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International information flows, sentiments and cross-country business cycle fluctuations*

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Abstract

Business cycles are strongly correlated between countries. One possible explanation (beyond traditional economic linkages like trade or finance) is that consumer or business sentiments spread over boarders and affect cyclical fluctuations in various countries. We first lend empirical support to this concept by showing that sentiments travel between countries at a speed much higher than can be explained by traditional linkages. Then we construct a two-economy new Keynesian model where noisy international information can generate cyclical fluctuations (comovement of GDP, consumption, investment and inflation) in both countries. Estimation with US and Canadian data reveals a significant role of international noise shocks in generating common fluctuations - they explain between 15-30% of consumption variance in the US and Canada and raise the correlation between these variables by up to unity in periods of sentiment breakdowns. We also show that our estimated noise shock has a clear interpretation as a sentiment shock.

JEL: C32, E32, F44

Keywords: International spillovers, animal spirits, sentiments, business cycle

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1 Introduction

Three findings from the earlier literature form the founding stones of our paper. First, business cycle fluctuations can result from (possibly nonfundamental) swings in confidence, related inter allia to flows of imperfect information. Second, business cycles (all main macroeconomic variables) are strongly correlated between countries. Third, in spite of much effort (and some progress) we still lack a good explanation of international business cycle spillovers being as high as observed in the data. Our paper tries to help out. First, we show on empirical grounds that shocks to sentiments travel across borders at a speed that leaves information flows as the most likely transmission channel. Then we construct a two-economy structural model where business cycle fluctuations can i.a. result from imperfect (noisy) information flows. This information is assumed to have a global character and can, thus, affect both economies. Estimation with US and Canadian data reveals that noisy information can indeed contribute to correlated business cycle fluctuations. Below we elaborate on the above in more detail.

Let us start with swings in sentiments (also called moods, confidence or animal spirits) and their impact on economic fluctuations. The idea is by far not new, already Pigou (1927) and Keynes (1936) postulated that waves of optimism or pessimism might influence current economic conditions. More recently this concept found support in a number of empirical and theoretical studies. It must be admitted upfront that neither from the theoretical nor from the measurement side the concept of sentiments has found a unique definition. Several approaches have been introduced and we do not attempt to systematize them.[1] On the theoretical front Beaudry and Portier (2004) were the first to consider a signal about future technology and show in an RBC framework that it causes a boom in investment and consumption. Eusepi and Preston (2011) develop a model that departs from the rational expectations assumption towards learning, in which self-fulfilling expectations arise in response to technology shocks. Angeletos and La'O (2013) provide a model in which limited communication between agents provides an environment in which shocks to believes (sentiments) have real effects that resemble boom-bust phenomena. Blanchard et al. (2013), to which our paper comes closest, derive sentiment-related business cycle fluctuations from noisy signals about productivity.

Several other papers approached the topic from an empirical perspective. Beaudry and Portier (2006) document the existence of a shock (derived from stock price data) that causes a boom in investment and consumption and significantly precedes the growth of productivity (and thus resembles their earlier theoretical concept). Kamber et al. (2017) estimate VAR models for four economies and document that technology news shocks explain between 6%

[1] Consequently we will use the terms "confidence" and "sentiments" interchangeably.

and 40% of output fluctuations. Barsky and Sims (2011) estimate a new Keynesian model that allows for technology news shocks and show that their contribution to explaining the variance of consumption and investment, while negligible in the short run, increases to 50% in the long run. Milani (2017) estimates a general equilibrium model with learning and shows that sentiment fluctuations are responsible for over 40% of cyclical fluctuations in the US. Levchenko and Pandalai-Nayar (2019) estimate the spillover of confidence shocks between the US and Canada. They identify sentiment shocks in a VAR model and show that the Canadian business cycle is driven to a large extent by US sentiments. In a similar approach Brzoza-Brzezina and Kotłowski (2018) show that euro area sentiment shocks strongly affect the business cycle in Poland. Moreover, this paper shows that approximately half of this effect is due to transmission unrelated to economic linkages between the euro area and Poland, but is most probably due to information spillovers.[2]

The second issue does not require a long introduction. It is common knowledge that business cycles comove strongly between countries. For instance DeGrauwe and Ji (2017) report the correlation of the cyclical component of GDP to be on average 0.8 between euro area member countries and 0.6 between other OECD countries. Only slightly lower numbers are reported by Gong and Kim (2018) for groups of East Asian, Central European and Latin American countries. Not only GDP but also its main components (investments, consumption) fluctuate together, so do also inflation and main financial variables (money, credit, stock prices).

Last but not least there is the question to what extent existing theories and models can explain this comovement. Some explanations exist: trade links have been considered the main culprit for many years. After the global financial crisis another explanation - financial linkages - has found large support. Technological spillovers could be considered another reason. Not questioning the role of these factors they are most probably not able to explain as much comovement as we observe in the data. For instance, according to Gong and Kim (2018) not more than 15% of the observed correlation can be explained by trade and financial linkages. Structural models that explicitly take into account international trade explain only a tiny fraction of business cycle fluctuations in small open economies to shocks stemming from their large neighbors (see e.g. Justiniano and Preston, 2010). Adding financial linkages improves the situation only partially (Olivero, 2010).

As mentioned above, in this paper we foster the idea that business cycle comovement can be also generated by international information flows that affect sentiments in various countries and, as a consequence, makes their cycles comove. We proceed as follows. First,

[2] Several other papers could (and should) be mentioned in this section, e.g. Jaimovich and Rebelo (2009), Schmitt-Grohe and Uribe (2012), Beaudry et al. (2011), Angeletos et al. (2014), Hürtgen (2014) or DeGrauwe and Ji (2017).

we provide empirical evidence that sentiment shocks indeed travel at high speed between countries. To that end we calculate the phase shift of sentiment indicators for a panel of 28 EU countries and show that for most the shift of confidence vs. the euro area is faster than the shift of GDP. Having found this we construct a structural DSGE model in the spirit of Blanchard et al. (2013), which however, in contrast to the original paper, comprises two economies. In this model noisy information about productivity is assumed to be of global nature. Consumers in both economies attempt to disentangle the signal. Mistakes in doing this result in nonfundamental fluctuations in demand in both economies. To the best of our knowledge this is the first attempt to incorporate and estimate spillovers resulting from international sentiment flows in a structural macroeconomic framework, and this is where we see our biggest contribution to the literature.

The model is then estimated on US and Canadian data. We show that: (i) shocks to US noise generate comovement in main macroeconomic variables (GDP, consumption, investments, inflation) - after a positive noise shock all increase and return to baseline only when agents discover the true nature of the shock, (ii) their role is particularly significant for consumption, on average15%-30% of consumption variance can be explained with US noise shocks in Canada and in the US respectively. However, more importantly the role of noise is not uniformly spread over time. Periods of relative tranquility are interrupted by sharp sentiment breakdowns. It is during these periods that noise generates strong comovement between US and Canadian consumption. For instance bad information shocks deepened the slowdown in consumption during the oil shocks or the global financial crisis in both economies and (iii) the time series of noise shocks can be interpreted in terms of consumer sentiment shocks: periods of relative tranquility were interrupted by noise (sentiment breakdowns) i.a. after oil shocks, the Gulf war, dot-com bubble or financial crisis.

The rest of the paper is organized as follows. Section 2 presents the empirical evidence on sentiment spillovers, Section 3 discusses the model and its estimation, Section 4 the results, Section 5 offers a number of explanatory exercises and robustness checks and Section 6 concludes.

2 Empirical evidence on sentiment spillovers

In this Section we show that spillovers of sentiment across countries are faster and of larger magnitude than GDP spillovers. For that purpose we analyze the co-movements of confidence and business cycles between the euro area and the individual member states of the European Union and investigate whether the cross-border transmission of confidence follows or leads the co-movement of business cycles. We focus on European Union (EU) countries (instead of the US-Canada pair for which the structural model will be estimated) for two reasons. First the results based on the panel of 28 countries are more robust than the findings related to a pair of countries only. Second, when focusing on EU countries we can use a comprehensive and unified measure of sentiments based on the survey conducted among consumers and firms from different sectors using the same methodology for all UE member states.

On the other hand the estimation of the DSGE model in Section 3 requires long data series for a pair of two (large and small) economies. Such series are available for the US-Canada pair. This explains our choice of data for the two estimation exercises.

2.1 Spectral analysis

We identify the time lags in the spillovers of confidence and business cycles between the euro area and individual EU member states and the relative strength of these spillovers using spectral analysis tools. We use quarterly data for 28 member states of the European Union and for the euro area economy as a whole. The measure of countries' economic activity is real GDP. The confidence of economic agents is represented by the Economic Sentiment Indicator (ESI) published on monthly basis by the European Commission. The ESI is a composite indicator, which covers all main sectors of the economy and is computed using surveys conducted among both producers and consumers.[3] We aggregate monthly observations to quarterly data to work with time series comparable with GDP figures. The length of the times series differs across countries. For most economies the sample for ESI and GDP data starts in Q1 1995, but some countries joined the ESI survey later. Also the GDP time series for selected countries begins at a later date. All time series run through Q2 2019.

While examining spillovers we treat the euro area as a reference point. We do not exclude that shocks to confidence and to economic activity move not only from the euro area to individual EU economies but also in the opposite direction. We expect that the time lags in GDP and ESI spillovers for the largest euro area economies (Germany, France, Italy) towards the euro area should be close to zero since these countries contribute to the largest extent to the euro area aggregate.

We use spectral analysis tools to investigate the strength and the speed of spillovers. Let $\{y_t\}_{t=-\infty,\infty}$ and $\{x_t\}_{t=-\infty,\infty}$be a pair of zero mean, covariance-stationary time series processes. The cross-spectrum for processes y_t and x_t is defined as (Hamilton, 1994):

$$s_{yx}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \lambda_k^{yx} \left(cos(\omega k) - i \cdot sin(\omega k) \right) \text{ for } \omega \in [-\pi, \pi], \tag{1}$$

where $\lambda_k^{yx} = Cov(y_t, x_{t-k})$.

[3]The detailed methodological guidelines are published on the European Commission website: https://ec.europa.eu/info/files/user-guide-joint-harmonised-eu-programme-business-and-consumer-surveys_en

The cross-spectrum can be decomposed into real and imaginary components as follows:

$$s_{yx}(\omega) = c_{yx}(\omega) + i \cdot q_{yx}(\omega), \tag{2}$$

where $c_{yx}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\lambda_k^{yx}cos(\omega k)$ is called co-spectrum between x_t and y_t while the element $q_{yx}(\omega) = -\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\lambda_k^{yx}sin(\omega k)$ is the quadrature spectrum from x_t to y_t. The co-spectrum at frequency ω can be interpreted as the portion of covariance between x_t and y_t attributed to common cycles for given frequency ω. The quadrature spectrum refers also to the covariance between processes x_t and y_t but with one of them shifted in cycle (Hamilton, 1994). We examine the strength as well as direction and time lags in spillovers using two measures popularized by Sargent (1987) and widely used in the empirical literature on business cycles synchronization (see e.g. Skrzypczyński, 2010, Skrzypczyńska, 2014): coherence and phase shift. Coherence is defined as:

$$K_{yx}^2 = \frac{c_{yx}^2(\omega) + q_{yx}^2(\omega)}{s_{yy}(\omega)\, s_{xx}(\omega)} \text{ for } \omega \in [-\pi, \pi] \tag{3}$$

and measures the strength of the contemporaneous relationship between y_t and x_t for cycles of frequency ω. Coherence is the degree of fit in the regression of y_t on x_t at frequency ω analogously to R-squared in the time domain. It assumes values between 0 and 1 for covariance-stationary processes.

The phase shift is defined as:

$$\phi_{yx} = arctan\left(\frac{q_{yx}(\omega)}{c_{yx}(\omega)}\right) \text{ for } \omega \in [-\pi, \pi]. \tag{4}$$

and determines the shift (lead or lag) in cyclical fluctuations of y_t relative to x_t at frequency ω. A negative (positive) value of ϕ_{yx} means that the cycle for x_t leads (lags) the cycle for y_t.

The sample counterpart of the cross-spectrum is a cross-periodogram, for which the theoretical covariances between x_t and y_t in (1) are substituted by the sample covariances. A consistent estimation of the cross-spectrum in a finite sample requires also the sequence of covariances in (1) to be truncated and smoothed with a set of weights called lag window. In our work we use a rectangular lag window called Daniell window (see Priestley, 1981 for details)[4].

[4] We use the modified double Daniell smoother with m=5, which places half weights at the end points.

2.2 Results

We analyze spillovers of the cyclical components of GDP and economic sentiment indicator instead of raw data to filter out the noise related to low frequencies. Therefore we first extract the cyclical components of GDP and ESI using the Christiano-Fitzgerald asymmetric band-pass filter (Christiano and Fitzgerald, 2003). We isolate the cyclical components, which refer to the cycles from 1.5 to 10 years. For GDP time series we account for a unit root and a drift. We also assume a unit root for ESI data. Figure 1 shows the co-movement of business cycles for the European Union member states. We plot the cyclical fluctuations of GDP in the EU countries in the form of the inter-percentile 80% central range of GDP cyclical components extracted individually for all 28 EU economies. We find that the business cycle synchronization is moderate with two clear peaks in the common cycle: at the break of 1997/1998 and in 2007. The latter peak ends the long lasting Great Moderation period, which was accompanied by the ongoing globalization process and growing housing markets. The most severe trough occurs during the global financial crisis in 2008/2009 which affected strongly and rapidly all EU economies. The second deep trough in 2011/2012 is related to the onset of the sovereign debt crisis in the euro area.

The cyclical fluctuations of the economic sentiment indicator plotted on Figure 2 are more volatile than GDP fluctuations. The most severe decline in confidence was also caused by the global financial crisis. However as opposed to the relatively slow and moderate rebound of economic activity the improvement in confidence was substantial and the cyclical component of ESI achieved its pre-crisis level in 2011. Two other apparent peaks in sentiment fluctuations, which occur in years 1997 and 2001 are also larger than the peaks of the business cycles. The timing of both peaks is however rather consistent with the timing of corresponding peaks in the GDP fluctuations.

Next we calculate the coherence statistics as defined by (3) to measure the contemporaneous correlation between GDP and ESI cyclical fluctuations for each country with reference to the euro area as a whole. We focus on aggregated coherence measures averaged over all examined frequencies (Figure 3). For 18 economies the strength of the contemporaneous correlation with the euro area is higher for the fluctuations of economic sentiment than for the GDP movements. For the remaining economies the coherence of GDP exceeds the ESI coherence by a small margin. The average value of ESI coherence amounts to 0.63 while average coherence for GDP is equal to 0.56. The stronger contemporaneous correlation between the swings in confidence allows to presume that confidence spreads across countries through an additional channel than only via GDP fluctuations. If the sentiment spillovers would be secondary to spillovers of economic activity occurring via trade or financial channels the contemporaneous correlation of sentiment would be at most as large as the correlation of GDP.

Next, we compare the leads and lags in the spillovers of sentiment and GDP fluctuations. Accordingly we calculate the phase shifts in the cycles, averaging over all frequencies and weighting by the value of coherence for the given frequency. Hence we impose higher weights to the frequencies for which the strength of the interdependence is higher. The estimated leads or lags are collected in Table 1. We find that the business cycles for most EU member states lead the cycle in the euro area. A small number of countries, in particular some new member states, lag the euro area cycle. The standard deviation of the shift amounts to 2.9 months. The dispersion of sentiment cycles is significantly lower - the standard deviation of the shift equals 1.9 months, which means that sentiment cycles are more synchronized with the euro area.

Finally, we investigate the relative shifts in GDP and sentiment cycles. For countries for which the GDP cycle lags the euro area we calculate the relative shift as the difference between the lag in the GDP cycle and the shift in the sentiment cycle. For countries which lead the euro area cycle the relative shift is calculated as the difference between the shift for the sentiment cycle and for the GDP cycle. Such measure takes positive (negative) values if the leads or lags towards the euro area are smaller (larger) for sentiment spillovers than for the transmission of GDP cycles. In other words a positive value indicates that sentiment spills over faster than economic activity does (from the euro area to the given country or in the opposite direction depending on the sign of the phase shift in GDP cycles). We plot the results on Figure 4. For 22 out of 28 EU economies the relative shift in spillovers of GDP fluctuations and the sentiment fluctuations is positive, for 15 of them it exceeds one month. Only for six EU economies is the lag in sentiment transmission higher than the lag in GDP spillovers.

These results lead to the conclusion that for most of the EU economies sentiment spillovers are faster than business cycle spillovers. Such outcome when combined with findings stemming from the comparison of the coherence statistics discussed above lends support to the existence of an additional channel of cross-border sentiment transmission, which acts faster and stronger than transmission of business cycles via traditional trade or financial linkages. Some of the sentiment is probably transmitted via media before the real effects of the transmission via trade or financial linkages materializes.

3 Model, data and estimation

3.1 Model

We construct a two-country DSGE model. Most features of the model are standard for the literature that followed Smets and Wouters (2003). In each economy there are infinitely lived households, final good producers, capital good producers, exporters and intermediate goods producers. There is a number of real and nominal frictions, including habit formation, investment adjustment costs, sticky wages and prices that have been broadly used to bring this class of models closer to the data. Each economy runs independent monetary policy, given by a Taylor-type rule and the exchange rate is flexible. In both economies there is a government with an exogenous spending pattern. As the framework is standard we delegate its detailed description to Appendix A. Here we concentrate on the non-standard feature of the model, related to imperfect information about technology.

Agents in both economies face a signal extraction problem concerning productivity of intermediate goods producers. In this setup, technology consists of two components: a permanent and a temporary one. Agents observe only aggregate productivity and receive a noisy signal about its permanent component. This way of modeling passive learning was applied i.a. by Blanchard et al. (2013); Hürtgen (2014) in a closed economy framework and discussed in the context of other ways of modeling beliefs by Chahrour and Jurado (2018). We add to the existing modeling frameworks an international dimension of noise - information about technology (the noisy signal) is available to agents in both economies. As a consequence noise from the large economy can impact - on top of standard trade and financial linkages - business cycles in both economies. Below we present the details of our framework.

Intermediate goods producer i rents capital $k_t(i)$ and labor $n_t(i)$ and utilizes a standard Cobb-Douglas production function with labor-augmenting technology a_t and fixed cost ϕ to generate output $y_{p,t}(i)$:

$$y_{p,t}(i) = k_t(i)^{\alpha}(a_t n_t(i))^{1-\alpha} - \phi$$

The crucial, nonstandard feature of the model is agents' imperfect information about the technology processes, a_t. In both countries productivity is assumed to consist of time-varying permanent (x_t) and transitory (z_t) components. We assume that permanent productivity in the large economy (foreign variables are denoted with an asterisk) may diffuse to the small economy to some extent (measured by parameter λ_x):

$$a_t = x_t^{(1-\lambda_x)}(x_t^*)^{\lambda_x} z_t \quad a_t^* = x_t^* z_t^* \tag{5}$$

Permanent components follow unit root processes:

$$\frac{x_t}{x_{t-1}} = \left(\frac{x_{t-1}}{x_{t-2}}\right)^{\rho^x} \exp(\varepsilon_{x,t}) \quad \frac{x_t^*}{x_{t-1}^*} = \left(\frac{x_{t-1}^*}{x_{t-2}^*}\right)^{\rho^{x*}} \exp(\varepsilon_{x,t}^*) \tag{6}$$

Stationary components follow AR(1) processes:

$$z_t = z_{t-1}^{\rho^x} \exp(\varepsilon_{z,t}) \quad z_t^* = (z_{t-1}^*)^{\rho^{x*}} \exp(\varepsilon_{z,t}^*) \tag{7}$$

Agents are assumed to observe a_t, a_t^* and noisy signals s_t and s_t^* about their unit root components:

$$s_t = x_t \exp(\varepsilon_{s,t}) \quad s_t^* = x_t^* \exp(\varepsilon_{s,t}^*) \tag{8}$$

Above $\varepsilon_{x,t}$, $\varepsilon_{z,t}$, $\varepsilon_{s,t}$, $\varepsilon_{x,t}^*$, $\varepsilon_{z,t}^*$ and $\varepsilon_{s,t}^*$ are normal i.i.d. shocks centered around zero. Following Blanchard et al. (2013) we assume that the persistence parameters of temporary and permanent productivity processes are equal and that technology is a unit root process implied by the following restriction: $\rho_x \sigma_x^2 = (1-\rho_x)^2 \sigma_z^2$, where σ_x denotes the standard deviation of the permanent productivity shock, while σ_z - its temporary counterpart. Shocks to the signal, $\varepsilon_{s,t}$ and $\varepsilon_{s,t}^*$ are called noise shocks. The impact of the foreign one on the domestic economy is the central issue we investigate in this article.

In order to solve the above system of equations we follow Blanchard et al. (2013) and Hürtgen (2014) who deal with similar problems. The system is linearized, then we apply the Kalman filter. Importantly, since the system is linear certainty equivalence holds and the Kalman filter provides the optimal forecast of the state variables. In other words, beliefs about productivity levels resulting from the Kalman filter are treated by agents as their true realizations. Appendix C contains a detailed presentation of the state space representation of the imperfect information problem.

3.2 Data, calibration and estimation

We utilize 13 time series for the DSGE model estimation. For each economy we include productivity, individual consumption, investments, wages, inflation and the nominal interest rate. Additionally, we take nominal bilateral exchange rate between US and Canadian dollars. All non-stationary time series (productivity, consumption, investments, wages) are expressed as rates of growth while interest rates and inflation rate are demeaned. The time span of the data goes from 1Q1960 to 1Q2014. Time series are downloaded from the Fred database and the Canadian Statistical Office. Our specific data choices are driven by data availability and choices made in the earlier literature, in particular Blanchard et al. (2013), Hürtgen (2014), Smets and Wouters (2007) and Justiniano and Preston (2010). National account data and wages are at constant prices and seasonally adjusted. In case of Canada we download wages from the statistical office (series Wages and salaries), whereas for the US we take hour non-farm business sector real compensation. Labor supply represents employment. As the indicator of price growth we take core inflation in order to reduce noise coming from short-lived supply shocks. Interest rates are central bank rates (for Canada) and Fed Funds rates (for the US).

We calibrate parameters that are well-established in the literature (table 2), while estimating remaining ones (tables 3 and 4). In line with GDP data Canadian output constitutes 7% of the world economy in the model. Discount rates in both economies imply a steady state annual real rate equal to 2%. Home bias in the final goods aggregate in Canada equals 70%, in line with OECD data corrected for the import content of exports. Quarterly capital depreciation is equal to 2.5% as commonly assumed in the literature. Elasticity of the exchange rate to foreign debt amounts to 0.13% as estimated by Brzoza-Brzezina and Kotłowski (2020).

We estimate a large set of structural parameters that allow us to fit the model to both nominal and real data applying Bayesian techniques. We set prior means of the parameters close to their values reported in the literature and we choose typical distribution types. The only exception are priors for parameters γ_π and γ_π^* in Taylor rules as we choose beta as their prior distribution in order to avoid too low values that make the model indeterminate.[5] In case of parameters that are specific for our information friction, their choice follows findings from the earlier studies on noise in the US economy.

We draw two chains of 200 000 draws, burn the first half of each and use the remaining draws to obtain posterior distributions. According to our estimation, the posterior variances are smaller than thosee assumed a priori indicating that the data was informative for finding parameter values. This is important particularly for parameters associated with signal extraction problems faced by agents in both economies that are specific to our model. In particular, the pass-through of the foreign permanent productivity to the domestic one λ_x has been estimated well above its prior mean and very close to 1. This indicates that for Canadian productivity the US permanent component is much more important than the domestic one. The standars deviation of technology processes estimate above their prior levels for both Canada and the US, but nevertheless remain in line with the earlier findings in the noise literature.

As for less model-specific parameters, we note that posterior modes of most of them are close to values reported in other papers. This refers in particular to monetary policy parameters whose values are important for our main results. Taylor rules display high inertia as the autoregressive parameter in the United States equals 0.80 and its Canadian counterpart 0.86. At the same time the responsiveness to inflation is very moderate (as Blanchard et al., 2013 found for the US case): $1 + \gamma_\pi^*$ is 1.07 while $1 + \gamma_\pi$ is 1.10. We also found strong price inertia that is also typical for these two economies. As for consumption smoothing, it

[5] Note that in contrast to the usual procedure we set the inflation elasticities in the Taylor rules to $1 + \gamma_\pi$ and $1 + \gamma_\pi^*$. Assuming a beta prior prevents the parameter from falling below unity.

is stronger in the US as the habit formation parameter equals there 0.76 as compared with 0.58 in Canada.

4 Noise, sentiments and spillovers

How does noise affect the US and Canadian economies? How important are its fluctuation for the comovement of business cycles? When did US noise affect the Canadian economy positively and when negatively? How did noise fluctuate over time and can it be interpreted in the spirit of the sentiment literature as nonfundamental fluctuations to consumer confidence? All these questions will be answered in the subsequent section.[6]

4.1 Impulse responses

In order to gain a better understanding of how our model (and in particular the information friction) works we investigate various impulse responses. Figure 6 shows how the US and Canadian economy reacts to a permanent US technology shock. Since the shock affects technology in both economies (the posterior mean of $\lambda = 0.95$), the reactions are similar. Productivity increases gradually, reflecting the aturogressive character of the process. Consumption rises initially in line with productivity and then faster. The initial reaction reflects the uncertainty whether the shock is permanent or temporary. Once agents realize that productivity increased permanently they start to smooth consumption intertemporarily.

A temporary technology shock in Canada (Figure 7) has only an impact on the Canadian economy. In spite of its temporary character, consumption increases quite significantly as agents are not sure whether the impulse is not permanent. When they recognize its character, the impact on consumption dies out. The effect of a shock in the US is similar (Figure 8), with the only difference being sizable spillovers to Canada. However, as has been recognized in the literature, as resources are diverted to the country with higher productivity, the spillover to GDP is in fact negative.

The most interesting thing, from this papers perspective, is the impact of US noise. The results are presented on Figure 9. Since agents in US and Canada have the same information set, their filtering problem is exactly the same and so are their conclusions about the type of shock. As agents attach some probability to the signal informing about productivity being permanently higher, they react with increased consumption and investment spending in both economies. Since in fact technology does not change, the innovation is a typical demand shock and hence inflation increases as well. The reactions in the US and Canada are

[6] The results presented below come from the equivalent, full information representation of the model. The equivalence result has been described in detail in Blanchard et al. (2013).

similar though not equal - this results from different structural features of the two economies (e.g. price or wage rigidities). The most important finding from our perspective is however, that shocks to common information generate a clear comovement of all variables at business cycle frequency.

For the sake of brevity we do not present impulse responses to Canadian noise and permanent productivity. As we show later these shocks have a negligible share in generating fluctuations.

4.2 Noise vs. sentiments

The motivation of our paper was centered around the idea of sentiments affecting cyclical fluctuations. However, the central concept of our paper is noise. How are these related? First, it should be clearly stated that no formal mapping can be applied here. This is because sentiments do not have a unique definition in the literature, but are rather a vague notion of nonfundamental feelings about the future course of events in the economy. Having this in mind, we have two arguments in favor of identifying noise with sentiments. First, in our framework noise is indeed a nonfundamental process that drives expectations about how bright the future is.

Our second argument is made on empirical grounds. To this end we take a look at the estimated (smoothed) noise shock and seek its interpretability in terms of economic developments. To what extent can noise be interpreted in the spirit drawn by the confidence/ sentiment literature presented in the Introduction? Figure 5 plots the US noise series upon which we point to several periods of sentiment turbulence that seem to have coincided with negative noise shocks. These are in particular the oil shocks of 1973 and 1979, the Gulf war of 1990-91, the dot-com crash of 2000-01 and the financial crisis 2007-08 - all of them periods characterized by deeply worsening sentiments. This leads us to the conclusion that our noise can be interpreted as shocks to sentiments and we will use these terms interchangeably in what follows.

4.3 The role of noise in generating comovement

We have already seen from the impulse responses that noisy information has the potential to generate a comovement across business cycle variables and across countries. A different question is how strong this effect is and when did it affect the economies under consideration. To show this we present variance decompositions and counterfactual scenarios.

Table 5 documents the forecast error variance decomposition of GDP and consumption growth in the US and Canada. GDP and Consumption in Canada are driven by productivity shocks stemming from both economies, while, not surprisingly, the US cycle is affected mainly by US shocks. Also, not surprisingly, do neither Canadian permanent productivity nor noise shocks have any significant impact on either Canada or the US. The reason is simple - given the estimated value of λ_x the role of Canadian permanent productivity is negligible. As a consequence agents do not pay much attention to signals about Canadian productivity and so noise becomes unimportant as well. In section 4.4 we show how changing the value of λ_x would affect this result. The most interesting question is related to the role of US noise shocks. These play a significant role primarily for consumption, determining between 22% and 38% of its variance in the US and between 13% and 17% in Canada. The numbers for GDP are lower and amount to 2-3% in both economies.[7]

Our estimated model allows to precisely determine when and how noise shocks mattered for macroeconomic variables and their comovement. Given their most pronounced role in driving consumption we concentrate on this variable. Figure 10 presents the historical impact of noisy information on consumption in the US and Canada: the red line presents consumption growth and the blue line the contribution of the US noise shock. Clearly the role of noise varies over the sample and is most pronounced in periods identified earlier as moments of deeply worsening sentiments. Noise had a clear and significant role in driving consumption down i.a. after oil price shocks, during the Gulf war, after the dot-com bubble burst and when the financial crisis erupted. This happened in both economies and we suppose that these were periods when sentiments led to a strong international comovement.

To formalize this idea on Figure 11 we plot the 12-quarter moving correlation between US and Canadian actual consumption series and between counterfactual consumption series generated under the assumption of no noise shocks occurring. The correlation of counterfactuals is on average lower than that of true data, meaning that noise shocks indeed contributed to comovement. Interestingly, periods when US noise did not play a significant role are interrupted by periods where the contribution of noise to comovement was particularly pronounced. The latter were - not surprisingly - the same moments as listed above. For instance, during the first oil shock the correlation between US and Canadian consumption was approximately 0.6. However, if there were no noise shocks, the correlation would have been a negative -0.3. A similar role was played by noise shocks during the financial crisis of 2008. Correlation of consumption was mildly positive and amounted to 0.2. This was due to noise shocks, as without them the variables would have been desynchronized with a correlation coefficient of -0.4.

[7]Interestingly, the shares of noise in variance decompositions for the US are lower than those obtained by Blanchard et al. (2013) or Hürtgen (2014). This happens because agents in our model solve a "richer" filtering problem: instead of two observable variables they have access to four variables (US and Canadian signals and productivities). As a consequence it is easier for them to separate noise from signal. We verified this concept by estimating the model with the US-only filtering problem like in Blanchard et al. (2013) and obtaining results close to the original study.

An important conclusion from this section is that while the overall role of sentiment spillovers is not huge, it rises significantly during episodes of deep sentiment deterioration.

4.4 What determines the role of noise?

It is known from the literature that a number of model features are important determinants of the role noise plays in generating fluctuations. Blanchard et al. (2013) and Hürtgen (2014) analyze the topic in detail and point out the role of Taylor rule coefficients (aggressive monetary policy has the potential to eliminate the impact of noise) as well as sticky wages and prices (the more rigidity, the more important is noise). Instead, we concentrate on two features characteristic for the open economy framework that have a potential to determine cross-border spillovers.

The first is related to the diffusion of the permanent technology component given by parameter λ_x. Clearly, if λ_x was low, US technology would not matter much for Canadians and we should expect US noise to become unimportant. To check this numerically we simulate our model for various levels of λ_x keeping other parameters fixed. Figure 12 presents the share of US noise in the variance decomposition of Canadian consumption at the 4-quarter horizon. As could be expected, for low levels of λ_x the share of noise is indeed close to zero. Interestingly the relationship is far from linear, and for diffusion parameters below 30% the role of noise remains close to zero.

The second is related to the exchange rate regime. So far we assumed a flexible exchange rate arrangement between our economies. It seems interesting to check how important this is for the international transmission of noise. To check this we counterfactually impose a fixed exchange rate between Canada and the US. Again, keeping all estimated parameters fixed, we simulate the model under the counterfactual assumption of fixed exchange rate. The results are not significantly affected, though in the short run the role of US noise in Canada increases slightly - its share in the variance decomposition of consumption rises from 16.9% to 20.8%. The most likely reason is that, according to our estimates, the Fed (which is now responsible for setting interest rates) reacts less aggressively to inflation and GDP developments than the Bank of Canada.

5 Conclusions

This paper deals with the following hypothesis: sentiment shocks are able to travel fast across borders and thus affect several economies instantly and, as a consequence, generate cyclical comovement between countries. While the idea of sentiment shocks affecting business cycles is by far not new, the notion that they contribute to international business cycle spillovers is relatively recent. Our contribution is twofold. First, we show empirically that sentiments spill swiftly over borders and, thus, have the ability to affect business cycles in various countries almost immediately. Then we embed this idea into a structural model and check how important sentiment shocks have been for generating international comovement.

Next, we use a two-economy DSGE model with information frictions that we estimate for the US and Canada. The friction is related to noisy information about productivity that is available in both economies. Agents use this information to guess the type of productivity shock that hits the economy. We show that US noise has the potential to generate demand-type cycylical comovement of consumption, investments, output and inflation in both economies in the same direction. Model estimation shows that noise bears strong simiraities with empirical measures of sentiments and thus, we interpret it in this spirit. It has been particularly important during large deteriorations of confidence, like the oil shocks or the financial crisis. In these periods US noise is a key driver of consumption in the US and Canada and is responsible for a significant increase of its international correlation.

Our model concentrates only on the impact of noisy information on the consumer's decission making process, and thus, mainly explains comovement in consumption. One could equally well think of mechanisms via which international information affects the problems of firms and, thus, has the potential to affect and correlate investment fluctuations to a larger extent. We leave this interesting topic for further research.

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Tables and Figures

Table 1: The phase shift in the GDP and ESI cycles of 28 EU countries with reference to the euro area

Country	GDP	ESI	Country	GDP	ESI
UK	-5.2	-3.0	SI	-0.6	-1.4
DK	-4.0	-1.9	IT	-0.4	0.8
EE	-3.9	0.1	CZ	-0.4	1.3
SK	-3.6	0.0	FR	-0.3	0.3
IE	-3.4	0.8	ES	-0.1	-2.3
BE	-3.1	-1.2	AT	0.1	-0.7
LT	-2.8	-0.2	MT	0.5	2.7
PL	-2.6	2.4	DE	0.7	0.3
SE	-2.5	-1.2	LV	0.8	2.1
FI	-2.4	-1.5	NL	1.0	1.3
EL	-2.1	2.5	HR	1.9	3.3
HU	-2.1	-2.0	CY	2.3	1.2
LU	-1.2	-1.1	RO	6.9	2.0
PT	-0.9	-0.2	BG	7.1	5.2

Note: The numbers are the phase shifts (in months) of GDP and Economic Sentiment Indicators (ESI) cycles for 28 EU economies with reference to the euro area averaged over all frequencies with weights proportional to coherence values related to subsequent frequencies. The negative (positive) value indicates that the cycle for the given country leads (lags) the cycle for the euro area.

Table 2: Structural parameteres calibrated in the DSGE model

name	value
β, discount rate CAN	0.995
β^*, discount rate US	0.995
η, home bias CAN	0.700
δ, depreciation rate CAN	0.025
δ^*, depreciation rate US	0.025
ξ, exchange rate elasticity w.r.t. foreign debt	0.0013
γ_{u1}, adjustment cost CAN	0.035
γ_{u1*}, adjustment cost US	0.035

Table 3: Structural parameteres estimated in the DSGE model

name	prior mean	post mean	90% HPD interval		pr. type	pr. std dev
λ^x, weight of US perm prod	0.800	0.9556	0.9208	0.9916	beta	0.1000
h, habit parameter CAN	0.500	0.5845	0.5682	0.6012	beta	0.1000
h^*, habit parameter US	0.500	0.7595	0.7322	0.7861	beta	0.1000
Θ, capacity cost CAN	5.000	4.3559	4.1020	4.5763	norm	0.5000
Θ^*, capacity cost US	5.000	5.0282	4.7967	5.2694	norm	0.5000
γ_U, adjustment cost CAN	0.150	0.0679	0.0237	0.1084	beta	0.0500
γ_{U*}, adjustment cost US	0.150	0.1518	0.1406	0.1626	beta	0.0500
γ_r, Taylor rule persistence CAN	0.700	0.8612	0.8464	0.8770	beta	0.1000
γ_{r*}, Taylor rule persistence US	0.700	0.8016	0.7772	0.8270	beta	0.1000
γ_π, Taylor rule inflation CAN	0.100	0.1057	0.0830	0.1261	beta	0.0500
$\gamma_{\pi*}$, Taylor rule inflation US	0.100	0.0748	0.0519	0.0945	beta	0.0500
γ_y, Taylor rule output CAN	0.100	0.1969	0.1646	0.2234	beta	0.0500
γ_{y*}, Taylor rule output US	0.100	0.0133	0.0056	0.0205	beta	0.0500
θ_H, Calvo domestic price CAN	0.750	0.7253	0.6567	0.7834	beta	0.1000
θ_F, Calvo imports price CAN	0.750	0.9806	0.9718	0.9891	beta	0.1000
θ_{F*}, Calvo domestic price US	0.750	0.8827	0.8365	0.9350	beta	0.1000
θ_{H*}, Calvo imports price US	0.750	0.4650	0.4261	0.5100	beta	0.1000
ζ_H, index. domestic price CAN	0.750	0.7428	0.6743	0.8073	beta	0.1000
ζ_F, index. imports price CAN	0.750	0.7448	0.6520	0.8329	beta	0.1000
ζ_{F*}, index. domestic price US	0.750	0.6657	0.6274	0.7057	beta	0.1000
ζ_{H*}, index. imports price US	0.750	0.8141	0.7507	0.8646	beta	0.1000

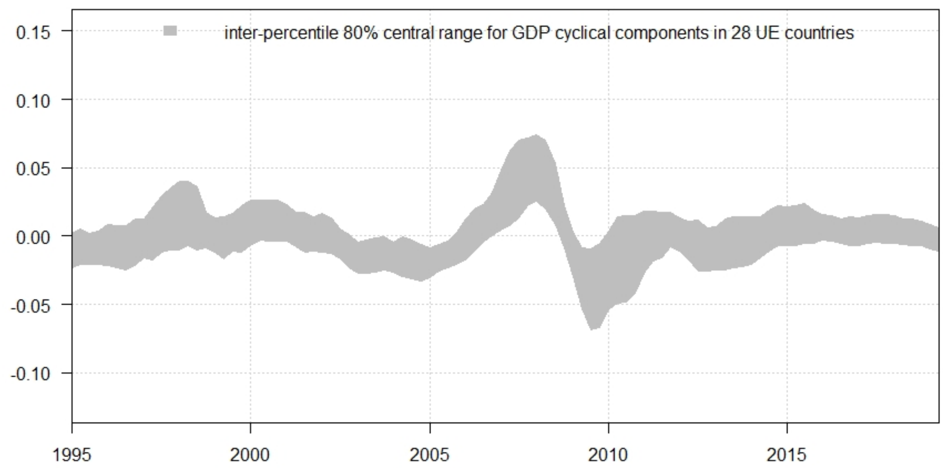
Table 4: Shock parameteres estimated in the DSGE model

name	prior mean	post mean	90% HPD interval		pr. type	pr. std dev
ρ^x, autocorrel. productivity CAN	0.900	0.9448	0.9277	0.9599	beta	0.0500
ρ^{x*}, autocorrel. productivity US	0.900	0.9678	0.9578	0.9774	beta	0.0500
ρ^i, autocorrel. investment CAN	0.700	0.4604	0.4381	0.4876	beta	0.0500
ρ^{i*}, autocorrel. investment US	0.700	0.4085	0.3808	0.4317	beta	0.0500
ρ^{μ_H}, autocorrel. price mark-up CAN	0.700	0.5455	0.5128	0.5743	beta	0.0500
ρ^{μ_H*}, autocorrel. price mark-up US	0.700	0.5604	0.5350	0.5852	beta	0.0500
ρ^{μ_W}, autocorrel. wage mark-up CAN	0.700	0.8626	0.8441	0.8839	beta	0.0500
ρ^{μ_W*}, autocorrel. wage mark-up US	0.700	0.7591	0.7323	0.7871	beta	0.0500
ρ^ρ, autocorrel. UIP shock	0.700	0.9196	0.9082	0.9313	beta	0.0500
σ^s, std dev noise shock CAN	0.010	0.0063	0.0032	0.0095	invg	0.0010
σ^{s*}, std dev noise shock US	0.010	0.0094	0.0054	0.0141	invg	0.0010
σ^x, std dev prod. shock CAN	0.005	0.0151	0.0141	0.0162	invg	0.0010
σ^{x*}, std dev prod. shock US	0.005	0.0235	0.0230	0.0239	invg	0.0010
σ^r, std dev monetary policy CAN	0.001	0.0024	0.0022	0.0027	invg	Inf
σ^{r*}, std dev monetary policy US	0.001	0.0023	0.0021	0.0025	invg	Inf
σ^i, std dev investment CAN	0.01	0.0915	0.0823	0.1003	invg	Inf
σ^{i*}, std dev investment US	0.01	0.2075	0.1879	0.2280	invg	Inf
σ^{μ_H}, std dev price mark-up CAN	0.01	0.0268	0.0192	0.0330	beta	Inf
σ^{μ_H*}, std dev price mark-up US	0.01	0.0230	0.0142	0.0324	beta	Inf
σ^{μ_W}, std dev wage mark-up CAN	0.01	0.0825	0.0684	0.0984	beta	Inf
σ^{μ_W*}, std dev wage mark-up US	0.01	0.0080	0.0025	0.0144	beta	Inf
σ^ρ, std dev UIP shock	0.01	0.0032	0.0027	0.0037	beta	Inf

Table 5: Variance decomposition in the estimated DSGE model

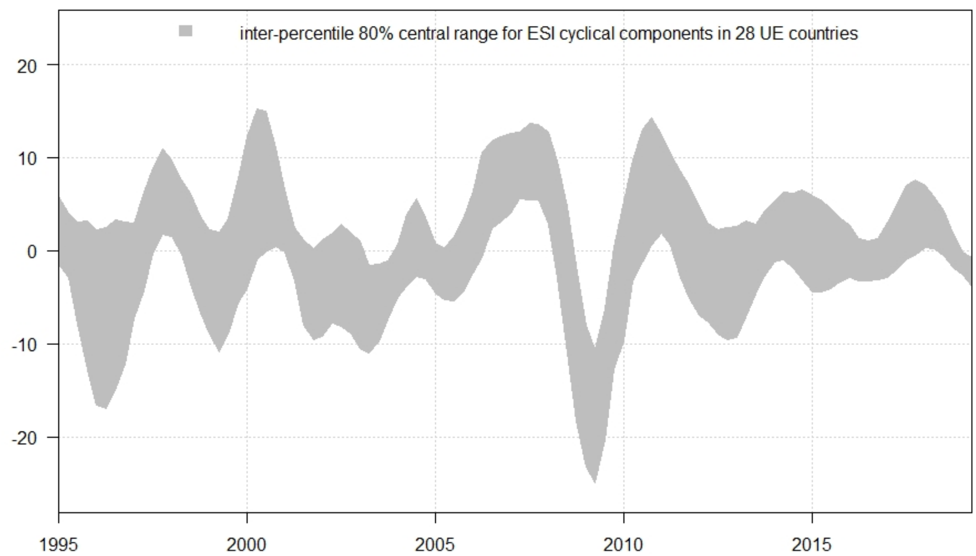
GDP in Canada						
Quarter	CAN perm prod	CAN temp prod	US perm prod	US temp prod	CAN noise	US noise
1	0.0	32.5	0.1	10.0	0.0	1.7
4	0.0	29.1	0.9	9.7	0.0	1.8
8	0.0	26.1	2.0	8.8	0.0	1.9
12	0.0	25.6	2.5	8.6	0.0	1.9
40	0.0	25.2	3.2	8.4	0.0	1.9
GDP in the US						
Quarter	CAN perm prod	CAN temp prod	US perm prod	US temp prod	CAN noise	US noise
1	0.0	0.0	0.1	16.1	0.0	1.8
4	0.0	0.0	0.9	14.4	0.0	1.8
8	0.0	0.1	2.6	14.1	0.0	2.5
12	0.0	0.1	3.4	13.9	0.0	2.5
40	0.0	0.1	4.3	13.6	0.0	2.5
Consumption in Canada						
Quarter	CAN perm prod	CAN temp prod	US perm prod	US temp prod	CAN noise	US noise
1	0.0	16.5	0.4	4.4	0.0	16.9
4	0.0	14.2	4.2	4.2	0.0	15.0
8	0.0	12.0	7.4	3.5	0.0	14.5
12	0.0	11.4	7.8	3.3	0.0	13.8
40	0.0	11.1	8.1	3.4	0.0	13.3
Consumption in the US						
Quarter	CAN perm prod	CAN temp prod	US perm prod	US temp prod	CAN noise	US noise
1	0.0	2.6	0.5	26.1	0.0	37.6
4	0.0	1.9	8.2	27.4	0.0	25.9
8	0.0	1.7	18.5	21.7	0.0	26.4
12	0.0	1.6	20.1	19.1	0.0	23.4
40	0.0	1.5	20.9	17.7	0.0	21.9

Figure 1: Cyclical components of GDP for EU countries



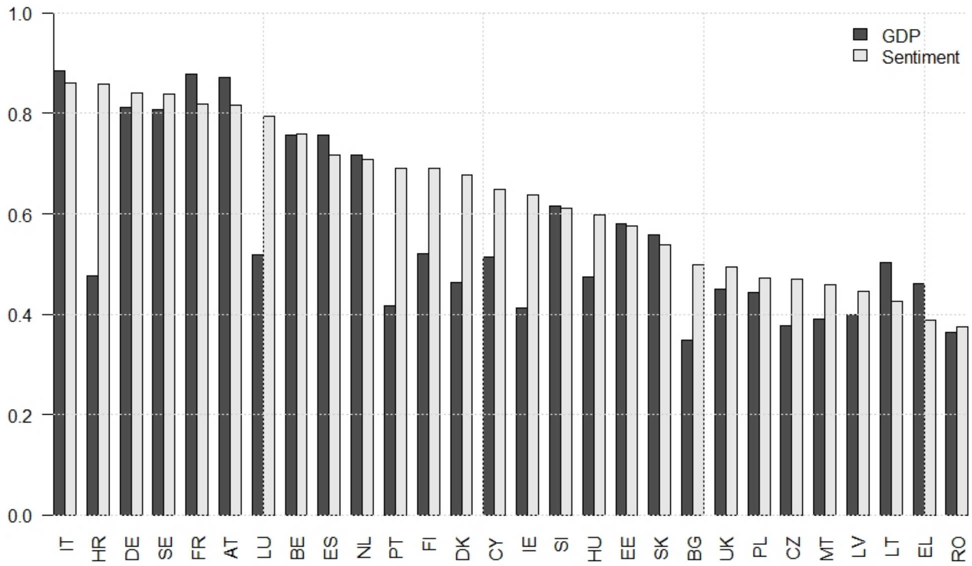
Note: The chart plots cyclical components of GDP for 28 member states of the European Union derived using Christiano-Fitzgerald asymmetric filter with modified Daniell smoother for window spanning from 6 to 40 quarters.

Figure 2: Cyclical components of Economic Sentiment Indicator (ESI) for EU countries



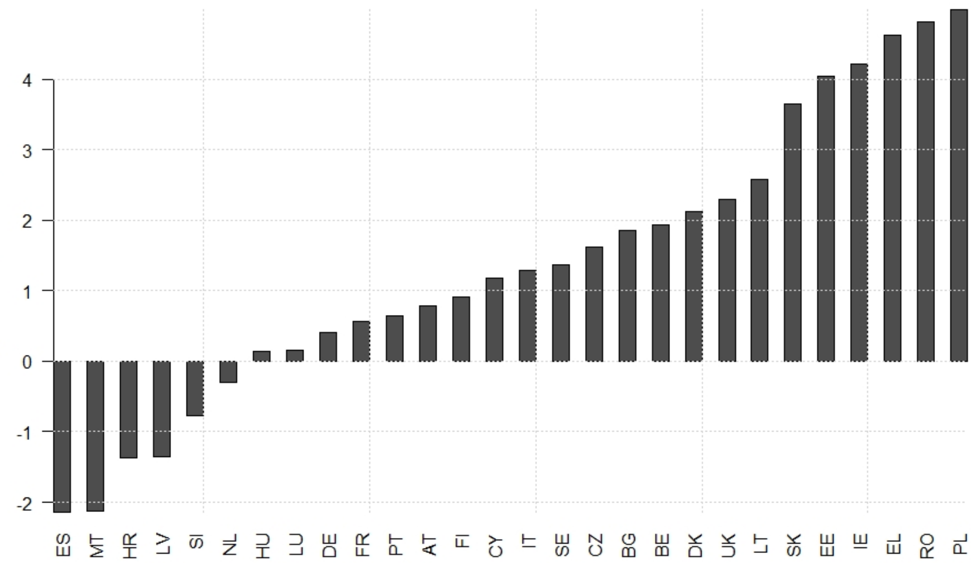
Note: The chart plots cyclical components of Economic Sentiment Indicators (ESI) for 28 member states of the European Union derived using Christiano-Fitzgerald asymmetric filter with modified Daniell smoother for window spanning from 6 to 40 quarters.

Figure 3: Coherence of GDP and ESI cyclical components of 28 EU economies with euro area aggregate.



Note: The chart plots the coherence between cyclical components of GDP and Economic Sentiment Indicators (ESI) for 28 EU economies and the euro area aggregate averaged over all frequencies.

Figure 4: Difference in phase shifts between GDP and ESI cyclical components of 28 EU economies in reference to the euro area.



Note: The chart plots the difference in phase shifts (in months) of GDP and Economic Sentiment Indicators (ESI) cycles for 28 EU economies with reference to the euro area averaged over all frequencies with weights proportional to coherence values related to subsequent frequencies. The positive value indicates that the sentiment spillovers faster than does the economic activity (from the euro area to the given country or in the opposite direction).

Figure 5: US noise shock

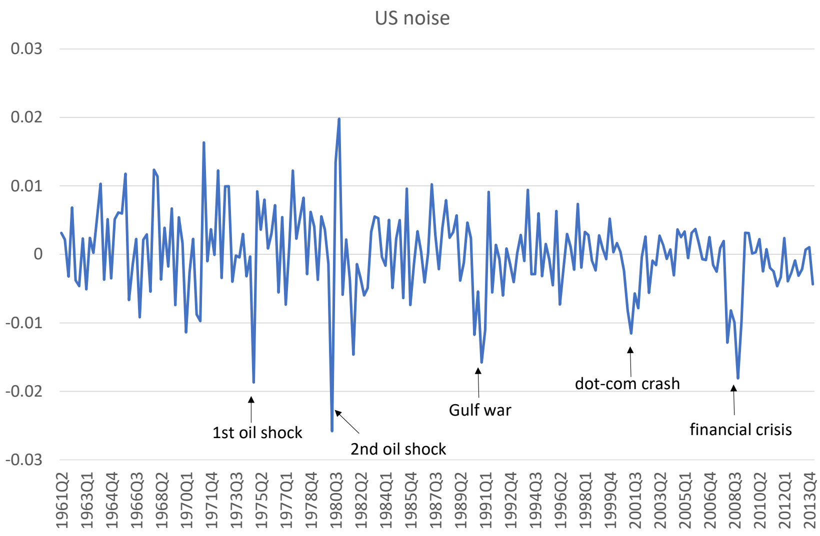


Figure 6: Impulse response functions to a permanent productivity shock in the US

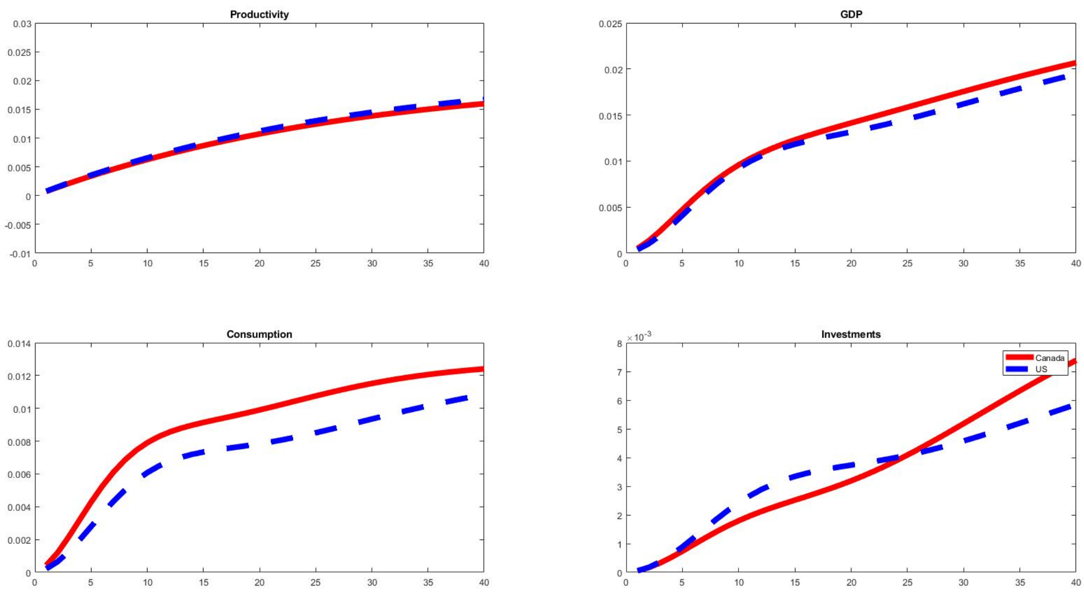


Figure 7: Impulse response functions to temporary productivity shock in Canada

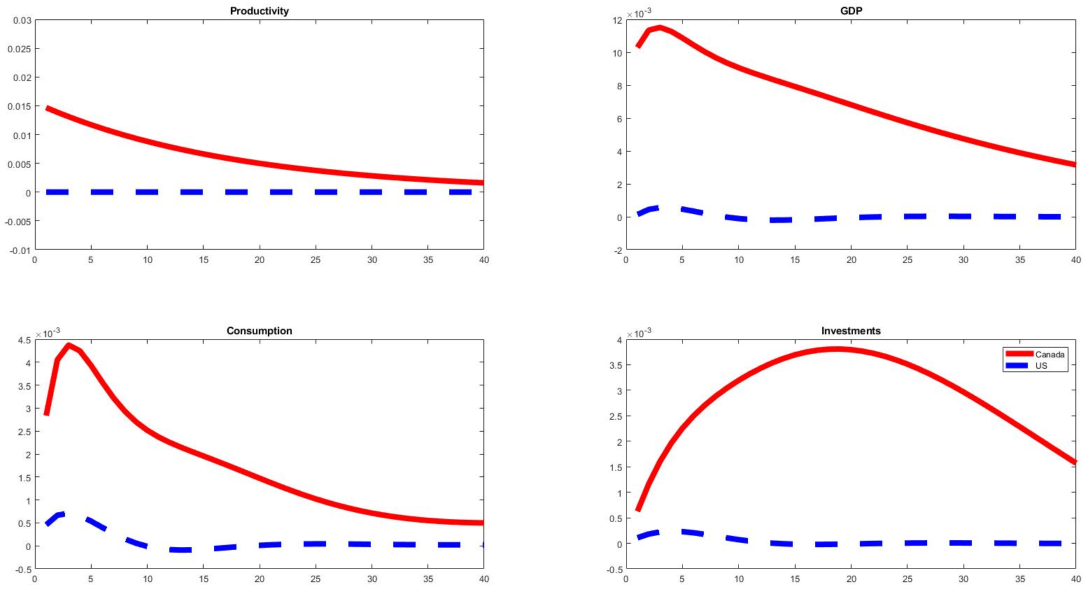


Figure 8: Impulse response functions to temporary productivity shock in the US

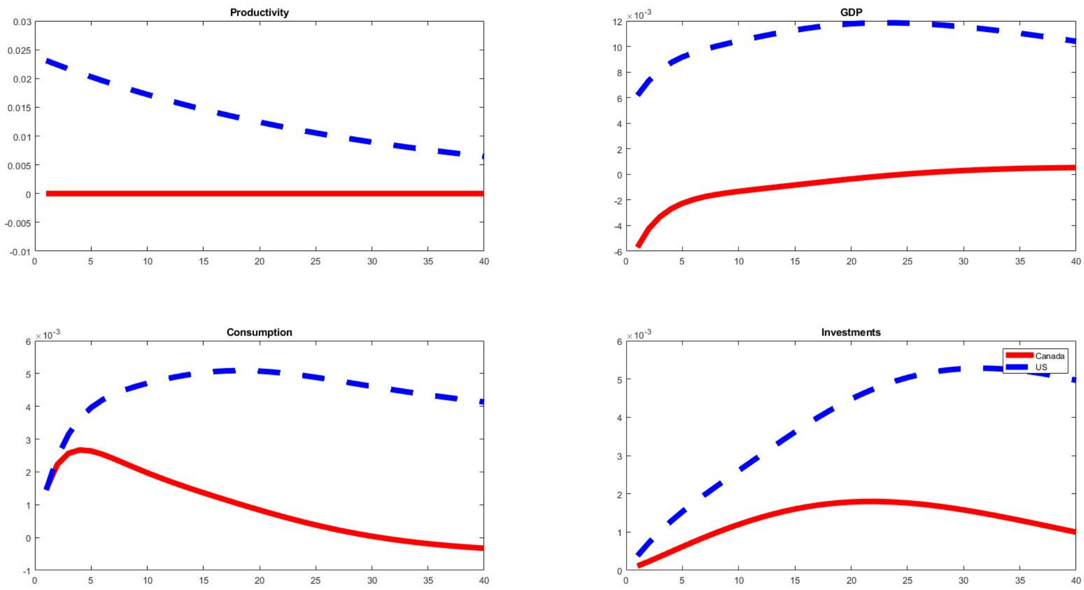


Figure 9: Impulse response functions to noise shock in the US

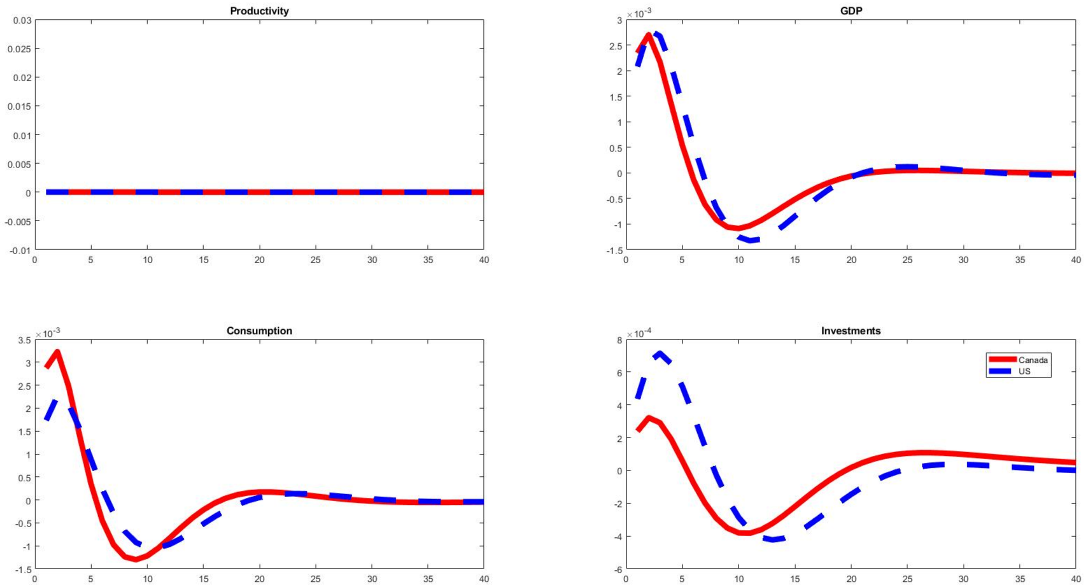


Figure 10: Consumption growth in Canada and US noise shock contribution

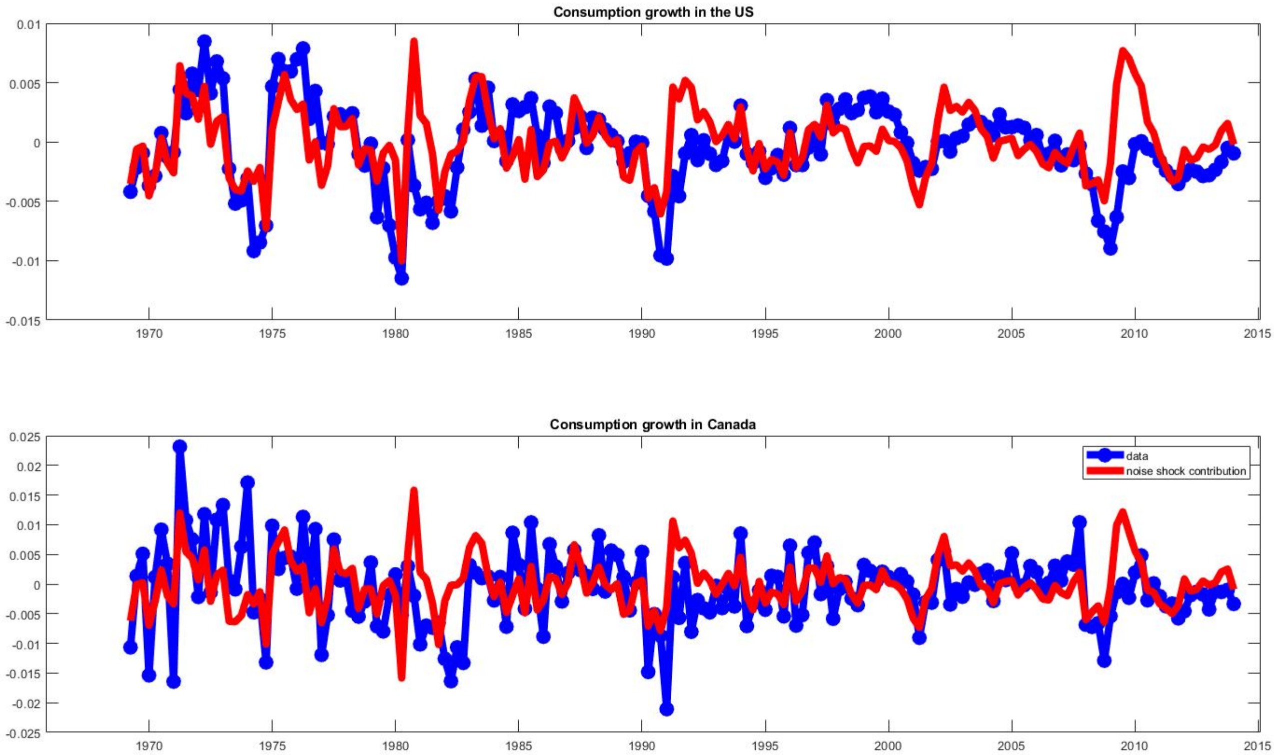


Figure 11: Rolling correlation (12-quarter window) of consumption in the US and Canada (with and w/o noise shocks)

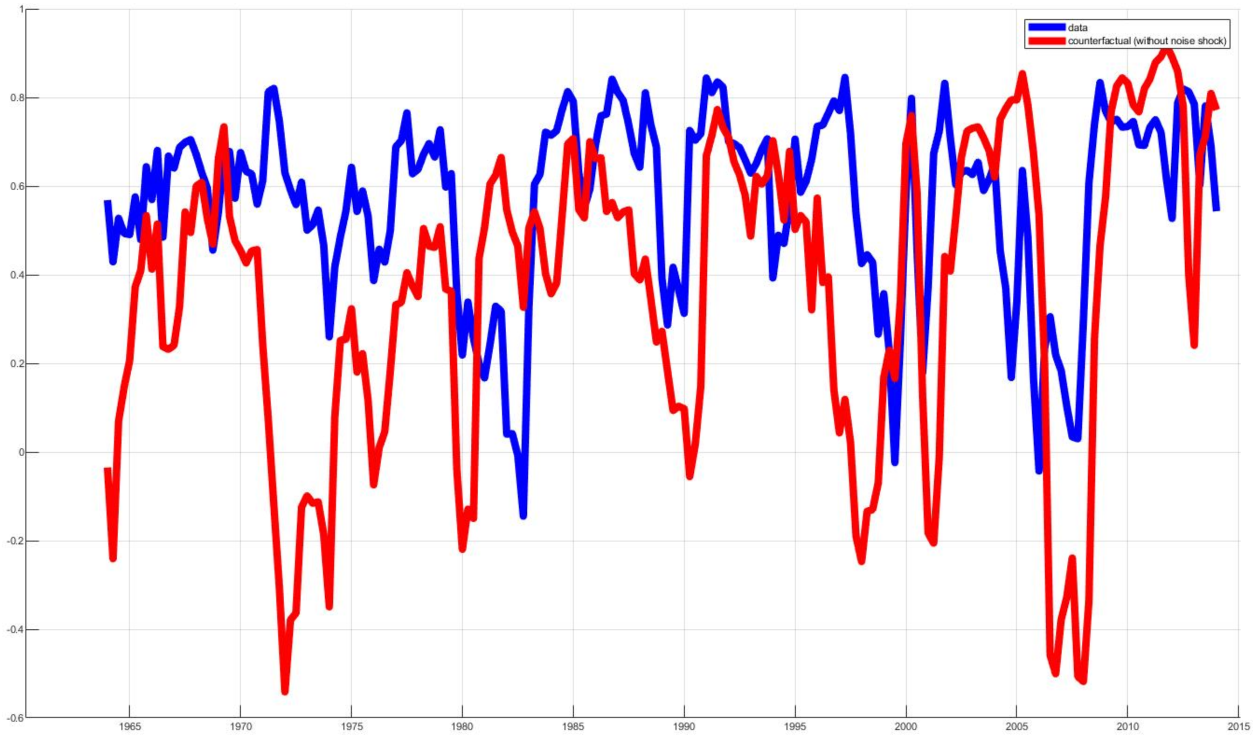
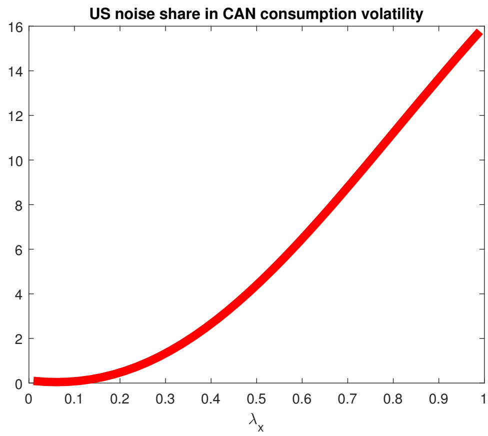


Figure 12: Role of technology diffusion in driving noise spillovers



Appendix A. DSGE model

This Appendix presents economic problems of the agents. Equations for both economies are broadly symmetric, therefore we present only problems of the small economy agents, reffering to their large economy counterparts where it is neccessary only. We use the convention that small letters denote real values of nominal variables that are denoted by capital letters. Variables without time indices denote steady state values.

A.1. Households

Households maximise expected lifetime utility U_0 being a function of consumption c_t and labour supply n_t:

$$\begin{aligned} U_0 &= E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t - hc_{t-1}) - \gamma \frac{1}{1+\varphi} n_t^{1+\varphi} \right] \end{aligned} \tag{9}$$

where E_0 denotes agents' expectations at the time 0, β - the discount rate, $u()$ - the period utility function and h captures internal habit formation. Utility maximization is subject to the household budget constraint:

$$P_t c_t + B_{H,t} + S_t B_{F,t} + T_t = R_{t-1} B_{H,t-1} + S_t R^*_{t-1} \Gamma_{t-1} B_{F,t-1} + W_t^h n_t + D_t \tag{10}$$

where $B_{H,t}$ stands for domestic bond holdings, S_t - the exchange rate vis-a-vis the foreign economy, $B_{F,t}$ - foreign bonds, T_t - lump-sum transfers from the domestic government, R_{t-1} - nominal domestic interest rate, R^*_{t-1} - nominal foreign interest rate, Γ_{t-1} - transaction cost on holding foreign bonds, W_t^h - wage, D_t - profits paid by intermediate firms and labour agencies. The transaction cost is given by $\Gamma_t = \exp\left(-\xi \left(nfa_t - nfa\right) \varepsilon_{\rho,t}\right)$ where nfa_t is the ratio of net foreign assets to GDP and $\varepsilon_{\rho,t}$ is a risk premium shock.

A.2. Labour market

We assume that households sell labour services to a labour agency at the nominal price W_t^h. Labour services are marked by the labour agency so that they become imperfect substitutes, $n_t(i)$. Therefore, the agency receives heterogenous wages $W_t(i)$ that are set according to the Calvo scheme with wage stickiness parameter equal θ_W. Labour is aggregated by a perfectly competetive labour aggregator who combines all labour types $n_t(i)$ into a homogenous labour service n_t according to the following formula:

$$n_t = \left(\int_0^1 n_t(i)^{\frac{1}{1+\mu_{w,t}}} di \right)^{1+\mu_{w,t}} \tag{11}$$

and sells it to intermediate goods producers at price W_t. We assume that if the agency is not allowed to optimise the wage for labour type i it indexes the price by a weigted average of steady state and last period inflation:

$$W_{t+1}(i) = \pi_{\zeta,w,t} W_t(i) \tag{12}$$

where:

$$\pi_{\zeta,w,t} = \bar{\pi}^{1-\zeta_W} \pi_{t-1}^{\zeta_W} \tag{13}$$

and where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is domestic inflation. If the labour agency is allowed to reset the price, it solves the following problem:

$$\max_{\tilde{W}_t(i), \{n_{t+s}(i)\}_{s=0}^{\infty}} E_t \sum_s (\beta\theta_W)^s \lambda_{t,t+s} \left(\tilde{W}_t(i) \pi_{\zeta,w,t,t+s} - W_{t+s}^h \right) n_{t+s}(i) \tag{14}$$

subject to the demand of labour aggregators:

$$n_{t+s}(i) = \left(\frac{\tilde{W}_t(i) \pi_{\zeta,w,t+s}}{W_{t+s}} \right)^{\frac{\mu_{w,t}}{1-\mu_{w,t}}} n_{t+s} \tag{15}$$

where $\pi_{\zeta,w,t,t+s} = \pi_{\zeta,w,t+1} \cdot \ldots \cdot \pi_{\zeta,w,t+s}$, $\lambda_{t,t+s} \equiv \frac{\Lambda_{t+s}}{\Lambda_t}$ where Λ_t is the household's stochastic discount factor for nominal payments and $\mu_{w,t}$ is a time-varying wage markup.

A.3. Firms

We consider saveral stages of the prodcution process. Capital producers use undepreciated capital from the previous period and investments as inputs in producing capital that is used by two types of intermediate goods producers: domestic and exporters. Intermediate goods producers produce differenctiated goods and sell them at home and abroad. Aggregators (domestic and foreign) bundle these goods into homogenous products. Final good producers combine goods sold by aggregators of domestic produced and imported goods into final goods that are subsequently used for consumption and investments.

Capital producers

Competetive capital producers decide on investments i_t, capital level $\bar{k}_{t-1}$ as well as capital utilization u_t. They rent effective capital k_t to intermediate good producing firms in order to maximize:

$$\max_{i_t,\bar{k}_{t-1},u_t} E_0 \sum_t \Lambda_t \beta^t \left(k_t \frac{R_t^k}{P_t} - i_t - C(u_t)\bar{k}_{t-1} \right) \tag{16}$$

subject to a capital accumulation rule with investment adjustment costs:

$$\bar{k}_t = (1-\delta)\bar{k}_{t-1} + \varepsilon_{i,t} \left(1 - S\left(\frac{i_t}{i_{t-1}} \right) \right) i_t \tag{17}$$

variable capital utilization:

$$k_t = u_t \bar{k}_{t-1} \tag{18}$$

and a related adjustment cost:

$$C(u_t) = \gamma_{u1}(u_t - 1) + \frac{\gamma_{u2}}{2}(u_t - 1)^2 \tag{19}$$

where$\varepsilon_{i,t}$ denotes investment specific shock, $C(u_t)$ - cost of capital utilization, R_t^k - return from renting capital and $S(\cdot)$ - an investment adjustment costs ($S'(\cdot) > 0$ and $S''(\cdot) > 0$). We assume $S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\Theta}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$.

Final good producers

Final good producers act in a perfectly competetive market. They combine domestic $y_{H,t}$and foreign output $y_{F,t}$ into homogenous goods y_t that are used for consumption, investment and government purchases. Thus, the domestic producer maximizes profits:

$$P_t y_t - P_{H,t} y_{H,t} - P_{F,t} y_{F,t} \tag{20}$$

subject to the following production technology

$$y_t = \left[\eta^{\frac{\mu-1}{\mu}} (y_{H,t})^{\frac{1}{\mu}} + (1-\eta)^{\frac{\mu-1}{\mu}} (y_{F,t})^{\frac{1}{\mu}} \right]^{\mu} \tag{21}$$

Aggregators

Aggregators buy goods from intermediate goods producers and bundle them into a homogenous product. We assume perfect competition at this stage of the production process. Domestic aggregators maximize profits, given by:

$$P_{H,t}y_{H,t} - \int P_{H,t}(i)y_{H,t}(i)di \tag{22}$$

subject to the technological constraint:

$$y_{H,t} = \left(\int y_{H,t}(i)^{\frac{1}{\mu_{H,t}}} di\right)^{\mu_{H,t}} \tag{23}$$

Foreign aggregators of exported goods maximize:

$$P^*_{H,t}y^*_{H,t} - \int P^*_{H,t}(i)y^*_{H,t}(i)di \tag{24}$$

subject to:

$$y^*_{H,t} = \left(\int y^*_{H,t}(i)^{\frac{1}{\mu^*_{H,t}}} di\right)^{\mu^*_{H,t}} \tag{25}$$

Intermediate goods producers

Intermediate goods producers are assumed to utilize a standard Cobb-Douglas production function:

$$y_{p,t}(i) = k_t(i)^{\alpha}(a_t n_t(i))^{1-\alpha} - \phi \tag{26}$$

where ϕ corresponds to the fixed cost of production that guarantees that economic profits are equal to zero in the steady state and a_t is a labour-augmenting technology process that we described in Section 3.1.

Cost minimization Intermediate goods producers act in a monopolistically competitive market. They solve a cost minimization problem (the problem is the same for all firms, thus we omit the subscript i):

$$\min_{k_t,n_t} TC = \frac{R^k_t}{P_t}k_t + w_t n_t \tag{27}$$

subject to the production technology. Then they solve price setting problems. We assume that prices are set according to the Calvo scheme with indexation, separately for the domestic and foreign markets.

Price setting (domestic) When selling to the domestic market firms set their price $\tilde{P}_{H,t}(i)$ to maximize:

$$\max_{\tilde{P}_{H,t}(i),\{y_{H,t+s}(i)\}_{s=0}^{\infty}} E_t \sum_s (\beta\theta_H)^s \Lambda_{t,t+s} \left(\frac{\tilde{P}_{H,t}(i)\,\pi_{\zeta,t,t+s}}{P_{t+s}} - p_{m,t+s} \right) y_{H,t+s}(i) \tag{28}$$

where $p_{m,t}$ is the real marginal cost, subject to the demand of domestic aggregators:

$$y_{H,t+s}(i) = \left(\frac{\tilde{P}_{H,t}(i)\,\pi_{\zeta,t+s}}{P_{H,t+s}} \right)^{\frac{\mu_{H,t}}{1-\mu_{H,t}}} y_{H,t+s} \tag{29}$$

where $\pi_{\zeta,t,t+s} = \pi_{\zeta,t+1} \cdot \ldots \cdot \pi_{\zeta,t+s}$.

Price setting (exports) When selling to the foreign market firms set their price $\tilde{P}^*_{H,t}(i)$ to maximize:

$$\max_{\tilde{P}^*_{H,t}(i),\{y^*_{H,t+s}(i)\}_{s=0}^{\infty}} E_t \sum_s (\beta\theta^*_H)^s \Lambda_{t,t+s} \left(S_{t+s} \frac{\tilde{P}^*_{H,t}(i)\,\pi^*_{\zeta_{H*},t,t+s}}{P_{t+s}} - p_{m,t+s} \right) y^*_{H,t+s}(i) \tag{30}$$

subject to an analogous demand function and indexation scheme.

A.4. Goods market clearing

Goods market clearing implies:

$$y_t = c_t + i_t + g_t - C(u_t)\bar{k}_{t-1} \tag{31}$$

and

$$gdp_t \equiv y_{H,t}\Delta_{H,t} + \frac{1-\omega}{\omega} y^*_{H,t}\Delta^*_{H,t} = {k_t}^{\alpha}(a_t n_t)^{1-\alpha} - \phi \tag{32}$$

where ω denotes the size of the small economy:

$$\Delta_{H,t} = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{\frac{\mu_{H,t}}{1-\mu_{H,t}}} di = \left(\frac{P_{H,t}}{P_{H,t-1}} \right)^{\frac{\mu_{H,t}}{\mu_{H,t}-1}} \theta_H \Delta_{H,t-1} \left(\frac{\pi_{\zeta_H,t}}{\pi_t} \right)^{\frac{-\mu_{H,t}}{\mu_{H,t}-1}} + (1-\theta_H) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{\frac{-\mu_{H,t}}{\mu_{H,t}-1}} \tag{33}$$

is price disperion and $\Delta^*_{H,t}$ is defined analogously.

A.5. Bond market clearing and public sector:

Bond markets clear:

$$B_{H,t} = 0 \tag{34}$$

$$\omega B_{F,t} = -(1-\omega) B^*_{F,t} \tag{35}$$

The government budget is always balanced:

$$T_t = G_t \tag{36}$$

Monetary policy is given by a standard Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_r} \left(\left(\frac{\pi_t}{\pi}\right)^{1+\gamma_\pi} \left(\frac{gdp_t}{gdp}\right)^{\gamma_y}\right)^{1-\gamma_r} \exp\{\varepsilon_{r,t}\} \tag{37}$$

A.5. Stochastic shocks:

The model features a number of stochastic shocks. Technology shocks $\varepsilon_{x,t}$ and $\varepsilon_{z,t}$ as well as the noise $\varepsilon_{s,t}$ shocks are assumed white noise. Price and wage markups $\mu_{w,t}$ and $\mu_{H,t}$ are assumed to follow ARMA(1,1) processes as in Smets and Wouters (2007). Government spending g_t, investment specific technology shocks $\varepsilon_{i,t}$ monetary policy $\varepsilon_{r,t}$ as well as risk premium $\varepsilon_{\rho,t}$ shock follow AR(1) processes.

Appendix B. Final model equilibrium conditions

Since technology contains a unit root, we transform the equations to make the model stationary. Variables corrected for technology will be denoted $\hat{x}_t \equiv \frac{x_t}{t}$. This applies to: c_t, w_t, b^H_t, b^F_t, d_t, y_t, y^H_t, y^F_t, i, g_t, k_t, $\bar{k}_t$, t_t, $gdp_t$and their foreign counterparts. Household's stochastic discount factor is redefined: $\hat{\Lambda}_t \equiv \Lambda_t a_t$. Denote technology growth: $DA_t \equiv a_t/a_{t-1}$.

Households Marginal utility

$$\hat{\Lambda}_t = (\hat{c}_t - hDA_t^{-1}\hat{c}_{t-1})^{-1} - \beta h(DA_{t+1}\hat{c}_{t+1} - h\hat{c}_t)^{-1}$$

$$\hat{\Lambda}^*_t = (\hat{c}^*_t - h^* DA_t^{*-1}\hat{c}^*_{t-1})^{-1} - \beta^* h^*(DA^*_{t+1}\hat{c}^*_{t+1} - h\hat{c}^*_t)^{-1}$$

Intratemporal optimality condition

$$\gamma(n_t)^{\varphi} = \hat{\Lambda}_t \hat{w}^h_t$$

$$\gamma^*(n^*_t)^{\varphi*} = \hat{\Lambda}^*_t \hat{w}^{h*}_t$$

Intertemporal optimality condition

$$\hat{\Lambda}_t = \beta E_t \left\{ \hat{\Lambda}_{t+1} DA_{t+1}^{-1} R_t \pi_{t+1}^{-1} \right\}$$

$$\hat{\Lambda}_t q_t = \beta E_t \left\{ \hat{\Lambda}_{t+1} DA_{t+1}^{-1} q_{t+1} \Gamma_t R_t^* \pi_{t+1}^{*-1} \right\}$$

$$\hat{\Lambda}_t^* = \beta^* E_t \left\{ \hat{\Lambda}_{t+1}^* DA_{t+1}^{*-1} R_t^* \pi_{t+1}^{*-1} \right\}$$

Budget constraint

$$\hat{c}_t + \hat{b}_{H,t} + q_t \hat{b}_{F,t} + \hat{t}_t = R_{t-1} \pi_t^{-1} \hat{b}_{H,t-1} DA_t^{-1} + q_t R_{t-1}^* \Gamma_{t-1} \pi_t^{*-1} \hat{b}_{F,t-1} DA_t^{-1} + \hat{w}_t^h n_t + \hat{d}_t$$

$$\Gamma_t = \exp\left(1 - \xi \left(nfa_t - nfa\right)\right) \varepsilon_{\rho,t}$$

Labour market Real wage

$$w_t^{\frac{1}{1-\mu_{w,t}}} = \theta_W \left(w_{t-1} \frac{\pi_{\zeta,w,t}}{\pi_t} \right)^{\frac{1}{1-\mu_{w,t}}} + (1-\theta_W) \left(\tilde{w}_t\right)^{\frac{1}{1-\mu_{w,t}}}$$

$$w_t^{*\frac{1}{1-\mu_{w*,t}}} = \theta_W^* \left(w_{t-1}^* \frac{\pi_{\zeta,w*,t}}{\pi_t^*} \right)^{\frac{1}{1-\mu_{w*,t}}} + (1-\theta_W^*) \left(\tilde{w}_t^*\right)^{\frac{1}{1-\mu_{w*,t}}}$$

$$\pi_{\zeta,w,t} = \bar{\pi}^{1-\zeta_W} \pi_{t-1}^{\zeta_W}$$

$$\pi_{\zeta,w*,t} = \bar{\pi}^{*1-\zeta_{W*}} \pi_{t-1}^{*\zeta_{W*}}$$

Optimal wage

$$\tilde{w}_t = \mu_{w,t} \frac{\Omega_{W,t}}{\Upsilon_{W,t}}$$

$$\Omega_{W,t} = \hat{\Lambda}_t w_t^h w_t^{\frac{\mu_{w,t}}{\mu_{w,t}-1}} n_t + \beta \theta_W E_t \left(\frac{\pi_{\zeta,w,t+1}}{\pi_{t+1}} \right)^{\frac{\mu_{w,t}}{1-\mu_{w,t}}} \Omega_{W,t+1}$$

$$\Upsilon_{W,t} = \hat{\Lambda}_t w_t^{\frac{\mu_{w,t}}{\mu_{w,t}-1}} n_t + \beta \theta_W E_t \left(\frac{\pi_{\zeta,w,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_{w,t}}} \Upsilon_{W,t+1}$$

$$\tilde{w}_t^* = \mu_{w*,t} \frac{\Omega_{W,t}^*}{\Upsilon_{W,t}^*}$$

$$\Omega^*_{W,t} = \hat{\Lambda}^*_t w_t^{h*} {w_t^*}^{\frac{\mu_{w*,t}}{\mu_{w*,t-1}}} n^*_t + \beta^* \theta^*_W E_t \left(\frac{\pi_{\zeta,w*,t+1}}{\pi^*_{t+1}} \right)^{\frac{\mu_{w*,t}}{1-\mu_{w*,t}}} \Omega^*_{W,t+1}$$

$$\Upsilon^*_{W,t} = \hat{\Lambda}^*_t {w_t^*}^{\frac{\mu_{w*,t}}{\mu_{w*,t-1}}} n^*_t + \beta^* \theta^*_W E_t \left(\frac{\pi_{\zeta,w*,t+1}}{\pi^*_{t+1}} \right)^{\frac{1}{1-\mu_{w*,t}}} \Upsilon^*_{W,t+1}$$

Profits of labour aggregator

$$\hat{d_t^L} = n_t \left(w_t - w_t^h \right)$$

$$\hat{d_t^{L*}} = n^*_t \left(w^*_t - w_t^{h*} \right)$$

Capital goods producers Capital law of motion

$$\hat{\hat{k}}_t = \frac{(1-\delta)}{DA_t} \hat{\hat{k}}_{t-1} + \varepsilon^i_t \left(1 - S\left(\frac{\hat{\hat{i}}_t}{\hat{\hat{i}}_{t-1}} \right) \right) \hat{\hat{i}}_t$$

$$\hat{\hat{k}}^*_t = \frac{(1-\delta)}{DA^*_t} \hat{\hat{k}}^*_{t-1} + \varepsilon^{i*}_t \left(1 - S\left(\frac{\hat{\hat{i}}^*_t}{\hat{\hat{i}}^*_{t-1}} \right) \right) \hat{\hat{i}}^*_t$$

Capital utilization

$$C(u_t) = \gamma_{u1}(u_t - 1) + \frac{\gamma_{u2}}{2}(u_t - 1)^2$$

$$\frac{dC(u_t)}{du_t} = \gamma_{u1} + \gamma_{u2}(u_t - 1)$$

$$\frac{R^k_t}{P_t} = \frac{dC(u_t)}{du_t}$$

$$C(u^*_t) = \gamma^*_{u1}(u^*_t - 1) + \frac{\gamma^*_{u2}}{2}(u^*_t - 1)^2$$

$$\frac{dC(u^*_t)}{du^*_t} = \gamma^*_{u1} + \gamma^*_{u2}(u^*_t - 1)$$

$$\frac{R^{k*}_t}{P^*_t} = \frac{dC(u^*_t)}{du^*_t}$$

Intertemporal optimal condition

$$\beta \frac{\Lambda_{t+1}}{DA_{t+1}} \left(\frac{R^k_{t+1}}{P_{t+1}} u_{t+1} - C(u_{t+1}) + \frac{(1-\delta)}{\varepsilon_{i,t+1} \left(1 - S\left(\frac{\hat{i}_{t+1}}{\hat{i}_t}\right) - \hat{i}_{t+1} DA_{t+1} \left(\frac{\Theta}{\hat{i}_t} \left(\frac{DA_t \hat{i}_{t+1}}{\hat{i}_t} - 1 \right) \right) \right)} \right) = \\ = \frac{\Lambda_t}{\varepsilon_{i,t} \left(1 - S\left(\frac{\hat{i}_t}{\hat{i}_{t-1}}\right) - \hat{i}_t DA_t \left(\frac{\Theta}{\hat{i}_{t-1}} \left(\frac{DA_t \hat{i}_t}{\hat{i}_{t-1}} - 1 \right) \right) \right)}$$

$$\beta^* \frac{\hat{\Lambda}^*_{t+1}}{DA^*_{t+1}} \left(\frac{R^{k*}_{t+1}}{P^*_{t+1}} u^*_{t+1} - C(u^*_{t+1}) + \frac{(1-\delta^*)}{\varepsilon^*_{i,t+1} \left(1 - S\left(\frac{\hat{i}^*_{t+1}}{\hat{i}^*_t}\right) - \hat{i}^*_{t+1} DA^*_{t+1} \left(\frac{\Theta}{\hat{i}^*_t} \left(\frac{DA_t \hat{i}^*_{t+1}}{\hat{i}^*_t} - 1 \right) \right) \right)} \right) = \\ = \frac{\hat{\Lambda}^*_t}{\varepsilon^*_{i,t} \left(1 - S\left(\frac{\hat{i}^*_t}{\hat{i}^*_{t-1}}\right) - \hat{i}^*_t DA^*_t \left(\frac{\Theta}{\hat{i}^*_{t-1}} \left(\frac{DA_t \hat{i}^*_t}{\hat{i}^*_{t-1}} - 1 \right) \right) \right)}$$

Profits of capital goods producers

$$\hat{d^c_t} = \frac{R^k_t}{P_t} \hat{k}_t - \hat{i}_t - C(u_t) \hat{k}_{t-1}$$

$$\hat{d^{c*}_t} = \frac{R^{k*}_t}{P^*_t} \hat{k}^*_t - \hat{i}^*_t - C(u^*_t) \hat{k}^*_{t-1}$$

Definitions

$$\hat{k}_t = \frac{u_t \hat{k}_{t-1}}{DA_t}$$

$$S\left(\frac{\hat{i}_t}{\hat{i}_{t-1}}\right) = \frac{\Theta}{2} \left(\frac{DA_t \hat{i}_t}{\hat{i}_{t-1}} - 1 \right)^2$$

$$\hat{k}^*_t = \frac{u^*_t \hat{k}^*_{t-1}}{DA^*_t}$$

$$S\left(\frac{\hat{i}^*_t}{\hat{i}^*_{t-1}}\right) = \frac{\Theta^*}{2} \left(\frac{DA_t \hat{i}^*_t}{\hat{i}^*_{t-1}} - 1 \right)^2$$

Final goods producers

$$\hat{y}_t = \left[\eta^{\frac{\mu-1}{\mu}} \left(\hat{y}_{H,t} \right)^{\frac{1}{\mu}} + (1-\eta)^{\frac{\mu-1}{\mu}} \left(\hat{y}_{F,t} \right)^{\frac{1}{\mu}} \right]^{\mu}$$

$$\hat{y}^*_t = \left[\eta^{*\frac{\mu^*-1}{\mu^*}} \left(\hat{y}^*_{H,t} \right)^{\frac{1}{\mu^*}} + (1-\eta^*)^{\frac{\mu^*-1}{\mu^*}} \left(\hat{y}^*_{F,t} \right)^{\frac{1}{\mu^*}} \right]^{\mu^*}$$

Intermediate goods producers

Marginal cost

$$\frac{\frac{R_t^k}{P_t}}{\hat{w}_t} = \frac{\alpha}{1-\alpha}\frac{n_t}{\hat{k}_t}$$

$$p_{m,t} = \frac{1}{\alpha}\frac{R_t^k}{P_t}(\frac{1-\alpha}{\alpha}\frac{\frac{R_t^k}{P_t}}{\hat{w}_t})^{\alpha-1}$$

$$\frac{r_t^{k*}}{\hat{w}_t^*} = \frac{\alpha^*}{1-\alpha^*}\frac{n_t^*}{\hat{k}_t^*}$$

$$p_{m,t}^* = \frac{1}{\alpha^*}r_t^{k*}(\frac{1-\alpha^*}{\alpha^*}\frac{r_t^{k*}}{\hat{w}_t^*})^{\alpha^*-1}$$

Demands for final goods

$$\begin{aligned} \hat{y}_{F,t} &= (1-\eta)\left(p_{F,t}\right)^{\frac{\mu_{F,t}}{1-\mu_{F,t}}} \hat{y}_t \\ \hat{y}_{H,t} &= \eta\left(p_{H,t}\right)^{\frac{\mu_{H,t}}{1-\mu_{H,t}}} \hat{y}_t \end{aligned}$$

$$\begin{aligned} \hat{y}_{H,t}^* &= \eta^*\left(p_{H,t}^*\right)^{\frac{\mu_{H*,t}}{1-\mu_{H*,t}}} \hat{y}_t^* \\ \hat{y}_{F,t}^* &= (1-\eta^*)\left(p_{F,t}^*\right)^{\frac{\mu_{F*,t}}{1-\mu_{F*,t}}} \hat{y}_t^* \end{aligned}$$

Real prices

$$p_{H,t}^{\frac{1}{1-\mu_{H,t}}} = \theta_H\left(p_{H,t-1}\frac{\pi_{\zeta,t}}{\pi_t}\right)^{\frac{1}{1-\mu_{H,t}}} + (1-\theta_H)\left(\tilde{p}_{H,t}\right)^{\frac{1}{1-\mu_{H,t}}}$$

$$p_{F,t}^{\frac{1}{1-\mu_{F,t}}} = \theta_F\left(p_{F,t-1}\frac{\pi_{\zeta_F,t}}{\pi_t}\right)^{\frac{1}{1-\mu_{F,t}}} + (1-\theta_F)\left(\tilde{p}_{F,t}\right)^{\frac{1}{1-\mu_{F,t}}}$$

$$p_{H,t}^{*\frac{1}{1-\mu_{H*,t}}} = \theta_H^*\left(p_{H,t-1}^*\frac{\pi_{\zeta_{H*},t}}{\pi_t^*}\right)^{\frac{1}{1-\mu_{H*,t}}} + (1-\theta_H^*)\left(\tilde{p}_{H,t}^*\right)^{\frac{1}{1-\mu_{H*,t}}}$$

$$p_{F,t}^{*\frac{1}{1-\mu_{F*,t}}} = \theta_F^*\left(p_{F,t-1}^*\frac{\pi_{\zeta_F^*,t}}{\pi_t^*}\right)^{\frac{1}{1-\mu_{F*,t}}} + (1-\theta_F^*)\left(\tilde{p}_{F,t}^*\right)^{\frac{1}{1-\mu_{F*,t}}}$$

Indexation

$$\pi_{\zeta,t} = \pi_{\zeta,t}^{\zeta_H} \pi^{1-\zeta_H}$$

$$\pi_{\zeta_F,t} = \pi_{\zeta_F,t}^{\zeta_F} \pi^{1-\zeta_F}$$

$$\pi_{\zeta_{H*},t} = \pi_{\zeta_{H*},t}^{\zeta_{H*}} \pi^{1-\zeta_{H*}}$$

$$\pi_{\zeta_F^*,t} = \pi_{\zeta_F^*,t}^{\zeta_F^*} \pi^{1-\zeta_F^*}$$

Optimal price of intermediate goods

$$\tilde{p}_{H,t} = \mu_{H,t} \frac{\Omega_{H,t}}{\Upsilon_{H,t}}$$

$$\Omega_{H,t} = \hat{\Lambda}_t p_{m,t} p_{H,t}^{\frac{\mu_{H,t}}{\mu_{H,t}-1}} y\hat{_{H,t}} + \beta \theta_H E_t \left(\frac{\pi_{\zeta,t+1}}{\pi_{t+1}} \right)^{\frac{\mu_{H,t}}{1-\mu_{H,t}}} \Omega_{H,t+1}$$

$$\Upsilon_{H,t} = \hat{\Lambda}_t p_{H,t}^{\frac{\mu_{H,t}}{\mu_{H,t}-1}} y\hat{_{H,t}} + \beta \theta_H E_t \left(\frac{\pi_{\zeta,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_{H,t}}} \Upsilon_{H,t+1}$$

$$\tilde{p}_{F,t} = \mu_F, t \frac{\Omega_{F,t}}{\Upsilon_{F,t}}$$

$$\Omega_{F,t} = \hat{\Lambda}_t Q_t p_{m,t}^* p_{F,t}^{\frac{\mu_F,t}{\mu_F,t-1}} y\hat{_{F,t}} + \beta \theta_F E_t \left(\frac{\pi_{\zeta_F,t+1}}{\pi_{t+1}} \right)^{\frac{\mu_F,t}{1-\mu_F,t}} \Omega_{F,t+1}$$

$$\Upsilon_{F,t} = \hat{\Lambda}_t p_{F,t}^{\frac{\mu_F,t}{\mu_F,t-1}} y\hat{_{F,t}} + \beta \theta_F E_t \left(\frac{\pi_{\zeta_F,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_F,t}} \Upsilon_{F,t+1}$$

$$\tilde{p}_{H,t}^* = \mu_{H*,t} \frac{\Omega_{H,t}^*}{\Upsilon_{H,t}^*}$$

$$\Omega_{H,t}^* = \hat{\Lambda_t^*} \frac{p_{m,t}}{q_t} p_{H,t}^{*\frac{\mu_{H*,t}}{\mu_{H*,t}-1}} y\hat{_{H,t}^*} + \beta^* \theta_H^* E_t \left(\frac{\pi_{\zeta_{H*},t+1}}{\pi_{t+1}^*} \right)^{\frac{\mu_{H*,t}}{1-\mu_{H*,t}}} \Omega_{H,t+1}^*$$

$$\Upsilon_{H,t}^* = \hat{\Lambda_t^*} p_{H,t}^{*\frac{\mu_{H*,t}}{\mu_{H*,t}-1}} y\hat{_{H,t}^*} + \beta^* \theta_H^* E_t \left(\frac{\pi_{\zeta_{H*},t+1}}{\pi_{t+1}^*} \right)^{\frac{1}{1-\mu_{H*,t}}} \Upsilon_{H,t+1}^*$$

$$\tilde{p}^*_{F,t} = \mu_{F*,t}\frac{\Omega^*_{F,t}}{\Upsilon^*_{F,t}}$$

$$\Omega^*_{F,t} = \hat{\Lambda}^*_t p^*_{m,t} p_{F,t}^{*\frac{\mu_{F*,t}}{\mu_{F*,t-1}}} y^{\hat{*}}_{F,t} + \beta^*\theta^*_F E_t\left(\frac{\pi_{\zeta^*_F,t+1}}{\pi^*_{t+1}}\right)^{\frac{\mu_{F*,t}}{1-\mu_{F*,t}}}\Omega^*_{F,t+1}$$

$$\Upsilon^*_{F,t} = \hat{\Lambda}^*_t p_{F,t}^{*\frac{\mu_{F*,t}}{\mu_{F*,t-1}}} y^{\hat{*}}_{F,t} + \beta^*\theta^*_F E_t\left(\frac{\pi_{\zeta^*_F,t+1}}{\pi^*_{t+1}}\right)^{\frac{1}{1-\mu_{F*,t}}}\Upsilon^*_{F,t+1}$$

Profits of intermediate goods producers

$$\hat{d}^I_t = p_{H,t}\hat{y}_{H,t} + \frac{1-\omega}{\omega}q_t p^*_{H,t}\hat{y}^*_{H,t} - \hat{w}_t n_t - \frac{R^k_t}{P_t}\hat{k}_t$$

$$\hat{d}^{I*}_t = \frac{\omega}{1-\omega}p_{F,t}\hat{y}_{F,t}\frac{1}{q_t} + p^*_{F,t}\hat{y}^*_{F,t} - \hat{w}^*_t n^*_t - \frac{R^{k*}_t}{P^*_t}\hat{k}^*_t$$

Goods market clearing

$$\hat{y}_t = \hat{c}_t + \hat{i}_t + \hat{g}_t - C(u_t)\hat{k}_{t-1}$$

$$\hat{y}^*_t = \hat{c}^*_t + \hat{i}^*_t + \hat{g}^*_t - C(u^*_t)\hat{k}^*_{t-1}$$

$$g\hat{d}p_t = \hat{y}_{H,t}\Delta_{H,t} + \frac{1-\omega}{\omega}\hat{y}^*_{H,t}\Delta^*_{H,t} \quad = \quad \hat{k}_t{}^{\alpha} n_t{}^{1-\alpha} - \hat{\phi}$$

$$g\hat{d}p^*_t = \frac{\omega}{1-\omega}\hat{y}_{F,t}\Delta_{F,t} + \hat{y}^*_{F,t}\Delta^*_{F,t} \quad = \hat{k}^{*\alpha*}_t (n^*_t)^{1-\alpha*} - \hat{\phi}^*$$

Price dispersion

$$\Delta_{H,t} = \left(\frac{p_{H,t}}{p_{H,t-1}}\right)^{\frac{1}{\mu_{H,t-1}}}\theta_H\Delta_{H,t-1}\left(\frac{\pi_{\zeta,t}}{\pi_t}\right)^{\frac{1}{1-\mu_{H,t}}} + (1-\theta_H)\left(\frac{\tilde{p}_{H,t}}{p_{H,t}}\right)^{\frac{1}{1-\mu_{H,t}}}$$

$$\Delta^*_{H,t} = \left(\frac{p^*_{H,t}}{p^*_{H,t-1}}\right)^{\frac{1}{\mu_{H*,t-1}}}\theta^*_H\Delta^*_{H,t-1}\left(\frac{\pi_{\zeta_{H*},t}}{\pi^*_t}\right)^{\frac{1}{1-\mu_{H*,t}}} + (1-\theta^*_H)\left(\frac{\tilde{p}^*_{H,t}}{p^*_{H,t}}\right)^{\frac{1}{1-\mu_{H*,t}}}$$

$$\Delta_{F,t} = \left(\frac{p_{F,t}}{p_{F,t-1}}\right)^{\frac{1}{\mu_{F,t-1}}}\theta_F\Delta_{F,t-1}\left(\frac{\pi_{\zeta_F,t}}{\pi_t}\right)^{\frac{1}{1-\mu_{F,t}}} + (1-\theta_F)\left(\frac{\tilde{p}_{F,t}}{p_{F,t}}\right)^{\frac{1}{1-\mu_{F,t}}}$$

$$\Delta^*_{F,t} = \left(\frac{p^*_{F,t}}{p^*_{F,t-1}}\right)^{\frac{1}{\mu_{F*,t}-1}} \theta^*_F \Delta^*_{F,t-1} \left(\frac{\pi \zeta^*_{F},t}{\pi^*_t}\right)^{\frac{1}{1-\mu_{F*,t}}} + (1-\theta^*_F) \left(\frac{\tilde{p}^*_{F,t}}{p^*_{F,t}}\right)^{\frac{1}{1-\mu_{F*,t}}}$$

Bond markets and NFA

$$\omega \hat{b}_{F,t} + (1-\omega)\hat{b}^*_{F,t} = 0$$

$$nfa_t \equiv \frac{\omega q_t \hat{b}_{F,t}}{g\hat{d}p_t}$$

Public sector

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_r} \left(\left(\frac{\pi_t}{\pi}\right)^{1+\gamma_\pi} \left(\frac{gdp_t}{gdp}\right)^{\gamma_y}\right)^{1-\gamma_r} \exp\{\varepsilon_{r,t}\}$$

$$\frac{R^*_t}{R^*} = \left(\frac{R^*_{t-1}}{R^*}\right)^{\gamma^*_r} \left[\left(\frac{\pi^*_t}{\pi^*}\right)^{1+\gamma^*_\pi} \left(\frac{g\hat{d}p^*_t}{g\hat{d}p^*}\right)^{\gamma^*_y}\right]^{1-\gamma^*_r} \exp\{\varepsilon_{r*,t}\}$$

$$\hat{t}_t = \hat{g}_t = \hat{g}\varepsilon_{g,t}$$

$$\hat{t}^*_t = \hat{g}^*_t = \hat{g}^*\varepsilon_{g*,t}$$

Appendix C. Kalman filter and model solution

As described in Section 3.1, consumers in both economies face a signal extraction problem which they solve by running the Kalman filter. Below this problem and its solution are presented. Let $\boldsymbol{X_t} = \begin{bmatrix} \mathring{x}_t & \mathring{x}_{t-1} & \mathring{z}_t & \mathring{x}^*_t & \mathring{x}^*_{t-1} & \mathring{z}^*_t \end{bmatrix}'$ denote the state vector, $\boldsymbol{S_t} = \begin{bmatrix} \mathring{a}_t & \mathring{s}_t & \mathring{a}^*_t & \mathring{s}^*_t \end{bmatrix}'$ the vector of observables and $\boldsymbol{\varepsilon_t} = \begin{bmatrix} \varepsilon_{x,t} & \varepsilon_{z,t} & \varepsilon_{s,t} & \varepsilon^*_{x,t} & \varepsilon^*_{z,t} & \varepsilon^*_{s,t} \end{bmatrix}'$ the vector of shocks (rings above variables denote log-deviations from steady state). After log-linearization the system (5)-(8) can be presented in matrix notation as follows.

$$\boldsymbol{X_t} = \boldsymbol{A X_{t-1}} + \boldsymbol{B \varepsilon_t} \tag{38}$$

$$\boldsymbol{S_t} = \boldsymbol{C X_t} + \boldsymbol{D \varepsilon_t} \tag{39}$$

where:

$$\boldsymbol{A} = \begin{bmatrix} 1+\rho & -\rho & 0 & & & \\ 1 & 0 & 0 & & 0 & \\ 0 & 0 & \rho & & & \\ & & & 1+\rho^* & -\rho^* & 0 \\ & 0 & & 1 & 0 & 0 \\ & & & 0 & 0 & \rho^* \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 & & & \\ 0 & 0 & 0 & & 0 & \\ 0 & 1 & 0 & & & \\ & & & 1 & 0 & 0 \\ & 0 & & 0 & 0 & 0 \\ & & & 0 & 1 & 0 \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} 1-\lambda^x & 0 & 1 & \lambda^x & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \ \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 & & & \\ 0 & 0 & 1 & & 0 & \\ & 0 & & 0 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}.$$

The variance-covariance matrices $\boldsymbol{S_1} = E_t\left[\boldsymbol{B\varepsilon_t\varepsilon_t'B'}\right]$ and $\boldsymbol{S_2} = E_t\left[\boldsymbol{D\varepsilon_t\varepsilon_t'D'}\right]$ are given by:

$$\boldsymbol{S_1} = \begin{bmatrix} (1-\rho)^2\sigma_u^2 & 0 & 0 & & & \\ 0 & 0 & 0 & & 0 & \\ 0 & 0 & \rho\sigma_u^2 & & & \\ & & & (1-\rho^*)^2(\sigma_u^*)^2 & 0 & 0 \\ & 0 & & 0 & 0 & 0 \\ & & & 0 & 0 & \rho^*(\sigma_u^*)^2 \end{bmatrix},$$

$$\boldsymbol{S_2} = \begin{bmatrix} 0 & 0 & & \\ 0 & \sigma_s^2 & & 0 \\ & 0 & 0 & 0 \\ & & 0 & (\sigma_s^*)^2 \end{bmatrix}.$$

Agents form expectations based on the Kalman filter. Hence, the evolution of the expected state vector follows:

$$\boldsymbol{X_{t|t}} = \boldsymbol{AX_{t-1|t-1}} + \boldsymbol{K}\left(\boldsymbol{S_t} - \boldsymbol{S_{t|t-1}}\right) \tag{40}$$

where

$$\boldsymbol{K} = \boldsymbol{PC'}\left(\boldsymbol{CPC'} + \boldsymbol{S_2}\right)^{-1}$$

is the Kalman gain matrix and

$$\boldsymbol{P} = \boldsymbol{A}[\boldsymbol{P} - \boldsymbol{PC'}\left(\boldsymbol{CPC'} + \boldsymbol{S_2}\right)^{-1}\boldsymbol{CP}]\boldsymbol{A'} + \boldsymbol{S_1}$$

captures uncertainty of the state vector (see Hamilton 1994, p. 380 for details).

Use (38) and (39) to derive $\boldsymbol{S}_{t|t-1} = \boldsymbol{C}\boldsymbol{X}_{t|t-1} = \boldsymbol{C}\boldsymbol{A}\boldsymbol{X}_{t-1|t-1}$ and substitute into (40):

$$\begin{aligned} \boldsymbol{X}_{t|t} &= \boldsymbol{A}\boldsymbol{X}_{t-1|t-1} + \boldsymbol{K}\left(\boldsymbol{S}_t - \boldsymbol{C}\boldsymbol{A}\boldsymbol{X}_{t-1|t-1}\right) \\ &= (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C}\boldsymbol{A})\boldsymbol{X}_{t-1|t-1} + \boldsymbol{K}\boldsymbol{S}_t \end{aligned} \tag{41}$$

Then substitute for

$$\boldsymbol{S}_t = \boldsymbol{C}\boldsymbol{X}_t + \boldsymbol{D}\boldsymbol{\varepsilon}_t = \boldsymbol{C}(\boldsymbol{A}\boldsymbol{X}_{t-1} + \boldsymbol{B}\boldsymbol{\varepsilon}_t) + \boldsymbol{D}\boldsymbol{\varepsilon}_t = \boldsymbol{C}\boldsymbol{A}\boldsymbol{X}_{t-1} + (\boldsymbol{C}\boldsymbol{B} + \boldsymbol{D})\,\boldsymbol{\varepsilon}_t$$

to get:

$$\boldsymbol{X}_{t|t} = (\boldsymbol{A} - \boldsymbol{K}\boldsymbol{C}\boldsymbol{A})\boldsymbol{X}_{t-1|t-1} + \boldsymbol{K}\left(\boldsymbol{C}\boldsymbol{A}\boldsymbol{X}_{t-1} + (\boldsymbol{C}\boldsymbol{B} + \boldsymbol{D})\,\boldsymbol{\varepsilon}_t\right) \tag{42}$$

Agents use this equation to form expectations of the state vector.

Since in the linearized model certainty equivalence holds, agents treat these expectations like true state variables. The model solution under imperfect information is based on the same laws of motion (policy functions) as the perfect information model, whereas the unobserved state variables are replaced by their estimates from the Kalman filter (see Hamilton 1994; Hürtgen 2014 for details).

Finaly, the imperfect information model is observationally equivalent to its perfect information counterpart with correlated shocks. This result is used to obtain impulse responses, variance and historical decompositions presented in the paper (see Lemma 2 in Blanchard et al., 2013).