COLLEGIUM OF ECONOMIC ANALYSIS
WORKING PAPER SERIES

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SGH KAE Working Papers Series Number: 2020/059 December 2020

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December 2020

Abstract

This paper reevaluates the complementary symmetry hypothesis and the supporting experimental evidence. Originally the hypothesis was stated for binary risky prospects. We generalize the hypothesis to arbitrary state-contingent real-valued acts, thus extending the domain from risk to uncertainty/ambiguity and allowing for multiple outcomes. Existing experiments tested the hypothesis using selling and buying prices and found systematic violations. We argue that in order to be consistent with the hypothesis one should replace selling with short-selling. We thus define a new elicitation task and run an experiment to test our conjecture. We replicate previously observed violations in the old setting and find strong support for the hypothesis in the new setting. In addition, our results shed new light on the validity of various reference point setting rules.

Keywords: complementary symmetry; short selling price; buying price; reference dependence.

1 Introduction

Birnbaum and Zimmermann (1998) and Birnbaum et al. (2016) introduced a complementary symmetry (CS) hypothesis for binary monetary prospects. This hypothesis holds if

*We want to thank Peter Wakker and Jacek Chudziak for their helpful suggestions on the previous draft of this paper. Special thanks to Joanna Sokołowska-Pohorille and Katarzyna Idzikowska for their help in preparing the experimental study whose results we refer to in this paper. All errors remain ours. The project was financed by the National Science Center: NCN grant number UMO-2019/35/B/HS4/00346.

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the sum of the buying price of the prospect in which \$$x$ is paid out with probability p and \$$y$ otherwise and the selling price of the complementary prospect in which \$$x$ is paid out with probability $1-p$ and \$$y$ otherwise equals $x+y$. Our paper extends this hypothesis to arbitrary prospects in $\mathbb{R}^N$, i.e. with multiple instead of binary outcomes. Moreover, we show that the property does not rely on the existence of a probability distribution over outcomes, and thus not only holds under the conditions of risk, but also under uncertainty and ambiguity. For this reason, we will henceforth refer to a prospect as a real-valued random variable defined on some finite state space N. Now, let L be a prospect and suppose $-L$ is the same as L but with prizes being the negative of those in L. Note that "relinquishing L" is consequentially equivalent as "acquiring $-L$", just as losing 100 dollars is the same as receiving -100 dollars. We will refer to this basic fact as the symmetry. In this context, our general question is how to use this symmetry in eliciting appropriate buying and selling prices, test CS experimentally, and finally, validate some of the rules of setting the relevant reference points. When doing so we need to address two challenges that we discuss now.

First, we need to relate the actual elicitation tasks with the appropriately chosen buying and selling prices. The propensity to acquire a prospect is captured by the task of eliciting its buying price, i.e., the maximal monetary amount the decision maker is willing to pay to acquire a prospect. Our first question is: what is the corresponding task that would capture the propensity to relinquish a prospect in a way that would allow us to use the symmetry. The first natural candidate is elicitation of a selling price, i.e. the minimal monetary amount the decision maker is willing to accept to relinquish a prospect. Note however, that selling, as opposed to buying, implicitly assumes that the decision maker initially owns (the right to) the prospect. This difference in initial position may impact prospect valuation, especially when the prospect is nondegenerate, and thus destroy the symmetry between the two tasks. We thus borrow the notion of short-selling, well known in the context of financial assets. Short-selling a prospect, also known as taking a short position in it, means selling the prospect without having it at the time of transaction. In our context, selling and short-selling can be also distinguished by the person paying out the prizes. In case of selling, you sell the right to the prospect that you have been endowed with. In case of short-selling, on the other hand, the right is sold to the prospect whose prizes will be paid out by yourself.

Second, since we are interested in experimental testing, we should design the price elicitation tasks in a way that is understood by human subjects. Specifically, we want to ensure the prospects we ask people to consider present positive value to them. Otherwise, they might have difficulty in grasping the idea of paying or receiving a negative amount which would correspond to the value of an undesirable prospect. While it is safe to assume that a prospect L containing only nonnegative prizes is a valuable object, it is just as certain that almost nobody would find its negative $-L$ desirable. So, before we ask people to consider buying or short-selling $-L$ we will shift all of its prizes upwards to make sure people find it attractive. We will do it in a way that controls for possible framing effects to minimize their impact on the symmetry.

Studies on the complementary symmetry hypothesis were started by Birnbaum and Zimmermann (1998). Specifically, they defined an extension of cumulative prospect theory in which the reference point can be random (the idea later adopted by Schmidt et al., 2008 in the so called third-generation prospect theory, TGPT) and thus allowed them to model selling prices in a spirit consistent with prospect theory, i.e. the idea of evaluating net changes of wealth with respect to the status quo wealth. They thus defined buying and selling prices and obtained the complementary symmetry property relating these prices. Birnbaum et al. (2016) showed that the CS property holds for binary prospects under the extended version of prospect theory using some popular parametric assumptions. On the other hand, the property has been shown to fail in experimental settings (Birnbaum and Sutton, 1992; Birnbaum and Zimmermann, 1998; Birnbaum et al., 2016), the fact used by Birnbaum (2018) to question TGPT. Since then, in several subsequent theoretical work (Lewandowski, 2018; Chudziak, 2020; Wakker, 2020), the CS property was showed to hold more generally than implied by Birnbaum et al. (2016). In particular, Wakker (2020) showed that it holds for any binary relation on any subset of binary prospects, implying that the CS property is not a property of a specific preference model (TGPT, in particular), but a property of buying and selling prices as defined by Birnbaum and Zimmermann (1998), irrespectively of preferences.

Taking this background into account, the contribution of our paper is fourfold. First, the original CS property was only defined for probability distributions of binary prospects. We generalize it to *multiple outcome prospects*, with or without probability distribution. Thus, our approach is suitable for decisions under risk, but also naturally extends to the context of uncertainty or ambiguity, in which case probability distributions may either be unknown or not even well-defined. Second, Birnbaum and Zimmermann (1998) and Schmidt et al. (2008) defined the selling price of a prospect L as a scalar S satisfying the following indifference: $S - L \sim 0$, where $\sim$ denotes the symmetric part of a preference relation over prospects.[1] For the reasons outlined above, we will refer to such price as the *short-selling price* rather than the *selling price* of L. This parallels our previous discussion on defining a complementary (symmetric) task to buying. Consequently, one may question the validity of previous experimental tests of the CS property, that elicited selling instead of short-selling prices. In this paper, we design the task of short-selling and experimentally test the CS property in this new setting. This is our third contribution. Fourth and finally, to relate formulas such as $S-L \sim 0$ with the corresponding testable task, we need to show how the choice alternatives such as $S - L$ or 0, defined over wealth changes, are derived from a more basic choice alternatives over wealth levels. Consequently, in such richer reference dependent model, we show how the CS property can be used to test various reference point setting rules.

2 Generalized complementary symmetry

Let N be a finite set of states of nature. Object of choice are prospects – state contingent real-valued outcomes denoted by $L \in \mathbb{R}^N$. If there is an objective probability measure on $N = 2$, then objects of the form $(x, y; p)$ denote the probability distribution of a prospect with values x, y.[2] For a scalar λ and prospect L, we write $L + \lambda$ to denote $L + \lambda\mathbb{1}$, where

[1] $S - L$ and 0 denote prospects with prizes equal to, respectively, $S - L(n)$ and 0 for each state $n \in N$.

[2] More generally, given prospect $L : N \to \mathbb{R}$ and a probability measure π on N, one derives the probability distribution P_L of L by setting $P_L(x) = \sum_{n \in N: L(n)=x} \pi(n)$ for any $x \in \mathbb{R}$. Decisions under risk occur whenever the decision maker is indifferent between any two prospects that have identical probability distributions. Otherwise, i.e. either if there is no exogenously given probability measure π, or preferences depend on states, we refer to decisions under uncertainty or ambiguity.

$\mathbb{1}$ is a unit vector in $\mathbb{R}^N$. Preferences are given by a binary relation $\succcurlyeq$ on the set of prospects.

Following the literature, the buying price of prospect L, denoted by $B(L)$, and the complementary selling price of L, denoted by $B^*(L)$, are scalars satisfying:

$$L - B(L) \sim 0, \tag{1}$$

$$B^*(L) - L \sim 0. \tag{2}$$

In what follows we assume that for any prospect L, $B(L)$ and $B^*(L)$ exist and are unique.[3] Then, it is immediate to observe that both B and B^* satisfy the translation invariance property, i.e. for any prospect L and $\lambda \in \mathbb{R}$, $B(L+\lambda) = B(L) + \lambda$ (same for B^*) as well as the symmetry:[4] $B^*(-L) = -B(L)$. We say that prospects (L', L'') are perfect hedges if they satisfy $L' + L'' = \theta$ for some scalar θ. Note that L' and L'' exhibit maximal negative correlation with each other: accepting a portfolio of L' and L'' removes uncertainty completely. We now state our first result.

Proposition 1. *For perfect hedges (L', L'') the following holds:*

$$B(L') + B^*(L'') = \theta. \tag{3}$$

Proof. Using the symmetry property of B and B^* and the translation invariance of B^*, we have $0 = B(L') - B(L') = B(L') + B^*(-L') = B(L') + B^*[\theta - L'] - \theta$, thus obtaining (3). $\square$

Next observe that, if (L', L'') are perfect hedges, they can be written as $L' = L + \lambda$, $L'' = \theta - \lambda - L$ for any $\lambda \in \mathbb{R}$ and any prospect L. This observation allows us to state our main result on generalized complementary symmetry property.

Corollary 1 (Generalized Complementary Symmetry)**.** *Let L be a prospect. The following*

[3] Generalizations including non-uniqueness are possible. See Wakker (2020) for relevant results and ideas that can be extended to prospects in $\mathbb{R}^N$.

[4] Both follow from similar reasoning as in Chudziak (2020).

holds for any pair of scalars λ, θ:

$$B(L+\lambda) + B^*(\theta - \lambda - L) = \theta. \tag{4}$$

Proof. Note that the prospects $L + \lambda$ and $\theta - \lambda - L$ are perfect hedges for any pair of scalars θ, λ and hence the result is true by a direct application of Proposition 1. $\square$

This result is important for few reasons. First, it generalizes the previous results on CS in the literature from probability distributions of binary prospects to prospects over $\mathbb{R}^N$. Indeed, for $N = 2$ taking for example $L = (x, y)$ and letting $\theta = x + y$ with $\lambda = 0$ we obtain the standard CS. Second, the corollary shows that the complementary symmetry property follows immediately from general properties of translation invariance and symmetry. Third, as it is clear from the construction, complementary symmetry does not depend on the (existence of) underlying probability distribution. This shows its validity for arbitrary situations involving uncertainty or ambiguity. In what follows we will propose a way of selecting λ and θ so that complementary symmetry can be appropriately tested.

2.1 Defining the right task

To use (3) or (4) for testing, we need to propose an experimental task for eliciting prices that are represented by B and B^*. Put differently, we ask which observable choice tasks correspond to (3) or (4).

Birnbaum and Zimmermann (1998) and Schmidt et al. (2008) argued that $B(L)$ in (1) represents the buying and $B^*(L)$ in (2) the selling price[5] of L. While we agree with the first part concerning $B(L)$, we believe that (2) represents the task of *short-selling* rather than *selling*. $B^*(L)$ is thus the short-selling price of L, i.e. the minimal price that the decision maker would accept to take the short-position in L.

The intuition is simple. Consider a nondegenerate gain prospect L'. Most people like the (positive) prizes it offers but dislike the uncertainty involved in getting them. The

[5]The notions of buy and sell prices in the field of financial mathematics are also defined in analogous way. See e.g. Carmona (2008).

way to eliminate the uncertainty is to accept L' jointly with another gain prospect L'' such that $L' + L'' = \theta$ for some scalar θ. Then (L', L'') are perfect hedges. Suppose that we agree that the task of determining the buying price is represented by (1) but are not sure which observable (and testable) task corresponds to (2). Suppose we are looking for the complementary task to buying L' that would eliminate the uncertainty just as hedging, i.e. we are looking for B^* satisfying $\theta = B(L' + L'') = B(L') + B^*$. We will derive it from generalized complementary symmetry. For this reason, consider buying $-L''$ and observe this task is the same as short-selling L'', and so we set $B^* := -B(-L'')$. Substituting it into (1) we get $0 \sim -L'' - B(-L'') = B^* - L''$. This is precisely the definition of $B^*(L'')$ in (2), and so by Proposition 1 we obtain the desired $\theta = B(L') + B^*$.

2.2 Experimental design

Note that properties (3) and (4) hold irrespective of the value of θ or/and λ. However, it was demonstrated that framing effects may have an impact on the valuation of a prospect (e.g. Sayman and Öncüler, 2005). In particular, the relative attractiveness of a prospect may change depending on the choice task being presented either in the loss or in the gain frame (McClelland and Schulze, 1991; Irwin, 1994). As argued in the introduction, it is semantically awkward to ask for a buying or a short-selling price of something unwanted, i.e. the decision maker would prefer to opt out even if the object was for free. This is the case of a loss prospect that does not contain gains: experimental results document many subjects choose corner solutions in this case, to reflect their lack of acceptance of such a task *per se*. Mixed prospects are also problematic as judging their attractiveness depends on the decision maker attitude to gains vs. losses (a given mixed prospect may seem better than the status quo for one decision maker and worse for another). On the other hand, it is relatively safe to assume people like money so that they would not turn down the offer in which they cannot lose. These two observations a.o. suggest we shall use gain prospects when designing the experimental tasks of eliciting prices. We also want to control for possible range[6] effects that are known to have potential effects on the

[6]By range we mean the minimal and the maximal element of a prospect.

evaluation of prospects (Mellers et al., 1992; Kontek and Lewandowski, 2018).

Recall that B and B^* are shift-invariant so shifting all the outcomes of the evaluated prospects does not destroy the symmetry. However, we want to choose the values by which we shift the outcomes in a way that controls for the framing effects discussed above. For this reason we choose the values of θ and λ such that $L' = L+\lambda$ and $L'' = \theta-\lambda-L$ are both gain prospects and their ranges coincide, i.e. $\max(L') = \max(L'')$, $\min(L') = \min(L'') \geq 0$, where $\max(L) = \max_{n\in N} L(n)$, $\min(L) = \min_{n\in N} L(n)$. It is easily verified that the latter two conditions imply $\theta - 2\lambda = \max(L) + \min(L)$ and the former imply $\lambda \geq -\min(L)$ and $\theta - \lambda \geq \max(L)$.

For example, suppose L is a gain prospect, then setting $\lambda = 0$ gives $\theta = \max(L) + \min(L)$ and $L' = L$, $L'' = \max(L) + \min(L) - L$, precisely the choice of prospects used by Birnbaum and Zimmermann (1998) and others. If L is not a gain prospect, one may set $\lambda = -\min(L)$ and hence $\theta = \max(L) - \min(L)$ and $L' = L - \min(L)$ and $L'' = \max(L) - L$. To see that follow the example. For $N = 2$ (like in Birnbaum et al. (2016)) and $L' = (48, 60)$ we have $\theta = 108$ and $L'' := \theta - L' = (60, 48)$. For $N = 4$ with $L' = (80, 100, 120, 200)$ we should impose $\theta = 280$ and $L'' := \theta - L' = (200, 180, 160, 80)$.

3 Implications for reference point rules

So far, we have argued that the difference between selling and short selling tasks is the initial position. To observe and analyze that explicitly in formulas (1) and (2) we need to consider *wealth levels* and not only their changes. Specifically, until now preferences were defined on prospects, i.e. state contingent acts where prizes are interpreted as *changes of wealth* relative to *some* reference point. Clearly, wealth changes depend critically on choice of such reference wealth levels. Formal models of reference dependence have been developed theoretically by Sugden (2003), used by Schmidt et al. (2008) and tested by Baillon et al. (2020), a.o. In applications involving monetary prizes authors usually assume a special case of this general model in which prospects are determined as *differences* between considered acts and some reference acts.[7] Following that approach, we now consider various rules of setting the reference point, that are empirically grounded but also relevant for testing the CS property.

Specifically, Baillon et al. (2020) analyze six popular reference point setting rules. Here we take best four,[8] namely: Status Quo, MaxMin, MinMax and Prospect Itself. In the context of buying and selling task that we analyze, it is important to *specify* the status quo and the prospect itself rules. Both are derived from the current wealth position which may consist of two parts: one for which all previous uncertainty has been resolved and second for which uncertainty remains unresolved. We assume that all uncertainty and background risk *irrelevant* for the current decision has been resolved prior to making decision.

There might be uncertainty, however, that is *relevant* for the decision problem. In particular, this is the case of selling (the right to) prospect L. In this task the uncertainty concerning the prospect being sold is unresolved at the time of transaction and the initial wealth position is $W + L$, where the constant prospect W represents the wealth level. Then, in the case of selling, the prospect itself rule sets the reference at the whole current wealth position (i.e. $W + L$), while the status quo rule, sets it at the deterministic part (i.e. W). Henceforth we shall refer to the prospect itself rule as the "random status quo" rule, as the name captures the difference with respect to the "status quo" rule. We also combine the Maxmin and the Minmax rules into one since they coincide in our context.

According to the reference model (Sugden, 2003) for each task we must specify three acts, each of them defined over wealth levels: the two acts among which the choice is made and the reference act that is used to define (reference dependent) preferences. In what follows we consider three elicitation tasks, i.e.: buying price, selling price, and short-selling price. Following the previous discussion, we denote the initial wealth for which all previous uncertainty has been resolved at the time of decision by a constant act W.

[7] That is, prospects are derived from state-contingent acts where prizes are interpreted as *wealth levels*. Although formally indistinguishable, we refer to *acts* if the prizes are wealth levels and to *prospects* if the prizes are wealth changes.

[8] Excluding those that performed particularly bad as measured either by marginal posterior distribution or by a proportion of sharply classified respondents satisfying a particular reference point rule.

Table 1: Decision problems with different reference rules.

Tasks	Act 1	Act 2	SQ	RSQ	MM
Buying	$W+L-B$	W	W	W	W
Selling	$W+S$	$W+L$	W	$W+L$	$W+S$
Short-selling	$W-L+B^*$	W	W	W	W

Table 2: Evaluated prospects (wealth changes) under different reference point rules.

Tasks	SQ		RSQ		MM	
Buying	$L-B$	0	$L-B$	0	$L-B$	0
Selling	S	L	$S-L$	0	0	$L-S$
Short-selling	B^*-L	0	B^*-L	0	B^*-L	0

Table 1 lists the relevant acts for all three elicitation tasks and the three distinct reference point setting rules: (i) SQ: deterministic status quo wealth; (ii) RSQ: status quo wealth allowed to be random; and (iii) MM: the maxmin (or minmax) of the two acts. Note that the SQ rule does not differ in all considered tasks from a fixed reference point with no re-framing.[9]

Table 2 gives the corresponding prospects. Note that buying price and short-selling prices each have the same prospect representation under all three rules. On the other hand, selling price has a different prospect representation under each of these rules. We now present the testable predictions of each of these rules and discuss complementary symmetry properties between the three tasks.

Under the SQ rule B, S and B^* are different in general. Moreover, applying the same reasoning as in section 2, it is clear that the complementary symmetry property holds only for buying and short-selling prices, i.e. for the pair (B, B^*). The situation is different under the RSQ rule. In such case, not only $S = B^*$ but also complementary symmetry can be stated for *both pairs*: (B, S), (B, B^*). This is a stark difference to the SQ rule. Finally, under the MM rule we obtain that $S = B$ and the complementary symmetry can be tested for both pairs: (B, B^*), (S, B^*).

Observe, that the generalized complementary symmetry as defined in this paper is implied by all three models of setting the reference point rules, while the symmetry

[9]Fixed reference point is in fact equivalent to the expected utility of wealth model in which wealth is taken to be equal to 0. Such as model, unlike the reference-dependent models that allow for reframing, shares strong normative properties with the standard EU model.

between buying and selling only for the RSQ model. As a result, evidence on violations of the symmetry between buying and selling prices can be interpreted as a violation of the RSQ model.

4 Experimental results

In a series of experiments, Birnbaum and others (Birnbaum and Sutton, 1992; Birnbaum and Zimmermann, 1998; Birnbaum et al., 2016) have tested complementary symmetry prediction and found it to be systematically refuted. That is, the sum of the median reported buying price of $(x, y; p)$ and the median reported selling price of $(x, y; 1-p)$ was found to be always below $x + y$ and the deviation from this benchmark increased with the range, i.e. $|x - y|$. For example, for $x = 48, y = 60, p = 0.5$ the sum of buying and selling prices was 104, being slightly below $x + y = 108$, while for $x = 12, y = 96, p = 0.5$, the sum dropped to 75, significantly below $x + y = 108$.

These experimental findings, which were originally used by Birnbaum (2018) to question the third-generation prospect theory should be reevaluated in light of our findings. We have argued that (2) does not represent the selling but the short-selling price. This suggests the previous experimental results did not test the CS property. Indeed, as showed by Chudziak (2020) (theorem 2.2) after accepting definition of the buying price B, complementary symmetry holds if and only if one accepts definition of the (complementary) selling price.

Our experiment involved 38, mainly student, subjects from Warsaw School of Economics and University of Georgia. As our focus is on testing the CS property, we elicited from each subject three prices: buying, selling and short-selling for one of four available equal chance binary prospects: $(48, 60), (36, 72), (24, 84), (12, 96)$, outcomes measured in dollars. Observe that the outcomes in each gamble sum up to 108 dollars.

We first report the results of a between subject design to compare them to Birnbaum (2018) who also used such data. The results involving median prices are summarized in table 3 and show that the CS tested with the use of buying and short-selling prices is confirmed in all ranges (in fact is exactly confirmed in three out of four). In contrast, the

Table 3: CS with selling vs. short-selling prices.

ranges	$96-12$	$84-24$	$72-36$	$60-48$
$B+S$	73	101	100	108
$B+B^*$	103	108	108	108
number of subjects	11	6	10	11

CS tested with the use of buying and selling prices is violated. Observe, our results also replicate Birnbaum (2018) finding: the sum of median buying and selling prices is below $x+y$ for each x, y and decreases with $|y-x|$, while the same is not true if selling price is replaced by a short-selling price.

Next, in difference to Birnbaum (2018) we have elicited all three prices (B, S, B^*) for each subject. This within-subject design allows us to test the CS property at the individual level and not using the median prices only. The advantage is that we can capture the heterogeneity of subjects and check whether each subject fulfills the CS property of a given kind or not.

For 20 (out of 38) of our subjects the sum of reported buying and short selling prices equals exactly 108. Nevertheless, to avoid testing such knife-edge predictions, we decided to classify all cases with the sum of reported prices in $[100, 116]$ as fulfillment of the tested CS property, otherwise as the violation. Specifically, since we have all three prices for each subject, we can test each of the three possible CS properties, i.e. for pairs (B, B^*), (B, S) and (S, B^*) for each individual. We classified all observations into those satisfying the three complementary symmetry hypotheses: (B, B^*) only, (B, B^*), (B, S) only, and (B, B^*), (S, B^*) only; or none of them. These correspond to the three reference point setting rules developed in section 3. Indeed, the SQ rule implies the (B, B^*) complementary symmetry only; the RSQ rule implies (B, B^*) and (B, S) CS while the MM rule implies (B, B^*), (S, B^*) CS.[10]

Table 4 summarizes our results. There are three important conclusions in the view of the reference point setting rules. First, there is a significant number of subjects that supports only the SQ rule and not the others. Second, there is 45-50% of subjects that

[10] Note that the support for the RSQ or the MM rules is fully contained in the support of the SQ rule. This per se is an argument in favor of the latter rule.

Table 4: Validity of CS in a within-subject experiment. Clusters group subjects according to the particular CS being satisfied (1) or not (0).

subject cluster:	1	2	3	4	5	6	7	8	Sum
CS: $B+S$	1	1	0	1	1	0	0	0	
CS: $B+B^*$	1	1	1	0	0	1	0	0	
CS: $S+B^*$	1	0	1	1	0	0	1	0	
no. of subjects	11	7	6	0	1	8	3	2	38

support either the MM rule or the RSQ rule, meaning that none of them is clearly better than another. Importantly, around 85% of subjects satisfy the $B+B^*$ CS property (and thus supports the SQ rule). And third, out of 12 subjects satisfying only one out of three CS properties, 8 subjects (or 2/3) satisfied the $B+B^*$ CS property.

Concluding, in this paper we have tested the validity of the CS for gambles directly comparable to those of Birnbaum and Sutton (1992) or Birnbaum (2018). Further experimental tests using short selling prices are necessary to validate the CS hypothesis in more general contexts. Specifically, the generalized CS property, as defined in our paper, could be tested for more general prospect types e.g. multiple outcome ones with well-defined probability distribution but also including uncertainty or ambiguity contexts. We leave it for further studies.

References

Baillon, A., H. Bleichrodt, and V. Spinu (2020). Searching for the reference point. *Management Science 66*(1), 93–112.

Birnbaum, M. H. (2018). Empirical evaluation of third-generation prospect theory. *Theory and Decision 84*(1), 11–27.

Birnbaum, M. H. and S. E. Sutton (1992). Scale convergence and utility measurement. *Organizational Behavior and Human Decision Processes 52*(2), 183–215.

Birnbaum, M. H., S. Yeary, R. D. Luce, and L. Zhao (2016). Empirical evaluation of four models of buying and selling prices of gambles. *Journal of Mathematical Psychology 75*, 183–193.

Birnbaum, M. H. and J. M. Zimmermann (1998). Buying and selling prices of investments: Configural weight model of interactions predicts violations of joint independence. *Organizational Behavior and Human Decision Processes 74*(2), 145–187.

Carmona, R. (2008). *Indifference pricing: theory and applications*. Princeton University Press.

Chudziak, J. (2020). On complementary symmetry under cumulative prospect theory. *Journal of Mathematical Psychology 95*, 102312.

Irwin, J. R. (1994). Buying/selling price preference reversals: Preference for environmental changes in buying versus selling modes. *Organizational Behavior and Human Decision Processes 60*(3), 431–457.

Kontek, K. and M. Lewandowski (2018). Range-dependent utility. *Management Science 64*(6), 2812–2832.

Lewandowski, M. (2018). Complementary symmetry in cumulative prospect theory with random reference. *Journal of Mathematical Psychology 82*, 52–55.

McClelland, G. H. and W. D. Schulze (1991). The disparity between willingness-to-pay versus willingness-to-accept as a framing effect. In *Frontiers of Mathematical Psychology*, pp. 166–192. Springer.

Mellers, B. A., L. D. Ordoñez, and M. H. Birnbaum (1992). A change-of-process theory for contextual effects and preference reversals in risky decision making. *Organizational Behavior and Human Decision Processes 52*(3), 331–369.

Sayman, S. and A. Öncüler (2005). Effects of study design characteristics on the WTA–WTP disparity: A meta analytical framework. *Journal of Economic Psychology 26*(2), 289–312.

Schmidt, U., C. Starmer, and R. Sugden (2008). Third-generation prospect theory. *Journal of Risk and Uncertainty 36*(3), 203–223.

Sugden, R. (2003). Reference-dependent subjective expected utility. *Journal of Economic Theory 111*(2), 172–191.

Wakker, P. P. (2020). A one-line proof for complementary symmetry. *Journal of Mathematical Psychology 98*, 102406.