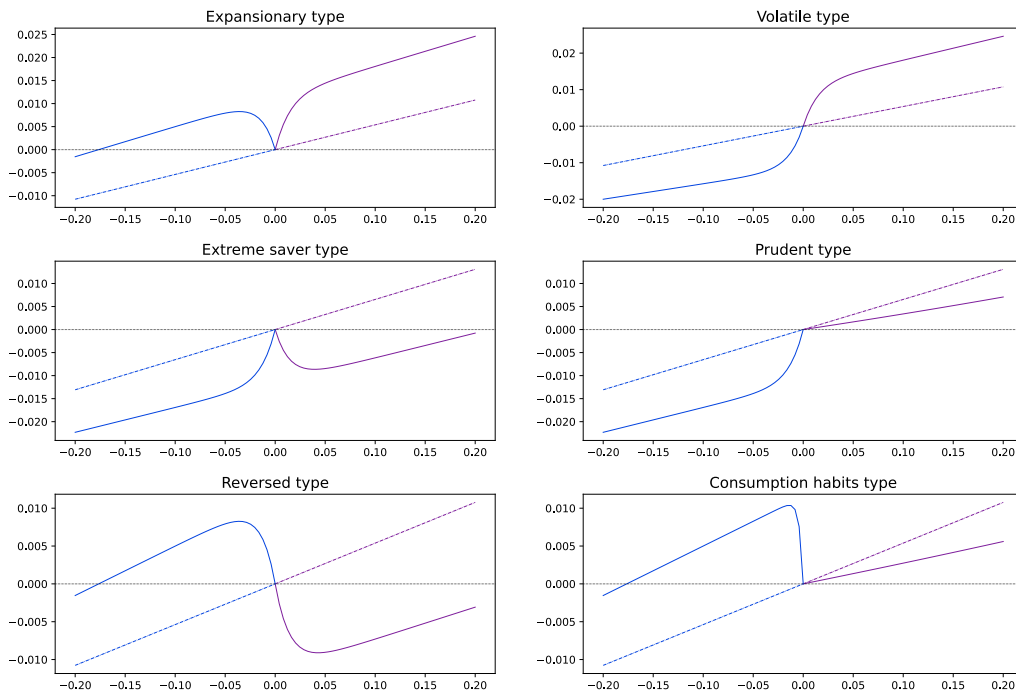
**Fig. 3** – The division-of-funds variables for various types of consumers, under strong non-fungibility, category: books. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\tilde{\Omega}$ , while the vertical one displays the values of the division-of-funds variable,  $\beta^b$ .



**Fig. 4** – The division-of-funds variables for various types of consumers, under strong non-fungibility, category: sports equipment. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\tilde{\Omega}$ , while the vertical one displays the values of the division-of-funds variable,  $\beta^b$ .

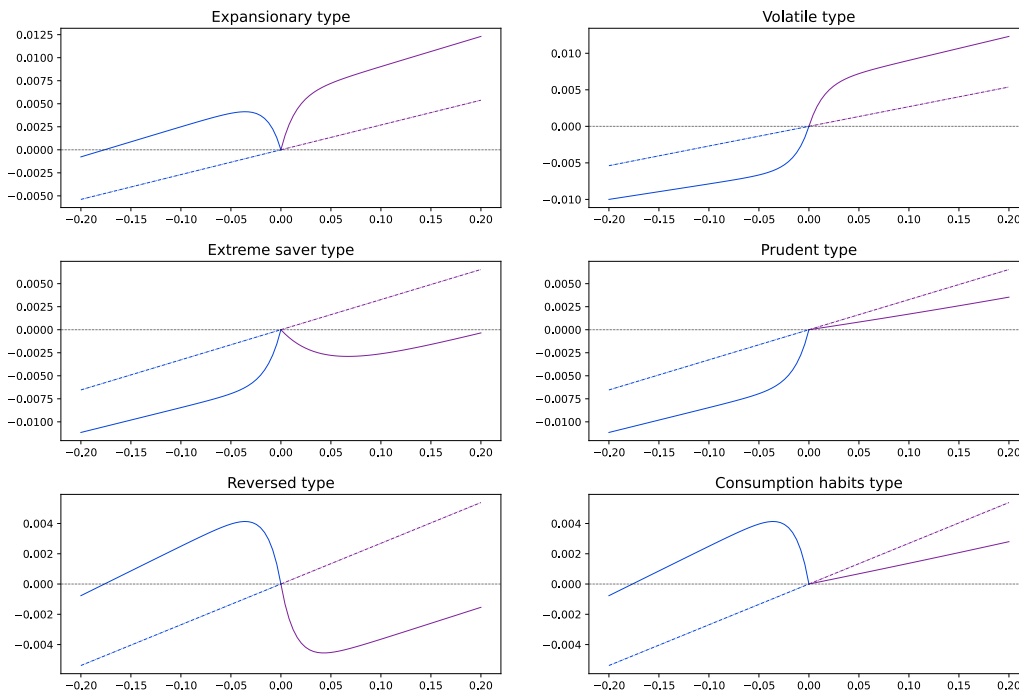
## 4.8. Expenditure change under categorisation-enhanced mental-accounting consumer behaviour

Figures 5, 6 demonstrate expenditure changes,  $y_{IN}^s, y_{DCR}^s$  for the six types of consumers described above but satisfying only weak nonfungibility and two frequently-bought goods categories: books, sports equipment (the remaining eleven categories are not shown to preserve the clarity of presentation, and due to space constraints). We may observe the proportionality of the behavioural responses (i.e. expenditure) by noting the same shapes of division-of-funds variables' changes for the weakly nonfungible behaviour.



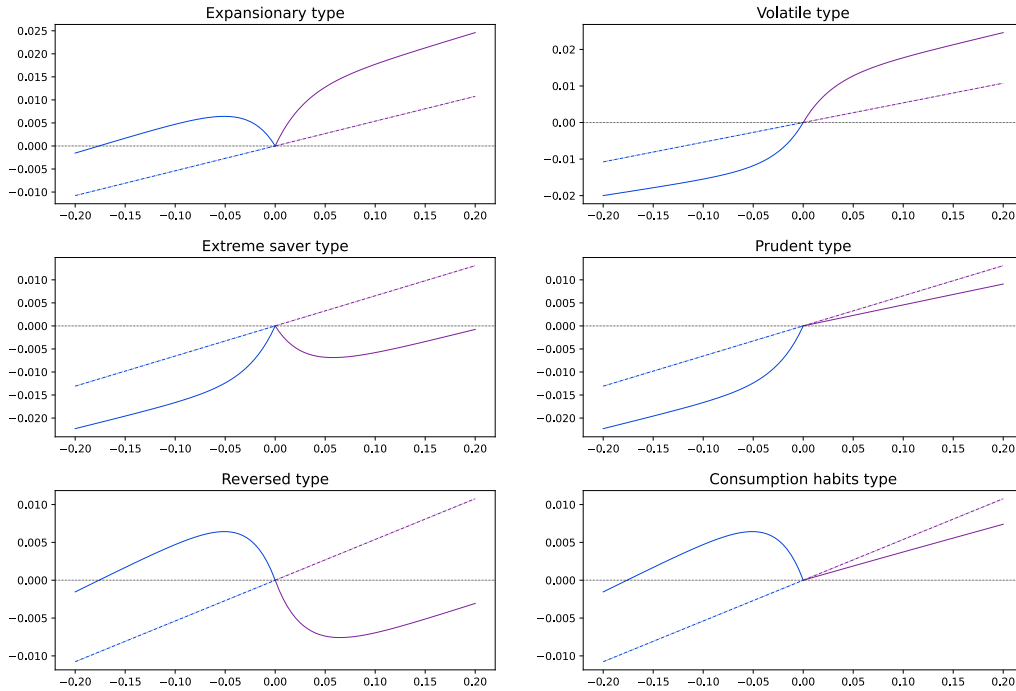
**Fig. 5** – The division-of-funds variables for various types of consumers, under weak nonfungibility, category: books. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\tilde{\Omega}$ , while the vertical one displays the induced change in the expenditure on a given category.

Figures 7, 8 depict expenditure changes,  $y_{IN}^s, y_{DCR}^s$  for the six types of consumers described above whose decision rules satisfy strong nonfungibility and two frequently-bought goods categories: books, sports equipment. We can see that the behavioural reactions (i.e. expenditure) are not proportional by comparing them with the counterparts of the weakly nonfungible rule. This is due to the differences in responses, described by  $\omega_s$  parameters

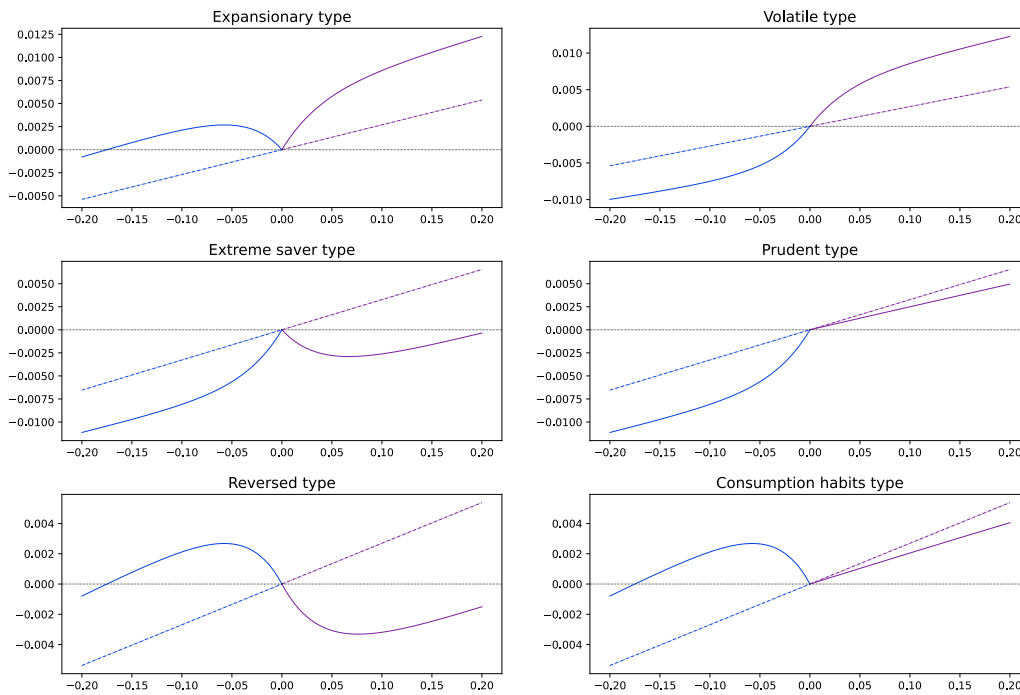


**Fig. 6** – The division-of-funds variables for various types of consumers, under weak non-fungibility, category: sports equipment. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\tilde{\Omega}$ , while the vertical one displays the induced change in the expenditure on a given category.

and  $y_{IN}^s, y_{DCR}^s$  quantities. In the present model, the only category for which weakly and strongly nonfungible types share the same behavioural parameters, i.e. the coefficients of the  $\beta_t^s$  variable (described, among else, by  $\omega_s$  parameters; see the above discussion in section 4) is ‘food and beverages’. See section 4 and appendix A for technical details.



**Fig. 7** – The division-of-funds variables for various types of consumers, under strong non-fungibility, category: books. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\hat{\Omega}$ , while the vertical one displays the induced change in the expenditure on a given category.



**Fig. 8** – The division-of-funds variables for various types of consumers, under strong non-fungibility, category: sports equipment. Starting from the upper left corner, the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows percentage changes of the disposable income  $\hat{\Omega}$ , while the vertical one displays the induced change in the expenditure on a given category.

## 5. Categorisation-enhanced mental-accounting consumption in the life-cycle of a worker

### 5.1. The model

The modelled consumer is assumed to spend on frequently bought goods using the behavioural decision rule described in section 3. As for houses ( $H$ ), flats ( $F$ ), vehicles ( $veh$ ) and other durable goods ( $dur$ ), the consumer follows the behavioural rules described in procedure 1.

Although in this section only one consumer's working-life-cycle is modelled, the agent has assigned the superscripts  $i, a$  to develop the notation sufficiently so that can be readily applied to models featuring many consumers. The index  $i$  is individual-specific, while the superscript  $a$  is equal to the agent's age. Moreover, it is assumed that the consumer tries to use only the funds on the housing good account,  $d_t^{CA,H,i,a}$  to buy a flat or a house. Only if this amount is insufficient the decision-maker takes into account spending money accumulated on the savings account. If these combined funds are still too small to buy a flat or a house, the agent considers taking out a multiperiod loan. The consumer is subject to credit constraints: the first instalment cannot be higher than the fraction  $crc = 0.3$  of the agent's net disposable income.

Possible states of a decision-maker's funds relative to the prices of flats and houses are can be described by establishing whether the following six equations, labelled  $(DRF_1, DRF_2, DRF_3)$  and  $(DRH_1, DRH_2, DRH_3)$  are true or not. Denote the index of a given agent by  $i$ , the agent's age by  $a$ , mental account devoted to the housing good (a house or a flat) by  $d_t^{CA,H,i,a}$ , net disposable income by  $\tilde{\Omega}_t^{i,a}$ , saving account by  $d_t^{SA,i,a}$  and the maximum admissible amount of new debt (only if there is no outstanding debt) by  $HD_t^{adm,i,a} = (r + \frac{1}{CD})^{-1} \cdot \tilde{\Omega}_t^{i,a} \cdot crc$ . The prices of a house and a flat are equal to  $P_t^H$  and  $P_t^F$ .

$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H + d_t^{SA,i,a} \cdot (1 + r) + HD_t^{adm,i,a} \geq P_t^F, \quad (DRF_1)$$

$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H + d_t^{SA,i,a} \cdot (1 + r) \geq P_t^F, \quad (DRF_2)$$

$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H \geq P_t^F, \quad (DRF_3)$$



$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H + d_t^{SA,i,a} \cdot (1+r) + HD_t^{adm,i,a} \geq P_t^H, \quad (DRH_1)$$

$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H + d_t^{SA,i,a} \cdot (1+r) \geq P_t^H, \quad (DRH_2)$$

$$d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^H \geq P_t^H. \quad (DRH_3)$$

A consumer spends on frequently bought nondurable and durable goods, and then considers whether to buy a house or a flat, and whether to spend on a new vehicle (according to its depreciation value and price) and other durable goods (similarly to vehicles, depreciation and price are considered). The order of needs concerning the housing good is as follows:

- 1) First priority: obtain a flat.
- 2) If at least one flat but no house is possessed, attempt to buy a house.
- 3) If one flat and one house are in a consumer's possession, buy a second flat.
- 4) If two houses but only one flat are owned by a consumer, buy a second flat
- 5) If two flats and two houses are possessed by a consumer, do not buy any more flats or houses.

The decision rule concerning frequently bought nondurable and durable goods is identical to the one described in section 3. As for infrequently bought durables, the behaviour governing their purchases is described in Procedures 1, 2, 3, 4, 5, 5, 6 where  $H_t^{i,a}$  and  $F_t^{i,a}$  denote the number of houses and flats in consumer's  $i, a$  possession, while  $dm_t^{i,a}$  is the indicator of debt maturity, where  $dm_t^{i,a} = 0$  means that no debt is held by the consumer at the beginning of a period. Debt are multiperiod, last for 60 periods, and can be taken out only for the purpose of buying a house or a flat. Periods are interpreted as quarters, which translates into mortgages having 15-years maturity. The cases of the decision-maker having two or more flats but no houses and the reversed situation can occur only in a multi-agent model with inheritance, but are included in this section for completeness.

Consumers are reluctant to spend using savings (Thaler 1990). They are most likely to use them only if they have to, e.g. for the purchases of expensive durable goods. Thus, a continuous decision rule would be inappropriate

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**Procedure 1** A consumer's decision rule for durable goods: part 1
 

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**if**  $F_t^{i,a} = 0 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
**if**  $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = True)$  :  
 $F_{t+1}^{i,a} = 1$   
 $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^F + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
 $HD_t^{i,a} = 0$   
 $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
 $dm_{t+1} * i, a = 0$   
**else if**  $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False)$  :  
 $F_{t+1}^{i,a} = 1$   
 $d_{t+1}^{CA,H,i,a} = 0$   
 $HD_t^{i,a} = 0$   
 $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a} + d_t^{CA,H,i,a} - P_t^F + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
 $dm_{t+1} * i, a = 0$   
**else if**  $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False)$  :  
 $F_{t+1}^{i,a} = 1$   
 $d_{t+1}^{CA,H,i,a} = 0$   
 $HD_t^{i,a} = P_t^F - d_t^{CA,H,i,a} - \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$   
 $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
 $dm_{t+1} * i, a = 1$   
**else**  
 $F_{t+1}^{i,a} = 0$   
 $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
 $HD_t^{i,a} = 0$   
 $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
 $dm_{t+1} * i, a = 0$   
 $H_{t+1}^{i,a} = 0$   
**else if**  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
 ...  
**else if**  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
 ...  
**else if**  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
 ...  
**else if**  $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
 ...  
**else**  
 ...

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**Procedure 2** A consumer's decision rule for durable goods: part 2
 

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if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
 ...  
 else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
   if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True)$  :  
      $H_{t+1}^{i,a} = 1$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^H + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False)$  :  
      $H_{t+1}^{i,a} = 1$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a} + d_t^{CA,H,i,a} - P_t^H + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False)$  :  
      $H_{t+1}^{i,a} = 1$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = P_t^H - d_t^{CA,H,i,a} - \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$   
      $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 1$   
   else  
      $H_{t+1}^{i,a} = 0$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
      $F_{t+1}^{i,a} = 1$   
   else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
     ...  
   else if  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
     ...  
   else if  $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
     ...  
   else  
     ...

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**Procedure 3** A consumer's decision rule for durable goods: part 3
 

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if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
 ...  
 else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
 ...  
 else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
   if  $(DRF_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True)$  :  
      $F_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^F + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False)$  :  
      $F_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a} + d_t^{CA,H,i,a} - P_t^F + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRF_1 = True) \wedge (DRF_2 = True) \wedge (DRF_3 = False)$  :  
      $F_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = P_t^F - d_t^{CA,H,i,a} - \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$   
      $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 1$   
   else  
      $F_{t+1}^{i,a} = 1$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
      $H_{t+1}^{i,a} = 1$   
   else if  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
     ...  
   else if  $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
     ...  
   else  
     ...

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---

**Procedure 4** A consumer's decision rule for durable goods: part 4
 

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if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
   ...  
 else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
   ...  
 else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
   ...  
 else if  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
   if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = True)$  :  
      $H_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} - P_t^H + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False)$  :  
      $H_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a} + d_t^{CA,H,i,a} - P_t^H + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $dm_{t+1} * i, a = 0$   
   else if  $(DRH_1 = True) \wedge (DRH_2 = True) \wedge (DRH_3 = False)$  :  
      $H_{t+1}^{i,a} = 2$   
      $d_{t+1}^{CA,H,i,a} = 0$   
      $HD_t^{i,a} = P_t^H - d_t^{CA,H,i,a} - \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a} - d_t^{SA,i,a} \cdot (1+r)$   
      $d_{t+1}^{SA,i,a} = \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 1$   
   else  
      $H_{t+1}^{i,a} = 1$   
      $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
      $HD_t^{i,a} = 0$   
      $d_{t+1}^{SA,i,a} = d_t^{SA,i,a} \cdot (1+r) + \sigma_{sr,t}^{SA,i,a} \cdot \tilde{\Omega}_t^{i,a}$   
      $dm_{t+1} * i, a = 0$   
      $F_{t+1}^{i,a} = 2$   
   else if  $dm_t^{i,a} > 0 \neg (F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
     ...  
   else  
     ...

---

---

**Procedure 5** A consumer's decision rule for durable goods: part 5

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if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $dm_t^{i,a} > 0 \wedge \neg(F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
     $F_{t+1}^{i,a} = F_t^{i,a}$   
     $H_{t+1}^{i,a} = H_t^{i,a}$   
     $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
    if  $dm_t^{i,a} < CD$  :  
         $dm_{t+1}^{i,a} = dm_t^{i,a} + 1$   
    else  
         $dm_{t+1}^{i,a} = 0$   
else  
    ...  
...

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**Procedure 6** A consumer's decision rule for durable goods: part 6

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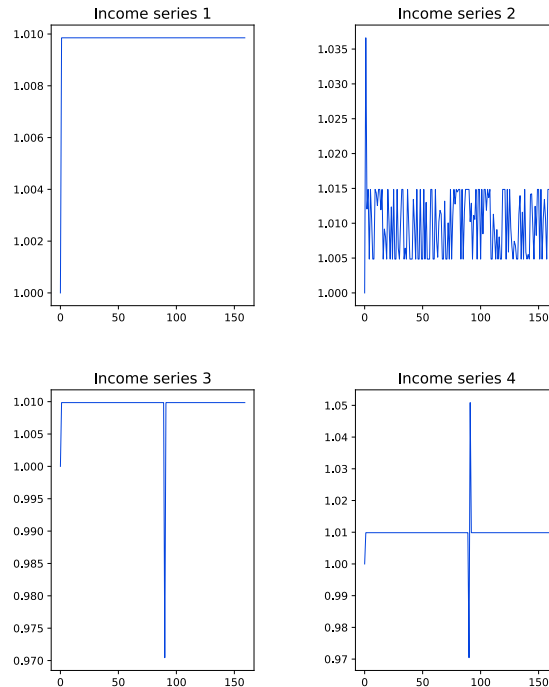
if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 0 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 1 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $F_t^{i,a} = 2 \wedge H_t^{i,a} = 1 \wedge dm_t^{i,a} = 0$  :  
    ...  
else if  $dm_t^{i,a} > 0 \wedge \neg(F_t^{i,a} = 2 \wedge H_t^{i,a} = 2)$  :  
    ...  
else  
     $F_{t+1}^{i,a} = F_t^{i,a}$   
     $H_{t+1}^{i,a} = H_t^{i,a}$   
     $d_{t+1}^{CA,H,i,a} = d_t^{CA,H,i,a} + \tilde{\Omega}_t^{i,a} \cdot \sigma_{sr,t}^{CA,i,a} \cdot \beta^{h,i,a}$   
    if  $dm_t^{i,a} < CD$  :  
         $dm_{t+1}^{i,a} = dm_t^{i,a} + 1$   
    else  
         $dm_{t+1}^{i,a} = 0$   
...

---

for modelling such decisions. On the other hand, algorithmic methods can easily represent such behaviour.

In the presented framework, only retired consumers use savings also for the purchases of nondurable and frequently bought durable goods. These expenditures are subject to the same rules as spending from retirement pensions that the old receive, but the expenditure rates are much lower.

## 5.2. Results: the working-life-cycle consumption of a single consumer



**Fig. 9** – Gross percentage changes of income for the four considered income processes.

For each of the six consumer types two versions of mental accounting are compared, one featuring only weakly nonfungible behavioural rule (with respect to frequently bought durable and nondurable goods), the other characterised by strong nonfungibility. Furthermore, four income processes are considered: the first one with a constant, positive growth rate (equal to  $1.04^{\frac{1}{4}} - 1$ ), the second featuring stochastic shocks to this income growth rate in every period, the third with a single negative shock to the growth rate in the 89th period (with the initial period having the index 0), and the fourth featuring the same shock as the third one, which is counteracted by a

positive shock in the next period, so that income growth returns on the original path. Figure 9 displays the growth rates of the four considered income processes. While they might

All of the series are characterised by a growth or fall of income in each period. Such a representation, of course, does not exactly represent a single worker’s wage dynamics in the real-world, but it serves two purposes. The first is developing a benchmark model, which shares at least some features with the existing models of consumption and remuneration. Second, it may be interpreted as a form of making the analysed agent to some extent an ‘average’ or ‘representative’ individual.

It is worth underlining at this point that the form of the working-life-cycle outcomes for a consumer whose behaviour is characterised by weak nonfungibility is one of many possible. Depending on the adopted values of  $\omega_s^{11}, \omega_s^{12}, \omega_s^{21}, \omega_s^{22}$  parameters, outcomes for such an agent will be more alike to those of category-related results for an agent characterised by strong nonfungibility for which the counterpart parameters have similar values.

### 5.2.1. Constant durable goods prices

Except for variations in the income’s growth rate, the smoothness of a consumer’s expenditure on frequently bought goods is disturbed by credit-taking and the ensuing necessity of repaying debt. Consumer types matter for the timing of the purchases of a flat and a house through different consequences of the agent’s behaviour on the resultant saving rate out of net disposable income. This affects the pace of accumulation of funds on accounts devoted for the purchases of a housing good, vehicles, or other infrequently purchased durable goods.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	45	45	45
Strong fungibility, CP	43	43	48
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	56	33	38
Strong fungibility, CP	56	34	38

**Table 3** – The timing of the purchase of a flat in the single-agent life-cycle model for the first version of the income process, with constant prices.

A single consumer’s life-cycle expenditure is disrupted by taking out debt for the purchases of a flat and a house, and the resulting debt payments in the following periods. They lower the net disposable income of a consumer,



<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	126	126	126
Strong fungibility, CP	124	124	129
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	137	114	119
Strong fungibility, CP	137	115	119

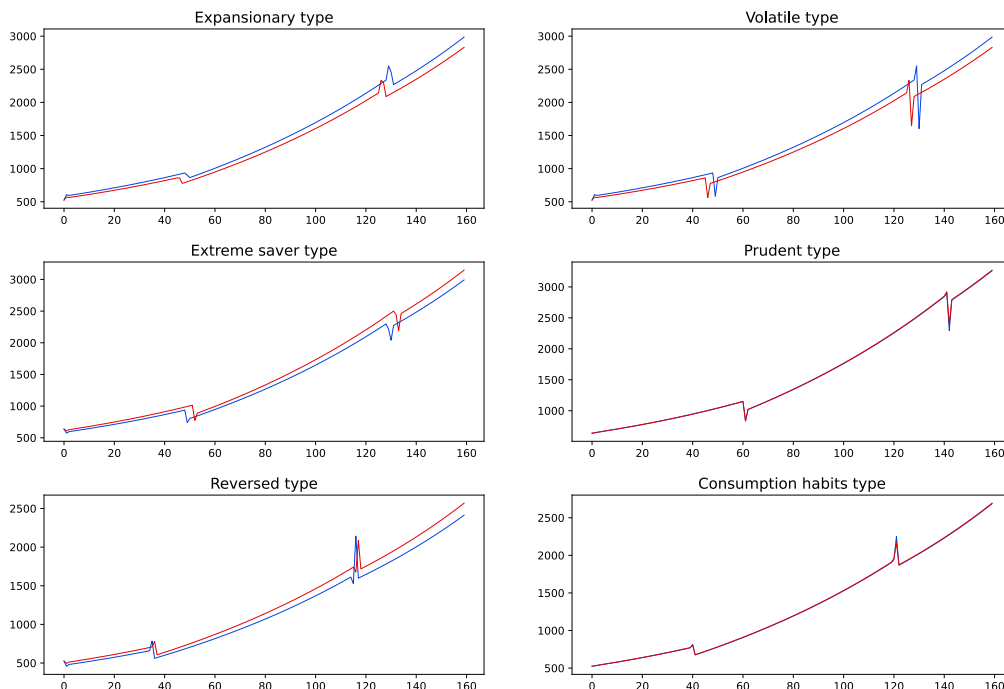
**Table 4** – The timing of the purchase of a house in the single-agent life-cycle model for the first version of the income process, with constant prices.

which is the basis (the reference point) for a consumer’s spending. Thus, consumption tracks income, but the appearance of additional financial burden acts as a shock and alters the consumer’s behaviour. Note, however, that the behavioural change is only temporary and does not last for the entire period of the debt’s duration. Moreover, income growth alleviates the loss of net disposable income. This, of course, is the result of the assumption of positive income growth in every period. The first income series version of the individual life-cycle simulation illustrates clearly the timing and effects of taking out debt for a flat and a house. Purchases of a vehicle and other infrequently bought durable goods do not cause such disturbances because of the assumptions of separate mental accounts and that the consumer does not take out debt for these purchases.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	43	43	46
Strong fungibility, CP	42	42	47
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	55	33	37
Strong fungibility, CP	55	33	37

**Table 5** – The timing of the purchase of a flat in the single-agent life-cycle model for the second version of the income process, with constant prices.

The results for the second income series, featuring a stochastic disturbance in every period, demonstrate that prudent and consumption-habits consumer types smooth out small income changes, or other possible small stimulus, such as firms’ marketing. The consumption of these types does not follow the erratic pattern of income. Moreover, as can be seen from tables 3, 4, 5 and 6, the second income process altered the timing of the flat and house purchases relative to the first case. Conversely, for the third and fourth versions, featuring a one-period shock and a one-period shock with a correction,

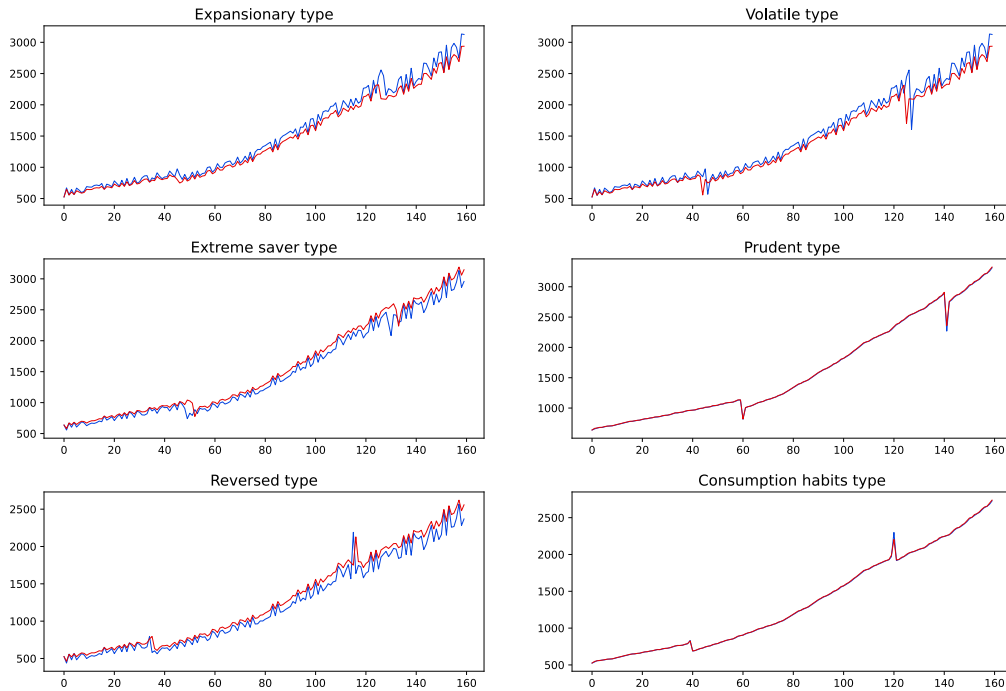


**Fig. 10** – Expenditure in the life-cycle for various types of consumers, for the first version of the income process, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	124	124	127
Strong fungibility, CP	123	123	128
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	124	124	127
Strong fungibility, CP	123	123	128

**Table 6** – The timing of the purchase of a house in the single-agent life-cycle model for the second version of the income process, with constant prices.

respectively, the periods of flat and house purchases are the same as for the constant income growth rate case.



**Fig. 11** – Expenditure in the life-cycle for various types of consumers, for the second version of the income process, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

Comparison of the third and fourth series of income with the first one show that in this framework temporary income shocks have permanent effects on the growth path, but not on its further shape, which remains very similar to the one that characterises the first version, without any disturbances. The impact of the shock, which causes income to decline with a rate equal to minus three times the growth rate, is clearly visible for all consumer types, and remains lower than the effect of the appearance of debt payments, for both flat and house purchases.

In the third and fourth versions of income growth, featuring a transitory one-period shock, in the second case followed by a correction which restores the original growth path, the consumption of all of the frequently bought goods categories follows the same pattern as the counterpart series of the

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	45	45	45
Strong fungibility, CP	43	43	48
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	56	33	38
Strong fungibility, CP	56	34	38

**Table 7** – The timing of the purchase of a flat in the single-agent life-cycle model for the third version of the income process, with constant prices.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	126	126	126
Strong fungibility, CP	124	124	129
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	137	114	119
Strong fungibility, CP	137	115	119

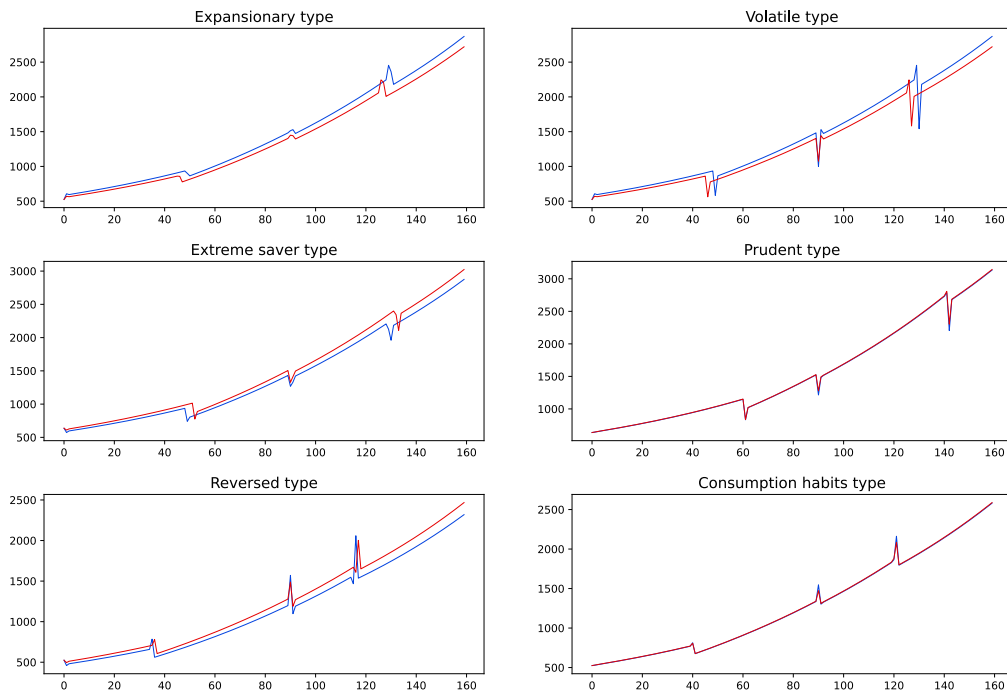
**Table 8** – The timing of the purchase of a house in the single-agent life-cycle model for the third version of the income process, with constant prices.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	45	45	45
Strong fungibility, CP	43	43	48
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	56	33	38
Strong fungibility, CP	56	34	38

**Table 9** – The timing of the purchase of a flat in the single-agent life-cycle model for the fourth version of the income process, with constant prices.

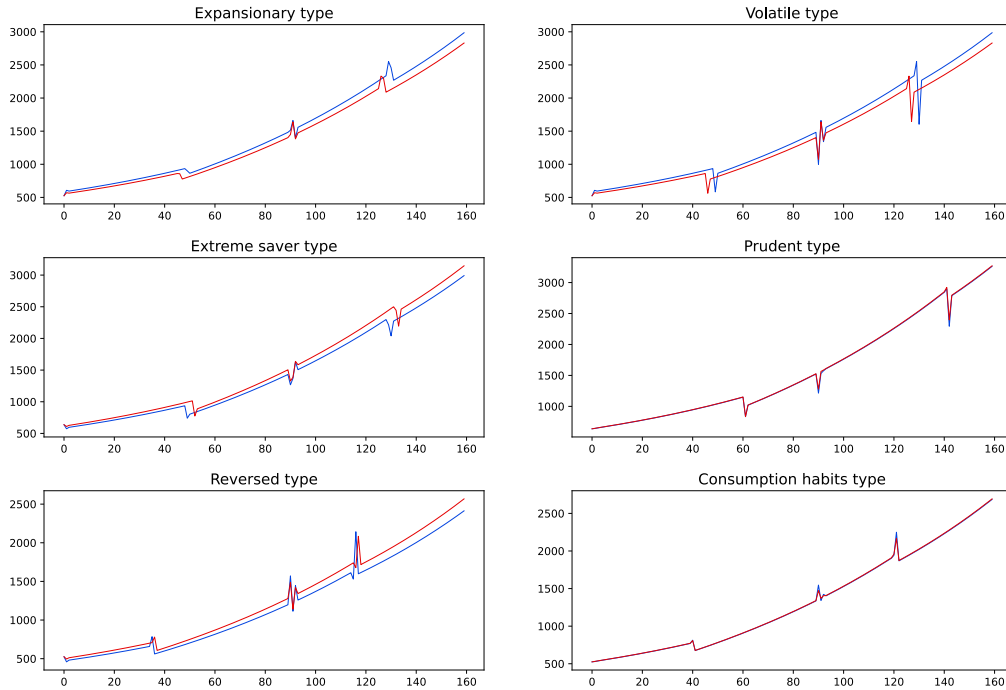
<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, CP	126	126	126
Strong fungibility, CP	124	124	129
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, CP	137	114	119
Strong fungibility, CP	137	115	119

**Table 10** – The timing of the purchase of a house in the single-agent life-cycle model for the fourth version of the income process, with constant prices.



**Fig. 12** – Expenditure in the life-cycle for various types of consumers, for the third version of the income process, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

first version. The only exceptions are the periods of the disturbance and the correction ( $t = 89$  and  $t = 90$ ).



**Fig. 13** – Expenditure in the life-cycle for various types of consumers, for the fourth version of the income process, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

Consumer types differ with respect to their spending rates on nondurable and frequently bought durable goods. This affects their saving rates and delays or accelerates their spending on timing of flat, house, vehicle and other durable goods purchases. The results of the single worker's working-life-cycle show that for small and smooth changes of income and no growth of prices of infrequently bought durable goods, the differences between weak and strong nonfungibility are very small, except for a few periods following taking out debt. In the stochastic income case, the differences in expenditure are slightly more pronounced.

### 5.2.2. Growing durable goods prices

As can be seen from tables 11, 12, 13, 14, 15, 16, 17 and 18, rising prices of durable goods not only delay the purchases of a flat and a house, but also increase the differences in their timing between consumer types.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	67	67	67
Strong fungibility, VP	63	63	72
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	88	44	53
Strong fungibility, VP	88	46	53

**Table 11** – The timing of the purchase of a flat in the single-agent life-cycle model for the first version of the income process, with growing durable goods’ prices.

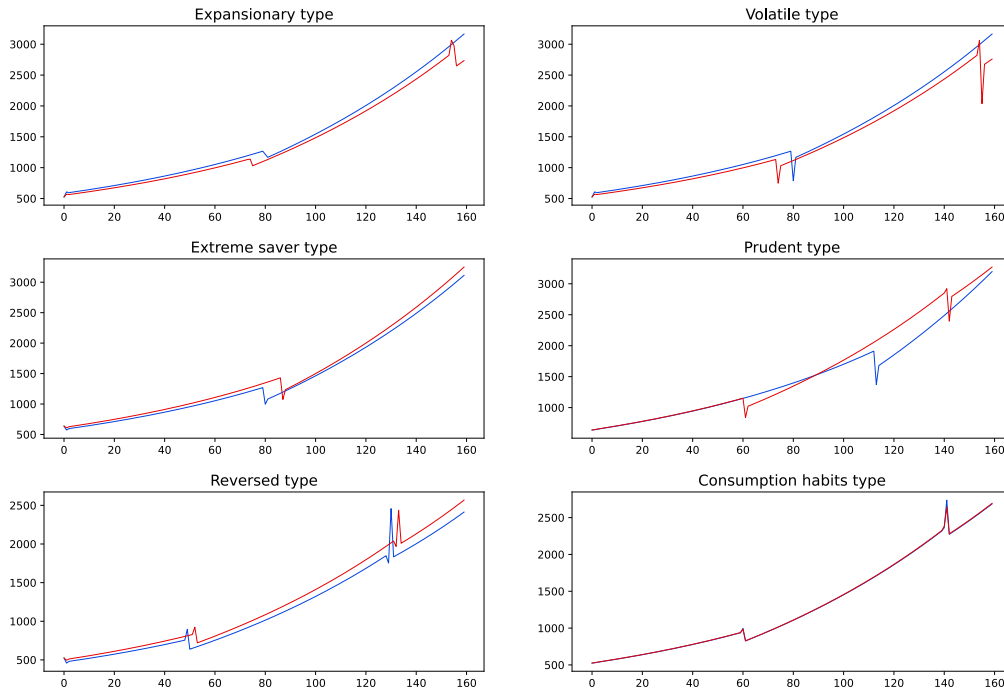
<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	148	148	148
Strong fungibility, VP	144	144	153
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	na	125	134
Strong fungibility, VP	na	127	134

**Table 12** – The timing of the purchase of a house in the single-agent life-cycle model for the first version of the income process, with growing durable goods’ prices.

A single consumer’s life-cycle expenditure in an environment with growing prices of flats, houses, vehicles and other durable goods is shifted relative to the model with constant durable goods’ prices. Moreover, the burden of debt payments causes larger drops in the consumption of nondurable and frequently bought durable goods, and lasts longer than in the versions without price growth. This causes significant disruptions in the path of category-related expenditures. Thus, it is argued that major changes to the life-cycle pattern of consumer expenses are caused by debt repayments.

Moreover, as can be observed by comparing the figures illustrating the output of the single-consumer working-life-cycle model featuring increasing prices of durable goods with its counterpart characterised by constant prices, in the presence of growing prices of expensive, infrequently-bought nondurable goods, the differences between weakly and strongly nonfungible

consumer behaviour become more visible for all consumer types but the consumption habits one. This feature is present for all four income processes.



**Fig. 14** – Expenditure in the life-cycle for various types of consumers, for the first version of the income process, with growing durable goods’ prices, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

The results of the single worker’s working-life-cycle show that for small and smooth changes of income and with increasing infrequently-bought durable goods prices, the differences between consumer expenditure under weak and strong nonfungibility are more pronounced than in the constant prices scenario. Thus, the discernment between the two notions is not negligible and strong nonfungibility may be a considerable cause for the differences in consumers’ propensities to consume for various categories of goods.

The differences in expenditure between consumers exhibiting weak and strong nonfungibility of funds from different, category-specific, accounts vary depending on how close are the parameters  $\omega_s^{11}, \omega_s^{12}, \omega_s^{21}, \omega_s^{22}$  of the strongly nonfungible behavioural rule to the weakly nonfungible counterpart. The reader is directed to appendix A to examine these values; of course,



<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	61	61	67
Strong fungibility, VP	59	59	70
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	85	43	50
Strong fungibility, VP	85	45	50

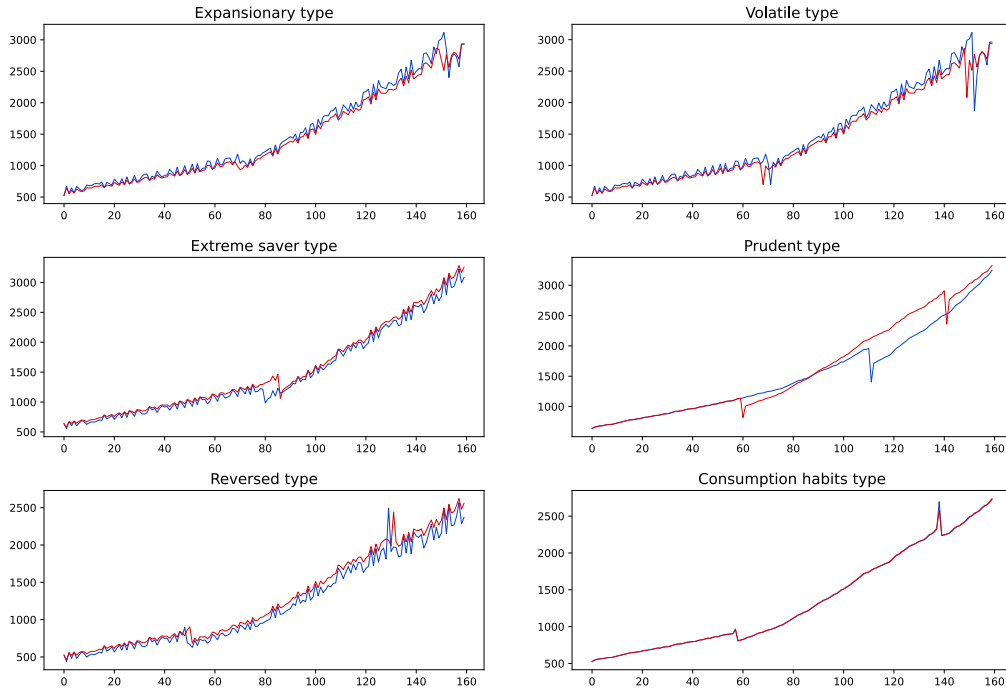
**Table 13** – The timing of the purchase of a flat in the single-agent life-cycle model for the second version of the income process, with growing durable goods’ prices.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	142	142	148
Strong fungibility, VP	140	140	na
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	na	124	131
Strong fungibility, VP	na	126	131

**Table 14** – The timing of the purchase of a house in the single-agent life-cycle model for the second version of the income process, with growing durable goods’ prices.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	67	67	67
Strong fungibility, VP	63	63	72
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	88	44	53
Strong fungibility, VP	88	46	53

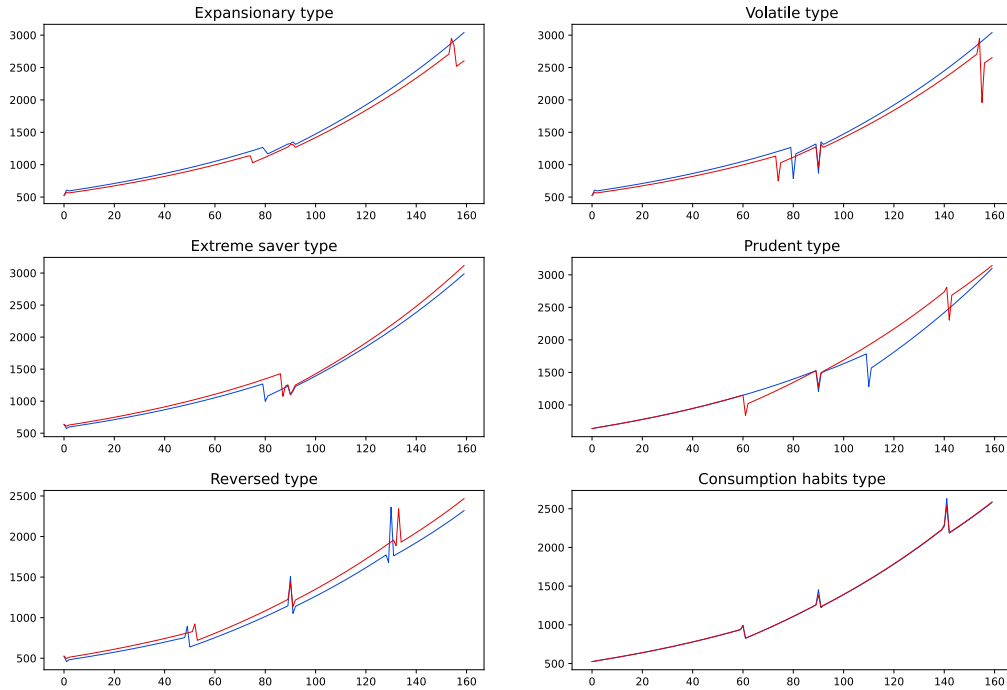
**Table 15** – The timing of the purchase of a flat in the single-agent life-cycle model for the third version of the income process, with growing durable goods’ prices.



**Fig. 15** – Expenditure in the life-cycle for various types of consumers, for the second version of the income process, with growing durable goods’ prices, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	148	148	148
Strong fungibility, VP	144	144	153
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	na	125	134
Strong fungibility, VP	na	127	134

**Table 16** – The timing of the purchase of a house in the single-agent life-cycle model for the third version of the income process, with growing durable goods’ prices.



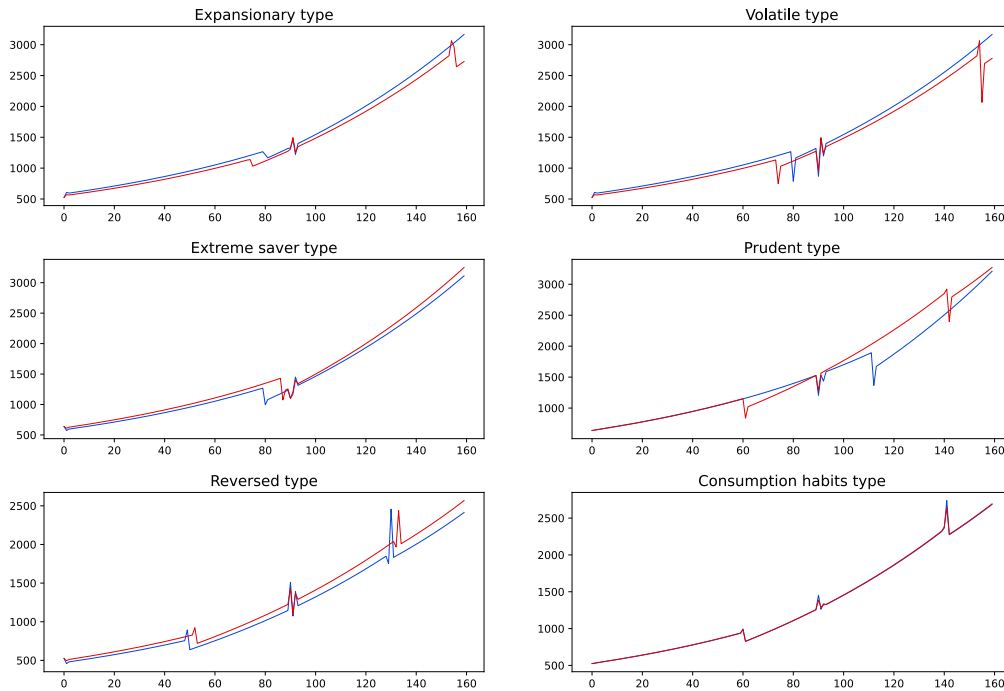
**Fig. 16** – Expenditure in the life-cycle for various types of consumers, for the third version of the income process, with growing durable goods’ prices, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	67	67	67
Strong fungibility, VP	63	63	72
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	88	44	53
Strong fungibility, VP	88	46	53

**Table 17** – The timing of the purchase of a flat in the single-agent life-cycle model for the fourth version of the income process, with growing durable goods’ prices.

<i>Version</i>	<i>Expansionary</i>	<i>Volatile</i>	<i>Extreme saver</i>
Weak fungibility, VP	148	148	148
Strong fungibility, VP	144	144	153
<i>Version</i>	<i>Prudent</i>	<i>Reversed</i>	<i>Consumption habits</i>
Weak fungibility, VP	na	125	134
Strong fungibility, VP	na	127	134

**Table 18** – The timing of the purchase of a house in the single-agent life-cycle model for the fourth version of the income process, with growing durable goods’ prices.



**Fig. 17** – Expenditure in the life-cycle for various types of consumers, for the fourth version of the income process, with growing durable goods’ prices, category: sports equipment. Starting from the upper left corner, the results for the following consumer types are displayed: expansionary, volatile, extreme saver, prudent, reversed, consumption habits. The horizontal axis shows time periods interpreted as quarters, while the vertical one displays the value of expenditures. Red lines show the results for strongly nonfungible behaviour while blue lines display the results for the weakly nonfungible version.

weakly nonfungible rule's parameters (recall section 3) could have been chosen so that the results for both kinds of nonfungibility were very different for all categories, but that was not the point of the study. Although the discrepancies may seem small for certain consumer types and some categories (not displayed for the sake of parsimony), these discrepancies will accumulate in a macroeconomic context with many heterogeneous agents, especially if consumers will face also market-specific stimulus (such as marketing) affecting their decisions in a fashion similar to changes of net disposable income.

The consumer expenditure model's dynamics are consistent with many facts that are unaccounted for by the permanent income model but were documented by microeconomic studies of consumption. Consistently with the findings of Zeldes (1989), debt-taking and repayment of loans affect the paths of consumption of goods from all categories of frequently bought goods. The presented model is also consistent with the implications of the study of Stephens (2003), i.e. the arrival of new income, higher than the past one, increases the level of consumption of frequently purchased products. Consistently with evidence provided by Stephens (2008), the modelled consumer reacts to the increase in discretionary income caused by paying off a loan and the disappearance of debt repayments. In line with the results of the study of D. S. Johnson et al. (2006), category-enhanced mental-accounting consumer framework does not feature forecasting future income or any anticipatory response to its variation. The finding of Parker (2017), that predictable changes in income have a significant impact on consumption at the time of their realisation, is a feature characterising the categorisation-enhanced mental-accounting consumer framework. It is manifested through the effect the non-stochastic parts of income series have on consumer's decisions (through the division-of-funds variables  $\beta_t^s$ ) and behaviour, i.e. expenditure changes that are not a result of smoothing, but a compound of behavioural reactions and income changes.

In reality, an individual does not (or not always) spend on some of the categories of goods that belong to the frequently bought goods in every quarter. The framework presented in this paper is an approximation of the actual process. Extensions, for example featuring infrequent purchases for more categories of goods are possible, but some degree of parsimony was sought in this paper.

## 6. Conclusions

It was demonstrated in this paper that mental accounting oriented at objects and categories of expenditure cannot be credibly represented using any

intertemporal optimisation approach. The notions of strong and weak non-fungibility were introduced, and the behavioural life-cycle model was shown to satisfy the principle of nonfungibility only in the weak sense. Intertemporal optimisation either leads to underdetermination of the variables constituting the consumer problem or to breaking the principle of nonfungibility (in the strong or weak sense), or both.

For the purpose of developing a theoretical model of category-of-goods mental accounting consumer expenditure, mental-accounting theory was merged with categorisation theories, following the call of Henderson and Peterson (1992). The resultant framework is characterised by procedural and limited rationality; objective computational, informational and cognitive limitations of consumers are mitigated by the categorisation and mental accounting behaviour. This facilitates the decision process, and thus is beneficial for a decision-maker.

The crucial features of the devised framework are disposable income, category-related mental budgets and division-of-funds variables, which determine how much of each budget is spent. A classification of six basic consumer types was devised, basing on the differences of changes of expenditure in response to variations of net disposable income and other possible stimuli. Individual spending rates out of each mental budget are time-variable, but the distinct frequency of purchases of durable and nondurable goods observed in real economies necessitate different decision processes for products that are bought infrequently, especially if such purchases are often supported by large amounts of consumer credit or housing loans. A comprehensive approach for modelling such decisions and debt-taking was devised.

The presented single-agent models of consumer behaviour are consistent with microeconomic evidence. A single consumer's working-life-cycle expenditure on nondurable and frequently bought durable goods tracks income, but is disrupted by debt taking and repayment of past loans. Moreover, if a consumer's income is subject to changes of various magnitude, his/her category-related expenditure may be amplified beyond income variations, or smoothed out, depending on a consumer's type.

Microeconomic and experimental empirical applications of the framework may be based solely on the estimation of the parameters of the behavioural division-of-funds variables. As for macroeconomic applications, some assumptions on the form of income and debt-taking distributions will have to be made. The development of such methods is out of the scope of this work.

This framework offers vast possibilities of theoretical investigations of issues that cannot be addressed using analytical methods due to the curse of dimensionality and the limitations of representations of behaviour based on intertemporal optimisation. For example, the investigation of the role of

demand in business cycle fluctuations on various markets and in economic growth is enabled by the devised approach. Moreover, the question of what impact of heterogeneity expressed as an age-income distribution on the behaviour of aggregate consumption are the natural questions that can be addressed using this framework. The devised decision rules of consumers may be further modified, for instance by investigating the possibility that a consumer's type changes throughout the life-cycle.

The author has also applied the framework in a multi-market income distribution overlapping generations model with multiple age cohorts, not presented in the current paper due to space limitations, which, among else, demonstrates that seemingly inconsistent microeconomic and macroeconomic evidence on consumption are reconciled thanks to categorisation-enhanced mental-accounting consumer framework. The results show the emergence of endogenous cyclical fluctuations on the housing market and strongly indicate that mental-accounting demand of heterogeneous agents with different spending rates is a significant factor augmenting the rate of economic growth, even if the range of variability for a single consumer is very small. While this investigation is beyond the scope of this paper, it shows that the framework has the potential of enriching behavioural agent-based investigations, which are oriented at the study of complex systems, and therefore require the analysis of models of various levels of richness of economic structure.

## Data and code availability

A detailed pseudocode is provided in the paper; it suffices to reproduce the presented framework. Original series of shocks used for the demonstration of the model's basic dynamics are available upon request from the author.

## Declarations of interest

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## A. The sizes of category-related mental accounts and reactions of expenditure rates to stimulus

Given the minimum values of the saving rates  $\sigma_{sr,min}^{CA,i} = 0.1$ ,  $\sigma_{sr,min}^{SA,i} = 0.05$ , we have that

$$\sum_s \eta_s = 1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i} \quad (57)$$

In this paper, for exposition purposes, the following assumptions were made:

$$\eta_b = \frac{2}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (58)$$

$$\eta_{seq} = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (59)$$

$$\eta_t = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (60)$$

$$\eta_{cl} = \frac{3}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (61)$$

$$\eta_{mfs} = \frac{2}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (62)$$

$$\eta_{fb} = \frac{5}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (63)$$

$$\eta_{csm} = \frac{2}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (64)$$

$$\eta_{cs:ent} = \frac{3}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (65)$$

$$\eta_{cs:sp} = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (66)$$

$$\eta_{cs:hc} = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (67)$$

$$\eta_{cs:tr} = \frac{3}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (68)$$

$$\eta_{cs:sti} = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}), \quad (69)$$

$$\eta_{cs:edu} = \frac{1}{26}(1 - \sigma_{sr,min}^{CA,i} - \sigma_{sr,min}^{SA,i}). \quad (70)$$

Furthermore, it was assumed that the sum of account-specific saving rates, corresponding to no change in net disposable income and no other stimuli, equals  $\sigma_{sr,min}^{CA,i} + \sigma_{sr,min}^{SA,i}$ , and that the proportions of these saving rates are equal to the ratios of the corresponding mental accounts, i.e.

$$\sigma_{sr,0}^b = \frac{2}{26}(\sigma_{sr,min}^{CA,i} + \sigma_{sr,min}^{SA,i}), \quad (71)$$

etc.

For the weak fungibility demonstrations and simulations it was assumed that for all categories  $s$  the coefficients  $\omega_s$ , described in the presentation of different possible consumer types in section 4, are all equal to 0.5. As for the strong nonfungibility versions, the values of  $\omega_s$  coefficients are given in table 19 (recall that  $\omega_s^{12} = 1 - \omega_s^{11}$ ,  $\omega_s^{22} = 1 - \omega_s^{21}$ ).

<i>Account</i>	Value of $\omega_s^{11}$ and $\omega_s^{21}$
<i>b</i>	0.5
<i>seq</i>	0.1
<i>t</i>	0.51
<i>cl</i>	0.3
<i>mds</i>	0.4
<i>fb</i>	0.6
<i>csm</i>	0.7
<i>cs : ent</i>	0.15
<i>cs : sp</i>	0.8
<i>cs : hc</i>	0.05
<i>cs : tr</i>	0.25
<i>cs : sti</i>	0.35
<i>cs : edu</i>	0.75

**Table 19** – Values of  $\omega_s^{11}$  and  $\omega_s^{21}$  for all categories of frequently-bought nondurable and durable goods.