COLLEGIUM OF ECONOMIC ANALYSIS
WORKING PAPER SERIES

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SGH KAE Working Papers Series Number: 2022/077 May 2022

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May 10, 2022

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Abstract

Research and development (R&D) requires not only skilled research work, but also dedicated machinery and equipment: R&D capital. In this paper I demonstrate that R&D, producing labor-augmenting ideas, and the accumulation of R&D capital can form a dual engine of economic growth. With R&D capital, balanced growth can be sustained even under decreasing returns and in the absence of population growth. This result contributes to the long-lasting debate on endogenous vs. semi-endogenous R&D-based growth and the likelihood of an upcoming secular stagnation.

Keywords: R&D capital, long-run economic growth, growth engine, endogenous growth, secular stagnation.

JEL codes: O30, O40.

*Financial support from the Polish National Science Center (Narodowe Centrum Nauki) under grant OPUS 19 No. 2020/37/B/HS4/01302 is gratefully acknowledged. I thank an anonymous Reviewer for helpful suggestions which helped substantially improve the paper. I also thank Aleksandra Parteka and Jakub Mućk for their discussions and comments.

1 Introduction

R&D, producing non-rivalrous technological ideas, is a key driver of long-run economic growth (Romer, 1990). Capital accumulation contributes to growth as well, albeit its powers are limited due to decreasing returns (Solow, 1956). Yet, we know surprisingly little about R&D capital: machinery and equipment used in R&D. This is because despite the early lab-equipment formulation of the R&D process (Rivera-Batiz and Romer, 1991), the consecutive growth literature has largely sticked to the assumption that ideas are produced with R&D labor only (e.g. Aghion and Howitt, 1992; Jones, 1995; Acemoglu, 2003; Ha and Howitt, 2007; Madsen, 2008; Kruse-Andersen, 2017).

But in reality R&D *does* use machines, such as computers providing general computing power or specialized machinery necessary to measure natural phenomena and perform experiments. The practicality, complexity and power of research equipment has undergone systematic, cumulative changes over the centuries. Specifically, R&D capital in the US has been growing at an average rate of 3.4% per annum in 1968-2018 (in constant prices), outpacing R&D labor which grew at only 2.1% (Growiec, McAdam, and Mućk, 2022).

In this paper I demonstrate that R&D and the accumulation of R&D capital have the potential to create a *dual growth engine* of endogenous growth. With R&D capital, a balanced growth path can exist also under decreasing returns and with constant R&D employment, i.e. in circumstances in which without R&D capital, growth would certainly disappear. There is even a possibility of accelerating growth.

The consequence is that, other things equal, inclusion of R&D capital accumulation in an R&D-based growth model reduces the likelihood that the model will predict secular stagnation. Without R&D capital, if growth in the number of researchers slows down – as it eventually must, given globally declining population growth – technical change and economic growth slow down, too, unless one arbitrarily imposes that new ideas depend linearly on the stock of old ones (Jones, 2002, 2005; Bloom, Jones, Van Reenen, and Webb, 2020). However, this conclusion no longer follows in models with unbounded accumulation of R&D capital.

2 R&D Capital and Long-Run Growth

Consider the following two-sector economic growth model:

$$Y = F(K, AL), \tag{1}$$

$$\dot{A} = \Phi(K, AL), \tag{2}$$

$$\dot{K} = sY - \delta K, \tag{3}$$

where $s \in (0,1)$ is the savings rate and $\delta > 0$ is the capital depreciation rate. The variable $Y(t)$ is output, $A(t)$ represents ideas, $K(t)$ is capital and L is labor. All technological progress is assumed to be labor-augmenting (Harrod-neutral), and capital K and labor L are used both in production and R&D. $K(0) > 0$ and $L(0) > 0$ are given.

The production function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ and the idea production function $\Phi : \mathbb{R}^2_+ \to \mathbb{R}_+$ are assumed to be continuous, differentiable, strictly increasing and strictly concave in both factors. Time is continuous, $t \in [0, +\infty)$.

To simplify calculations and accommodate the discussion about the potential of sustaining long-run growth after population growth has petered out, I assume that $L > 0$ is constant from the outset.

2.1 Balanced Growth

I obtain the following proposition and three corollaries.

Proposition 1 *The model (1)–(3) allows a balanced growth path if and only if at all $t \geq 0$ the following equation holds along the time path of the economy:*[1]

$$(F - F_K K)(\Phi - \Phi_{AL} AL) - F_{AL} AL \cdot \Phi_K K = 0. \tag{4}$$

If the lhs in (4) is negative, then the model admits paths with accelerating, super-exponential growth, and if the lhs in (4) is positive, then the model only allows paths with decelerating, sub-exponential growth.

Proof. I re-write the system (1)–(3) in growth rates:[2]

$$\hat{K} = \frac{sF(K, AL)}{K} - \delta, \tag{5}$$

$$\hat{A} = \frac{\Phi(K, AL)}{A}. \tag{6}$$

Differentiating these growth rates with respect to time and equating to zero yields:

$$\begin{pmatrix} \frac{s(F_K K - F)}{K} & \frac{sF_{AL} AL}{K} \\ \frac{\Phi_K K}{A} & \frac{\Phi_{AL} AL - \Phi}{A} \end{pmatrix} \begin{pmatrix} \hat{K} \\ \hat{A} \end{pmatrix} = \Xi \begin{pmatrix} \hat{K} \\ \hat{A} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{7}$$

which is satisfied if either $\hat{K} = \hat{A} = 0$ (no growth), or if the matrix Ξ is singular. This singularity requirement underscores the knife-edge character of balanced growth (Growiec, 2007). Positing that $\det \Xi = 0$ and rearranging yields (4).

The dynamics of the system are qualitatively different whether $\det X$ (and equivalently, lhs in (4)) is positive or negative. From strict concavity of F and Φ we know

[1] By G_x I denote the partial derivative of G with respect to x. I omit the arguments of functions for compact notation.

[2] By $\hat{x} = \dot{x}/x$ I denote the growth rate of $x(t)$.

that $F_K K - F$ and $\Phi_{AL} AL - \Phi$ are negative. Therefore if also $\det \Xi > 0$ then Ξ is negative definite. Because its both eigenvalues are negative, growth is then sub-exponential. Conversely, if $\det \Xi < 0$ then Ξ is indefinite. Then one of the two eigenvalues is positive, and consequently there exist time paths of the system which exhibit super-exponential growth. ■

Corollary 1 *If both production functions F and Φ are homogeneous (exhibit constant returns to scale, CRS), then a balanced growth path exists.*

Homogeneous functions satisfy the Euler theorem: $F = F_K K + F_{AL} AL$; $\Phi = \Phi_K K + \Phi_{AL} AL$. Inserting both identities into (4) makes the equation trivially satisfied.

Corollary 2 *If the idea production function is proportional to aggregate production ($\Phi = \kappa F$ for some $\kappa > 0$), then a balanced growth path exists if and only if F is homogeneous (exhibits CRS).*

This is the lab equipment R&D specification of Rivera-Batiz and Romer (1991). Inserting $\Phi = \kappa F$ into (4), the equation simplifies to $F = F_K K + F_{AL} AL$, which holds along the whole time path of the economy if and only if F is homogeneous.

Corollary 3 *Without the accumulation of R&D capital, i.e. with $\Phi_K = 0$, the model only allows paths with decelerating, sub-exponential growth.*

This conclusion is obtained by inserting $\Phi_K = 0$ into (4). Due to strict concavity, $F - F_K K > 0$ and $\Phi - \Phi_{AL} AL > 0$, so the formula in (4) is positive. With constant population, unbounded accumulation of R&D capital is necessary for sustaining positive, constant rates of R&D-based economic growth.

2.2 Growth Rate Under Constant Returns to Scale

Let us focus on the case with CRS in F and Φ. Then one may use the intensive-form notation $F(K, AL) = AL \cdot F\left(\frac{K}{AL}, 1\right) := ALf(k)$. For the idea production function, one writes analogously $\Phi(K, AL) = AL \cdot \Phi\left(\frac{K}{AL}, 1\right) := AL\phi(k)$. The functions $f, \phi : \mathbb{R}_+ \to \mathbb{R}_+$ are strictly increasing and strictly concave. Assume furthermore that $f(0) = \phi(0) = 0$.

Using this notation, the balanced growth rate $g = \hat{Y} = \hat{K} = \hat{A}$ satisfies:

$$g = L\phi(k) = s\frac{f(k)}{k} - \delta. \tag{8}$$

This is a system of two equations in two variables, g and k, underscoring that R&D producing labor-augmenting ideas and the accumulation of R&D capital constitute a *dual growth engine*. The positive feedback loop between both activities is necessary to sustain growth.

As ϕ is strictly increasing, we may invert it. Inserting $k = \phi^{-1}(g/L)$ into (8) yields the following implicit equation for the growth rate:

$$\Psi(g) = g + \delta - s\frac{f(\phi^{-1}(g/L))}{\phi^{-1}(g/L)} = 0. \tag{9}$$

The following proposition holds.[3]

Proposition 2 *If* $\lim_{k\to 0} sf(k)/k > \delta$*, then there exists a unique balanced growth rate* g^* *solving* (9)*.*

Furthermore, using the implicit function theorem it is verified that g^* increases with s and L, and decreases with δ.

2.3 The Cobb–Douglas Case

Cobb–Douglas functions constitute a particularly tractable and instructive application of the above propositions. Let us assume:

$$\begin{aligned} Y &= F(K, AL) = (uK)^{\alpha}(vAL)^{\beta}, && (10)\\ \dot{A} &= \Phi(K, AL) = \lambda((1-u)K)^{\gamma}((1-v)AL)^{\epsilon}, && (11)\\ \dot{K} &= sY - \delta K, && (12)\end{aligned}$$

where $u \in (0,1)$ is the fraction of capital used in the production sector rather than the R&D sector, and $v \in (0,1)$ is the analogous fraction of labor. The parameters $\alpha, \beta, \gamma, \epsilon \in (0,1)$, and $\lambda > 0$.

With these assumptions, the following corollary from Proposition 1 is obtained:

Corollary 4 *In the Cobb–Douglas case, a balanced growth path exists if*

$$(1-\alpha)(1-\epsilon) - \beta\gamma = 0. \tag{13}$$

If $(1-\alpha)(1-\epsilon) - \beta\gamma > 0$ *then growth is sub-exponential; if* $(1-\alpha)(1-\epsilon) - \beta\gamma < 0$ *then there exist paths with accelerating, super-exponential growth.*

Specifically, with CRS in both equations ($\beta = 1 - \alpha$ and $\epsilon = 1 - \gamma$), equation (4) is trivially satisfied and a balanced growth path exists.

Without R&D capital ($\gamma = 0$) growth is sub-exponential (Jones, 1995) – unless $\epsilon = 1$ and thus the idea production function is no longer strictly concave, but instead linear in AL as in Romer (1990).

[3] Proof available upon request.

3 Extensions

3.1 Knowledge Spillovers

Consider again the system (1)–(3) but replace the R&D equation (2) with

$$\dot{A} = \Gamma(A)\Phi(K, AL). \tag{2'}$$

The knowledge spillover function $\Gamma : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed differentiable and monotonic (increasing or decreasing). Increasing $\Gamma(A)$ represents standing-on-shoulders effects in R&D; decreasing $\Gamma(A)$ represents fishing out ideas. I find the following.

Proposition 3 *The model (1), (2'), (3) allows a balanced growth path if and only if at all $t \geq 0$ the following equation holds along the time path of the economy:*

$$(F - F_K K)\left((\Phi - \Phi_{AL}AL)\Gamma(A) - \Phi\Gamma'(A)A\right) - F_{AL}AL \cdot \Phi_K K \cdot \Gamma(A) = 0. \tag{14}$$

If the lhs in (14) is negative, the model admits super-exponential paths; if the lhs in (14) is positive, the model only allows sub-exponential growth.

Corollary 5 *If both production functions F and Φ are homogeneous (exhibit CRS), then (a) if $\Gamma'(A) > 0$, the model admits super-exponential paths; (b) if $\Gamma'(A) = 0$, a balanced growth path exists; (c) if $\Gamma'(A) < 0$, the model only allows sub-exponential growth.*

Corollary 6 *Without the accumulation of R&D capital, i.e. with $\Phi_K = 0$, we can define the threshold spillover strength $\tilde{\Gamma}'(A)$:*

$$\tilde{\Gamma}'(A) = \frac{\Gamma(A)}{A} \cdot \frac{\Phi - \Phi_{AL}AL}{\Phi} > 0. \tag{15}$$

(a) If $\Gamma'(A) > \tilde{\Gamma}'(A)$ then the model admits super-exponential paths; (b) if $\Gamma'(A) = \tilde{\Gamma}'(A)$ for all $t \geq 0$ then a balanced growth path exists; (c) if $\Gamma'(A) < \tilde{\Gamma}'(A)$ then the model only allows sub-exponential growth.

Overall, balanced growth is achieved only when the knowledge spillover effect *exactly balances* the effects of decreasing/increasing returns to scale in F and Φ.

Specifically for Cobb–Douglas F and Φ ((10)–(11)), equation (14) becomes

$$((1-\alpha)(1-\epsilon) - \beta\gamma) - (1-\alpha)\frac{\Gamma'(A)A}{\Gamma(A)} = 0, \tag{16}$$

implying balanced growth at all $t \geq 0$ if and only if:

$$\Gamma(A) = c \cdot A^{\tilde{\mu}}, \qquad \tilde{\mu} = \frac{(1-\alpha)(1-\epsilon) - \beta\gamma}{1-\alpha}, \; c > 0. \tag{17}$$

When $\Gamma(A) = c \cdot A^{\mu}$ with $\mu > \tilde{\mu}$ (strong knowledge spillovers), the model admits super-exponential paths; if $\mu < \tilde{\mu}$ (weak spillovers) then the model only allows sub-exponential growth.

3.2 Exogenous Population Growth

Consider again the original system (1)–(3) but now assume that population grows at a constant rate $n > 0$:

$$\dot{L} = nL. \tag{18}$$

Inclusion of an exogenously growing factor in the production of non-rivalrous ideas introduces a third growth engine into the model. Other things equal, growth is accelerated; changes are often qualitative.

Proposition 4 *If equation (4) holds, or if the lhs in (4) is negative, the model (1), (2), (3), (18) admits super-exponential paths.*

If the lhs in (4) is positive, a balanced growth path exists, satisfying:

$$\begin{pmatrix} \hat{K} \\ \hat{A} \end{pmatrix} = -\Xi^{-1} \cdot \begin{pmatrix} s\frac{F_{AL}AL}{K}n \\ \frac{\Phi_{AL}AL}{A}n \end{pmatrix}, \tag{19}$$

with Ξ defined in (7), provided that the functional forms of F, Φ admit the rhs of (19) to stay constant for all $t \geq 0$ along the time path of the economy.

Corollary 7 *If F and Φ are homogeneous (exhibit CRS), the model admits super-exponential paths. A balanced growth path may exist either if both F and Φ exhibit decreasing returns to scale (DRS), or if one of them has CRS while the other has DRS – for example without the accumulation of R&D capital ($\Phi_K = 0$).*

The intuition behind these results mirrors the analysis by Jones (1995, 1999). With CRS in production and DRS in R&D (among other cases), balanced growth is possible, but is no longer generated endogenously by R&D and the accumulation of R&D capital. Instead, following a *semi-endogenous growth* mechanism, exogenous population growth passes through to growth in GDP per worker because people are partly employed in the production of non-rivalrous ideas.

Specifically for Cobb–Douglas F and Φ ((10)–(11)), equation (19) becomes

$$\begin{pmatrix} \hat{K} \\ \hat{A} \end{pmatrix} = \frac{1}{(1-\alpha)(1-\epsilon) - \beta\gamma} \begin{pmatrix} 1-\epsilon & \beta \\ \gamma & 1-\alpha \end{pmatrix} \begin{pmatrix} \beta n \\ \epsilon n \end{pmatrix}. \tag{20}$$

Along the balanced growth path, the growth rates of capital and ideas are proportional to the population growth rate $n > 0$.

With $\gamma = 0$ (no R&D capital), balanced growth rates satisfy $\hat{K} = \frac{\beta n}{(1-\alpha)(1-\epsilon)}$ and $\hat{A} = \frac{\epsilon n}{1-\epsilon}$ (cf. Jones, 1995).

4 Conclusion

Accumulation of R&D capital, when introduced to an aggregative R&D-based growth model, has the potential to generate positive long-run growth even with decreasing returns and in the absence of population growth. Given the ubiquitous use of computers and specialized machinery in modern-day R&D, the considered mechanism appears highly plausible. Therefore, factoring it in lowers the probability of the scenario of secular stagnation in the future (Jones, 2002; Gordon, 2016). Conversely, it lends weight to the scenario of constant or even accelerating growth (Brynjolfsson and McAfee, 2014; Brynjolfsson, Rock, and Syverson, 2019; Growiec, 2022; Venturini, 2022).

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