SGH

COLLEGIUM OF ECONOMIC ANALYSIS
WORKING PAPER SERIES

Generation of regional input-output tables: a spatial econometric approach with illustrative simulations for France, Germany and Poland

Andrzej Torój

Generation of regional input-output tables: a spatial econometric approach with illustrative simulations for France, Germany and Poland

Andrzej Torój*

Abstract

This paper investigates the construction of multisector-multiregion input-output tables by using spatial econometric methods. I demonstrate that, under reasonable assumptions, the problem of finding Leontief's technical coefficients can be formulated as a modified multi-equation spatial Durbin model and the missing parameters can be estimated via maximum likelihood. The resulting coefficients are computed as a function of country-wide coefficients, as well as distance and regional-sectorial data on value added. The statistical performance of the model is scrutinized and the method is illustrated with simulations of regional (NUTS-3 level) economic impact assessment for generic companies located in Southern France, Germany and Poland.

JEL Classification: C31, C67, R12, R15.

Keywords: input-output modelling, GRIT (generation of regional input-output tables), spatial econometrics, SDM (spatial Durbin model), regional EIA (economic impact assessment).

1 Introduction

The economic footprint of an enterprise can be evaluated from various perspectives and, hence, by using multiple tools. Perhaps the most widespread approach is the Nobel-rewarded input-output (I-O) analysis by Leontief (1936; 1941). It takes into account both the supply chain of the enterprise (indirect effects) and the incremental demand in the economy created through the wage fund of the entire chain (induced effects; Miller and Blair cf. 2009, ch. 6). The necessary information set for using Leontief's I-O tool involves the input-output matrix, normally reported by statistical officies of many advanced economies at the national level, e.g. for Poland, France and Germany (Central Statistical Office in Poland, 2014; Pak and Poissonnier, 2017; Kuhn, 2010, respectively). As a consequence, one can compute various impact measures for the national economy. Answering the impact assessment questions on the sub-national level (say: region) is, however, more complicated.

---

*Warsaw School of Economics, Institute of Econometrics (andrzej.toroj@sgh.waw.pl), ul. Madalińskiego 6/8, 02-513 Warszawa, orcid.org/0000-0003-3623-168X

One could in principle think of two naïve, limiting cases: (i) assigning the whole economic footprint to the region of impluse (which implies that no resources are demanded from other regions) or (ii) allocating it uniformly, or in proportion to some broad measure of economic activity, to other regions (which implies that there will be no bias towards locally produced inputs). Both approaches are in obvious contrast to the principle formulated by Folmer and Nijkamp (1985) to use models incorporating a number of cross-regional feedbacks. The handbook solution is to run the I-O analysis in a multi-sector, multi-region model (Miller and Blair, 2009, ch. 3, pp. 76-101). With S sectors and R regions in question, one should feed this model with the $(S \cdot R) \times (S \cdot R)$ matrix of cross-region-cross-sector flows. Such matrices are normally not available from statistical offices.

The problem of generating regional input-output tables (GRIT – cf. West, 1990) is hence at the heart of regional economic impact assessment questions. The relevance of analysis in the regional dimension can manifest itself in multiple contexts, such as negotiating terms of public-private partnerships, setting local taxation rates, designing spatial development plans or, in general, pursuing a given regional development policy. The list of such regional investigations conducted in the literature is long and includes the following non-exhaustive list of examples: effects of natural disasters (Rose et al., 1997; Rose and Liao, 2005; Hallegatte, 2008), manmade disasters (Giesecke et al., 2012), establishment of recreational infrastructure (Steinback, 1999), ecological footprints and impacts of climate change (Easterling, 1997; Wiedmann et al., 2006; Cicas et al., 2007; Turner et al., 2007), tourism (Horváth and Frechtling, 1999), epidemics (Santos et al., 2013), location of infrastructural objects (Hakfoort et al., 2001), football World Cup events (Baade and Matheson, 2004) or national defence installations (Atkinson, 1993). Such problems usually share two common characteristics: (i) their absolute impact is relatively low on the national level, but relatively high at the regional level, and (ii) as a result, the resources that can be devoted to a case-specific investigation are limited (e.g. surveys are prohibitively expensive) and the need for relatively flexible, universal methodology arises.

In this paper, I explore the use of spatial econometric tools to solve the GRIT problem. This research avenue, initially advocated by i.a. Rey (2000) and Loveridge (2004), has been further explored by Torój (2016). He proposed the use of regional-sectorial data on value added, and the distance criterion, to formulate a maximisation problem on the likelihood function constructed in a similar way to the spatial Durbin model (SDM) and applied the framework to the Polish data. I add to this previous literature in three ways. Firstly, I extend the application to two other European economies of comparable size and also representing relatively compact shapes, i.e. Germany and France. This leads to additional insights, including international comparisons. Secondly, I re-design the likelihood function so as to take into account the correlations between residuals from different sectors, various functional forms, as well as the efficiency of the numerical likelihood maximisation procedure in this relatively complex problem. Thirdly, I propose a set of diagnostic tools that allow for the identification of statistical problems.

The rest of the paper is organized as follows. In Section 2, I discuss the previous literature related to GRIT problem and regional economic impact assessment. In Section 3, I recall the derivation of the likelihood function from the previous literature and propose the aforementioned extensions, both on

the analytical and the numerical level. . Section 4 presents the obtained estimation results along with relevant diagnostics. In Section 5, example simulations at the NUTS-3 level in France, Germany and Poland are demonstrated. Section 6 concludes.

2 GRIT problem and regional impact assessment: overview of the literature

Systematic, theoretical foundations for the regional I-O analysis were laid by Leontief and Strout (1963), and an extensive review of later methodological developments and improvements can be found e.g. in Miller and Blair (2009, ch. 3 and 8).

The earliest regional applications from 1950s were intended for single-region problems (see Isard and Kuenne, 1953), especially for Washington, and the tables were based predominantly on surveys. Multi-region analyses from 1950s involved the cases of USA and Italy (Chenery, 1953; Moses, 1955). The US regions (in different configurations) were subject to the most extensive research on the topic, and hence the most popular contemporaneous application of regional I-O tables to the USA economy is the US MRIO model by Polenske (2004), consisting of 51 regions and 79 sectors. A number of multiregion models was created for Asian economies. Okamoto and Ihara (2005) elaborated a model for China with 30 sectors and 8 regions. Sub-national trade patterns across Japanese islands were analysed by Sonis et al. (2000). A 9-region, 25-sector table for Japan was created by Akita and Kataoka (2002). There are also multinational applications of the multi-region framework: for the EU, the ASEAN-5 group, or the world's leading economies (WIOD database).

In the literature, the approach towards GRIT problem has finally diverged into survey methods (e.g. commodity inflow survey in the USA – see Liu and Vilain, 2004) and non-survey methods, for which the scarce cases of survey-based tables often serve as benchmarks (see e.g. Tohmo, 2004).

Among the non-survey methods, the most widespread mathematical technique to develop the multi-region multi-sector I-O tables is the location quotient (LQ) technique, allowing to approximate intra-regional cross-industry flows and cross-region flows from and to specific industries (see McCann, 2007, for an overview). LQ framework has been applied in multiple variations: simple location quotients (SLQ), Flegg's LQ (FLQ, see Flegg et al. 1995) and further variations referred to as augmented AFLQ, semi-logarithmic RLQ, industry-specific SFLQ and others. A number of researchers investigate their empirical performance against various benchmarks (see e.g. Kowalewski, 2015; Lamonica and Chelli, 2017). More sophisticated versions of the LQ method involve additional coefficients that need to be calibrated or estimated, such as the convexity parameter δ in FLQ (see e.g. Bonfiglio, 2009, for an extensive investigation).

Before the LQ techique dominated this strand of literature, Leontief and Strout (1963) applied gravity models to data from individual regions. Further methodological advancements, as well as the need to deal with different data coverage, brought about the development of hybrid methods (advocated i.a. by Harris and Liu, 1998), combining the gravity model approach with other data sources, including

expert estimates (e.g. West, 1990). Hulu and Hewings (1993), in a model for Indonesia, impose further balancing restrictions.

Canning and Wang (2005) consider the construction of multiregion I-O tables as a constrained optimization problem. Their approach requires a richer set of inputs (country-wide I-O table plus regional-sectorial data on: gross output, value added, exports, imports and final demand). Regional differentials between supply and use give rise to exchange and the model is empirically validated by simulating tables of international trade with countries cast in the role of regions.

An extensive overview of recent applications and modelling directions is provided by Wiedmann et al. (2011), and recent efforts to construct sub-national I-O tables for many countries were taken i.a. by Wang et al. (2017).

As regards the European countries, previous analyses of regional input-output tables were conducted for in an inter-country framework for Europe as a whole (van der Linden and Oosterhaven, 1995) and for individual countries. The model MulitREG for **Austria** (Fritz et al., 2001) is an example of the survey approach in construction of cross-regional flows. In **Finland**, Koutaniemi and Louhela (2006) use a hybrid of various approaches (described as bottom-up and top-down) to compile regional tables, and Flegg and Tohmo (2013) use the FLQ technique. Multi-region input-output analyses have also been pursued in Germany (see e.g. Funck and Rembold, 1975) and **France** (Cristina de la Rúa and Lechón, 2016). The attempts to apply a regional I-O analysis for **Poland** involve mostly recent cases of generating I-O table for a given region. Welfe et al. (2008, chapter 1) apply the multiplier analysis to identify locally dominant branches in łódzkie voivodship. Chrzanowski (2013) constructed an I-O table for lubelskie voivodship based on location quotient technique. Tomaszewicz and Trębska (2005) also apply the LQ approach. Most recently, Zawalińska and Rok (2017) were the first to construct comprehensive regional tables for 19 NACE sections and 16 voivodeships (European Union's NUTS-2 level).

A number of authors, including Fritz et al. (2001), acknowledge the advantages from combining the econometric approach and the input-output models. In a similar vein to estimating δ under FLQ, one can think of various parameters, functional forms and control variables that determine the trade flows for a given couple of regions and sectors.The previous literature has predominantly focused on the estimation of parameters in private consumption block (Fritz et al., 2001), dynamisation of input-output coefficients (Kratena and Zakarias, 2004) or the aforementioned δ.

Relatively few econometric studies, except Jackson et al. (2006) and Torój (2016), investigated the role of distance (though with a single functional form). However, the distance is the most intuitive criterion for inter-regional connectivity that supplements the criteria of regional and sectorial supply and demand, traditionally postulated in the literature (Round, 1978). Accordingly, our analysis builds upon the strand of literature on gravity approach, originated by Leontief and Strout (1963), further explored by Theil (1967) and generally positively validated by Polenske (1970). Later applications of this approach include Uribe et al. (1966), Gordon (1976) and Lindall et al. (2006), as well as the previous econometric study of Jackson et al. (2006).

At the same time, according to Rey (2000) and Loveridge (2004), spatial econometrics could be a

promising direction in the strand of the so-called integrated econometric-input-output models, and they appear to be well-equipped to address the specific small-area problems (judging by the list of example issues related to small-area problems, provided by Morrison and Smith, 1974). Torój (2016) demonstrated that spatial econometric modelling suggests a convenient formalisation framework for this problem. As it is demonstrated in Sections 3 and 5, demand- and supply-side constraints similar to those used under LQ approach can also be included. However, in principle, the spatial econometric formulation is flexible enough to capture any observable determinant of trade linkage between a given couple of sectors and/or regions.

To the best of our knowledge, there have not been any previous attempts to formalize the problem of estimating multiregion I-O tables based on spatial econometric tools for multiple European countries, for a relatively high disaggregation level (NUTS-3) and accompanied by relevant statistical diagnostics. This paper is intended to fill this gap.

3 Modelling assumptions and spatial econometric specification of GRIT problem

The solution to GRIT problem for a $S \times S$ country-wide I-O matrix $\mathbf{X}$ (S − number of sectors) is to generate the following $R \times R$ matrix of cross-regional flows for each pair of sectors (s, v), $s, v \in 1, ..., S$:

$$
\mathbf{x^{s;v}} = \begin{bmatrix}
x_{1;1}^{s;v} & x_{1;2}^{s;v} & \cdots & x_{1;R}^{s;v} \\
x_{2;1}^{s;v} & x_{2;2}^{s;v} & \cdots & x_{2;R}^{s;v} \\
\vdots & \vdots & \ddots & \vdots \\
x_{R;1}^{s;v} & x_{R;2}^{s;v} & \cdots & x_{R;R}^{s;v}
\end{bmatrix}
\tag{1}
$$

instead of a scalar element $x^{s;v}$ of the matrix $\mathbf{X}$, where R − number of regions. The element $x_{r;p}^{s;v}$ shall be understood as the use of products from sector s and region r in sector v and region p. For every pair (s, v) it holds that:

$$
\sum_{r,p} x_{r;p}^{s;v} = x^{s;v}.
\tag{2}
$$

The interpolation is split into two steps. Firstly, I assume that sectorial techonological structures do not vary from region to region. This is not to say that individual regions cannot be characterized by different propensities to import (from other countries or regions), but that the same sectorial structure of costs (regardless of their region or country of origin).[1] This leads to interpolating $x^{s;v}$ into column sums of $\mathbf{x^{s;v}}$ in proportion to the value added va_p^v of the recipient sector-region pair (v, p) by assuming constant cost-to-VA ratios across regions for a given demand-side sector:

---

[1]Note that it also implicitly assumes that labour-intensity and gross profitability of output is a sectorial, but not a regional attribute.

$$\sum_r x_{r;p}^{s;v} = \frac{va_p^v}{\sum_p va_p^v} \cdot x^{s;v}. \tag{3}$$

In step 2, column sums have to be translated into individual column elements. In other words, given the quantities from sector s that sector v in each region intends to order, one needs to allocate them between regions on the supply side. I describe this allocation by a supply-sector (s) specific $R \times R$ matrix $\mathbf{W^s}$ with real, non-negative elements, indicating the proportions in which the vector of column sums over r (elements of 3) are to be split between individual rows of $\mathbf{x^{s;v}}$ (i.e. between individual r). Dependence on s (but not on v) means that the exact impact of distance depends on the sort of good, i.e. the supplying sector, but not on the demanding sector. For example, an entrepreneur may prefer to buy agricultural products locally, but there may be no role for distance in manufacturing; and, at the same time, it does not matter whether the recipient is an entrepreneur from the food industry or from the chemical industry. Given the fact that only the wihin-column proportions between elements of $\mathbf{W^s}$ are interpret ed, it is sufficient to adopt a just-identifying assumption that, for each $s \in 1, ..., S$, each column of $\mathbf{W^s}$ sums to unity. The economic interpretation of this identification scheme will be discussed later on in this section. Denoting the (r, p)-th element of $\mathbf{W^s}$ as $\mathbf{W^s}(r, p)$, one can define:

$$x_{r;p}^{s;v} = \left(\sum_r x_{r;p}^{s;v} \right) \cdot \mathbf{W^s}(r, p). \tag{4}$$

Equation (4) may be viewed as a modification of Leontief's and Strout's (1963) gravity formula, in a variant that allows for a spatial empirical investigation, with $\mathbf{W^s}$ treated as unknown.

In line with the underlying principle of spatial econometrics, known as the Tobler's law (Tobler, 1970) and describing the role of physical proximity (distance $w_{r,p}^*$ between r and p) in determining economic linkages, one should assume that $\frac{\partial x_{r;p}^{s;v}}{\partial w_{r,p}^*} \leq 0$. Intiutively, *ceteris paribus*, the recipient of a good is less likely to select a supplier in a less distant region, e.g. due to lower transport cost. There is no substitution between goods from different sectors and physical locations. Hence, I do not take into account the fact that missing local availability of some product (e.g. local repair services) may create incentives to switch to a different sector's goods (e.g. ordering a new device from a remote manufacturer).

The econometric formulation starts with a demand-side decomposition of value added created in sector s (va^s) into driven by the intermediate demand from other sectors and the final demand (y^s):

$$va^s = \sum_{v=1}^{S} \beta_v^s va^v + \beta_0^s y^s. \tag{5}$$

The parameters β_v^s, $v = 1, ..., S$, and β_0^s cannot be estimated[2] and, in fact, shall be treated as known from country-wide input-output ratios. One can calibrate β_v^s using the following reasoning:

---

[2] Due to the presence of the same variable on both sides of the equation, the endogeneity problem (as equation (5) can be written for any s) and insufficient degrees of freedom.

- Δva^v generates global output in sector v equal to $\Delta x^v = \frac{x^v}{va^v}\Delta va^v$ (whereby x^v denotes total global output in sector v);

- Δx^v translates into intermediate demand for goods produced by sector s, equal to $a^{s,v}\Delta x^v$ (whereby $a^{s,v}$ denotes s,v-th element of cost structure $S \times S$ matrix $\mathbf{A}$);

- this becomes part of global output in sector s, x^s, hence $\Delta x^s = a^{s,v}\Delta x^v$;

- value added in sector s grows in respective proportion to the global output in the same sector: $\Delta va^s = \frac{va^s}{x^s}\Delta x^s$;

- collecting terms: $\Delta va^s = \underbrace{\frac{va^s}{x^s}a^{s;v}\frac{x^v}{va^v}}_{\equiv \beta_v^s}\Delta va^v$.

Computation of β_0^s is straightforward as y^s directly becomes part of global output in s, hence $\Delta va^s = \frac{va^s}{x^s}\Delta x^s = \underbrace{\frac{va^s}{x^s}}_{\equiv \beta_0^s}\Delta y^s$. For future reference, it is useful to define $\boldsymbol{\beta} \equiv \begin{bmatrix} \beta_1^1 & \cdots & \beta_S^1 \\ \vdots & \ddots & \vdots \\ \beta_1^S & \cdots & \beta_S^S \end{bmatrix}$ and $\boldsymbol{\beta_0} \equiv \begin{bmatrix} \beta_{0,1} \\ \vdots \\ \beta_{0,S} \end{bmatrix}$.

Mathematically, equations (5) − for every sector s − are identities for the period and country that served as a basis for calibrating β_v^s and β_0^s as above. However, any disaggregation or extrapolation, either in space or time, renders this equation stochastic. Let us consider the cross-regional, matrix version and focus on sector s, and let va_r^v denote the value added in sector v (including s) in region $r = 1, ..., R$:

$$\begin{bmatrix} va_1^s \\ \vdots \\ va_R^s \end{bmatrix} = \begin{bmatrix} va_1^1 & \cdots & va_1^S \\ \vdots & \ddots & \vdots \\ va_R^1 & \cdots & va_R^S \end{bmatrix}\begin{bmatrix} \beta_1^s \\ \vdots \\ \beta_S^s \end{bmatrix} + \begin{bmatrix} y_1^s \\ \vdots \\ y_R^s \end{bmatrix}\beta_0^s + \begin{bmatrix} \varepsilon_1^s \\ \vdots \\ \varepsilon_R^s \end{bmatrix} \tag{6}$$

Note that (6) is too straightforward as a disaggregation to be realistic, because the independence of observations implies the autarky of regions. To introduce cross-regional trade, let us use the previously mentioned weighting scheme $\mathbf{W^s}$, consisting of elements named $w_{r,p}^s$:

$$\begin{bmatrix} va_1^s \\ \vdots \\ va_R^s \end{bmatrix} = \begin{bmatrix} w_{11}^s & \cdots & w_{1R}^s \\ \vdots & \ddots & \vdots \\ w_{R1}^s & \cdots & w_{RR}^s \end{bmatrix}\begin{bmatrix} va_1^1 & \cdots & va_1^S \\ \vdots & \ddots & \vdots \\ va_R^1 & \cdots & va_R^S \end{bmatrix}\begin{bmatrix} \beta_1^s \\ \vdots \\ \beta_S^s \end{bmatrix} + \begin{bmatrix} w_{11}^s & \cdots & w_{14}^s \\ \vdots & \ddots & \vdots \\ w_{41}^s & \cdots & w_{44}^s \end{bmatrix}\begin{bmatrix} y_1^s \\ \vdots \\ y_4^s \end{bmatrix}\beta_0^1 + \begin{bmatrix} \varepsilon_1^s \\ \vdots \\ \varepsilon_R^s \end{bmatrix}. \tag{7}$$

By rearranging terms, one obtains:

$$\begin{bmatrix} va_1^s \\ \vdots \\ va_R^s \end{bmatrix} = \begin{bmatrix} w_{11}^s & \cdots & w_{1R}^s \\ \vdots & \ddots & \vdots \\ w_{R1}^s & \cdots & w_{RR}^s \end{bmatrix} \begin{bmatrix} va_1^s \\ \vdots \\ va_R^s \end{bmatrix} \beta_s^s + \begin{bmatrix} w_{11}^s & \cdots & w_{1R}^s \\ \vdots & \ddots & \vdots \\ w_{R1}^s & \cdots & w_{RR}^s \end{bmatrix} \begin{bmatrix} va_1^1 & \cdots & va_1^S & y_1^s \\ \vdots & \ddots & \vdots & \vdots \\ va_R^1 & \cdots & va_R^S & y_R^s \end{bmatrix} \begin{bmatrix} \beta_{v_1}^s \\ \vdots \\ \beta_{v_{S-1}}^s \\ \beta_0^s \end{bmatrix} + \begin{bmatrix} \varepsilon_1^s \\ \vdots \\ \varepsilon_R^s \end{bmatrix}.$$
(8)

whereby $v_1, ..., v_{S-1} \neq s$. The above formulation is a special case of the spatial Durbin model (SDM, cf. LeSage and Pace, 2009, p. 82):

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\alpha_1} + \mathbf{W}\mathbf{X}\boldsymbol{\alpha_2} + \boldsymbol{\varepsilon} \tag{9}$$

with $\mathbf{y} = \mathbf{va^s}$, $\boldsymbol{\alpha_1} = \mathbf{0}$ (there are no purely local regressors), $\rho = \beta_s^s$, $\boldsymbol{\alpha_2} = \begin{bmatrix} \beta_{v_1}^s \\ \vdots \\ \beta_{v_{S-1}}^s \\ \beta_0^s \end{bmatrix}$, $\mathbf{W} = \mathbf{W^s}$

and $\mathbf{X} = \begin{bmatrix} \mathbf{va^{v_1}} & ... & \mathbf{va^{v_{S-1}}} & \mathbf{y^s} \end{bmatrix}$. Note that $\mathbf{y^s} = \begin{bmatrix} y_1^s \\ \vdots \\ y_R^s \end{bmatrix}$ is the vector of final output values in

individual regions in sector s and that $\mathbf{va^s}$ is a column vector of va_r^s for different values of r Hence, the whole model (for each $s = 1, ..., S$) is a multi-equation version of SDM.

This analogy allows us to use the likelihood function for the Durbin problem, but with a different vector of unknown parameters. In the standard SDM, $\mathbf{W}$ is treated as known and exogenous while estimating ρ, $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$, while in our case, $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_0$ are given, while $\mathbf{W_s}$ remains unknown. Another notable difference consists in the normalisation technique of $\mathbf{W_s}$. As mentioned previously, the just-identifying assumption is the column-wise normalisation. It has been demonstrated by Torój (2016) that this leads to compliance with the I-O ratios and multiplier at the country-wide level and avoiding the well-known disaggregation problems in the input-output analysis. In other words, a unit increase in va^v should lead to a total, country-wide increase in va^s equal to of β_v^s, for any geographical distribution of the impulse in va^v and response in va^s.[3]

Still, there are far too many elements of matrices $\mathbf{W^s}$ to estimate them freely and one still needs

---

[3]To see this, sum the impacts of intermediate demand changes on $\mathbf{va^s}$ over regions:

$\Delta va^s = \Delta va_1^s + ... + \Delta va_R^s =$

$= \left(\beta_1^s \sum_r w_{1r}^s \Delta va_r^1 + ... + \beta_S^s \sum_r w_{1r}^s \Delta va_r^S \right) + ... +$

$+ \left(\beta_1^s \sum_r w_{Rr}^s \Delta va_r^1 + ... + \beta_S^s \sum_r w_{Rr}^s \Delta va_r^S \right) =$

$= \beta_1^s \sum_r \left[\left(w_{1r}^s + ... + w_{Rr}^s \right) \Delta va_r^1 \right] + ... + \beta_S^s \sum_r \left[\left(w_{1r}^s ... + w_{Rr}^s \right) \Delta va_r^S \right] =$

$= \beta_1^s \sum_r \left[\Delta va_r^1 \left(\sum_p w_{pr}^s \right) \right] + ... + \beta_S^s \sum_r \left[\Delta va_r^S \left(\sum_p w_{pr}^s \right) \right]$

Consider a unit change in any sectorial value added, $\Delta va^v = 1$. To comply with country-wide I-O ratios, Δva^s must be equal to β_v^s, and consequently:

a parsimonious functional representation. To take account of the spatial proximity, I use a symmetric distance matrix $\mathbf{W}^*$ sized $R \times R$, representing the physical distances between the centroids of regions implied from Eurostat's GIS maps of NUTS-3 level regions:

$$
\mathbf{W}^* = \begin{bmatrix} 0 & w_{1;2}^* & \cdots & & w_{1;R}^* \\ . & 0 & \ddots & & \vdots \\ . & . & 0 & w_{R-1;R}^* \\ . & . & . & 0 \end{bmatrix}.
\tag{10}
$$

Contrary to Torój (2016), individual elements (r, p) of $\mathbf{W^s}$ were computed in 4 alternative functional forms:

1. **power**. The vector of parameters $\boldsymbol{\theta^s}$ consists a single parameter per sector, $\theta^s \leq 0$, controlling the pace of power spatial decay:

$$
w_{exponential}^s (r, p) = \frac{\left(1 + w_{r,p}^*\right)^{\theta^s}}{\sum_p \left[\left(1 + w_{r,p}^*\right)^{\theta^s}\right]}.
\tag{11}
$$

2. **triangle**. $\boldsymbol{\theta^s}$ consists of a single parameter per sector, $\theta^s < 0$, taking account of the linear degression of relative importance of suppliers along with the distance between regions r and p (down to zero):

$$
w_{triangle}^s (r, p) = \frac{\max\left(0; 1 + \theta^s \cdot w_{r,p}^*\right)}{\sum_p \left[\max\left(0; 1 + \theta^s \cdot w_{r,p}^*\right)\right]}.
\tag{12}
$$

3. **interval-wise**. $\boldsymbol{\theta^s}$ consists of four unknown parameters per sector, $0 \leq \theta_1^s \leq 1$ to $0 \leq \theta_4^s \leq 1$, and four predefined thresholds c_1 to c_4 ($c_5 = \infty$, $\mathbf{1}$ – indicator function):

$$
\forall_v \ \sum_r \left[\Delta va_r^v \left(\sum_p w_{pr}^s\right)\right] = 1
$$

Consider the extreme case when the unit growth of va^v is concentrated in a single region r, i.e. $\Delta va_r^v = 1$. Then:

$$
\forall_{r,s} \ \sum_p w_{pr}^s = 1
$$

i.e. the sum of every column r in every matrix $\mathbf{W^s}$ must be equal to 1. If these equalities hold, the condition to preserve the country-level I-O ratios also holds in the case when growth of va^v is distributed over multiple regions.

$$w^s_{interval}(r,p) = \frac{\mathbf{1}_{[0;c_1)}\left(w^*_{r,p}\right) + \Sigma^4_{d=1}\theta^s_d \cdot \mathbf{1}_{[c_d;c_{d+1})}\left(w^*_{r,p}\right)}{\sum\limits_p \left[\mathbf{1}_{[0;c_1)}\left(w^*_{r,p}\right) + \Sigma^4_{d=1}\theta^s_d \cdot \mathbf{1}_{[c_d;c_{d+1})}\left(w^*_{r,p}\right)\right]}. \tag{13}$$

Note that this does not imply $\frac{\partial x^{s;v}_{r;p}}{\partial w^*_{r,p}} \leq 0$ (as stated before; this function is non-differentiable), but only that $w^s_{interval}(r,p) \geq w^s_{interval}(r,q)$ if distance between r and p is lower than c_1 and the distance between r and q is higher than c_1. Thresholds c_1 to c_4 are set for each country individually as quantiles of order 0.1, 0.3, 0.5 and 0.7 taken from the upper triangular part of $\mathbf{W}^*$.

4. **gamma**. Using the gamma cumulative distrubution function $\Gamma\left(.\right)$ parametrized for each sector s with shape $\theta^s_1 > 0$ and scale $\theta^s_2 > 0$ as:

$$w^s_{gamma}(r,p) = \frac{1 - \Gamma\left(\theta^s_1, \theta^s_2, w^*_{r,p}\right)}{\sum\limits_p \left[1 - \Gamma\left(\theta^s_1, \theta^s_2, w^*_{r,p}\right)\right]}. \tag{14}$$

The gamma-pdf approach (13) can be seen as the most general approach as this functional form transforms the distance very flexibly, while its parametrisation is highly parsimonious. For example, it can be fitted to three different situations: (i) when local suppliers are strongly preferred, and the demanding company is relatively indifferent between supplier located 50 km and 1000 km away, (ii) local suppliers are preferred, but not strongly, to distant suppliers, and the utility from distance supplies is decreasing very gradually, (iii) the demanding company is relatively indifferent between supplies up to some threshold, e.g. from 0 to 100 km, above which the preference for supplies is decreasing sharply (see Figure 1).

The system is built up of equations (7) for all sectors $s = 1, ..., S$:

$$\begin{bmatrix} \mathbf{I_R} & & \\ & \ddots & \\ & & \mathbf{I_R} \end{bmatrix}\mathbf{va} = \begin{bmatrix} \beta^1_1\mathbf{W^1} & \cdots & \beta^1_S\mathbf{W^1} \\ \vdots & \ddots & \vdots \\ \beta^S_1\mathbf{W^S} & \cdots & \beta^S_S\mathbf{W^S} \end{bmatrix}\mathbf{va} + \begin{bmatrix} \beta_{0,1}\mathbf{W^1} \\ \vdots \\ \beta_{0,S}\mathbf{W^S} \end{bmatrix}\mathbf{y} + \boldsymbol{\varepsilon}, \tag{15}$$

with $\mathbf{va} = \begin{bmatrix} \mathbf{va^1} \\ \vdots \\ \mathbf{va^S} \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} y^1 \\ \vdots \\ y^S \end{bmatrix}$ and $\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon^1 \\ \vdots \\ \varepsilon^S \end{bmatrix}$. Based on (15), the following $\mathbf{A}$ and $\mathbf{B}$ matrices can be defined:

Figure 1: Fuctional forms of spatial decay in region weights
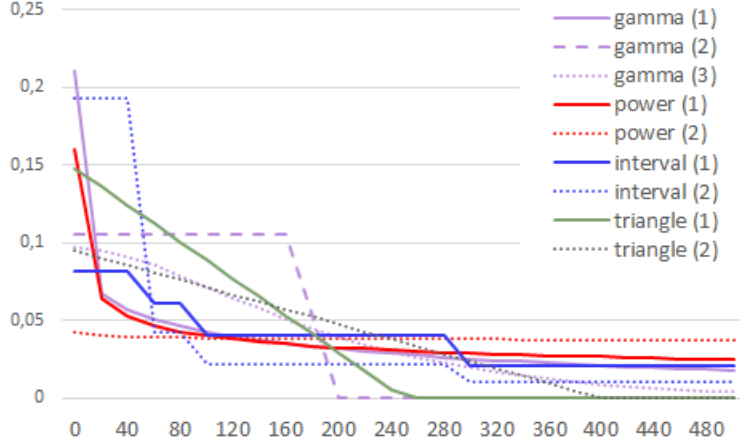
Source: own elaboration.

$$\left(\mathbf{I_{S \cdot R}} - \boldsymbol{\beta} \otimes \underbrace{\left[\begin{array}{ccc} \mathbf{W^1} & \cdots & \mathbf{W^1} \\ \vdots & \ddots & \vdots \\ \mathbf{W^S} & \cdots & \mathbf{W^S} \end{array} \right]}_{} \right) \mathbf{va} = \boldsymbol{\beta_0} \otimes \underbrace{\left[\begin{array}{c} \mathbf{W^1} \\ \vdots \\ \mathbf{W^S} \end{array} \right]}_{} \mathbf{y} + \boldsymbol{\varepsilon}. \tag{16}$$

$$\underbrace{}_{\equiv \mathbf{A}} \qquad \underbrace{}_{\equiv \mathbf{B}}$$

For the SDM like (16), the log-likelihood function reads as follows and is maximized with respect to $\mathbf{A}, \mathbf{B}$ and $\boldsymbol{\Sigma}$:

$$lnL = -\frac{n}{2} \ln(2\pi) + \ln|\mathbf{A}| - \frac{1}{2} \ln|\boldsymbol{\Sigma}| - \frac{1}{2} \boldsymbol{\varepsilon}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\varepsilon}, \tag{17}$$

whereby $\boldsymbol{\varepsilon} = \mathbf{A} \cdot \mathbf{va} - \mathbf{B} \cdot \mathbf{y}$, $n -$ length of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\Sigma} = E\left(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right) -$ variance-covariance matrix of zero-mean $\boldsymbol{\varepsilon}$.

In the discussed case, according to definition (16), $\mathbf{A} = \mathbf{A}\left[\boldsymbol{\beta}, \mathbf{W^1}\left(\boldsymbol{\theta^1}\right), ..., \mathbf{W^S}\left(\boldsymbol{\theta^S}\right)\right]$ and $\mathbf{B} = \mathbf{B}\left[\boldsymbol{\beta_0}, \mathbf{W^1}\left(\boldsymbol{\theta^1}\right), ..., \mathbf{W^S}\left(\boldsymbol{\theta^S}\right)\right]$ are both functions of the unknown vectors $\boldsymbol{\theta^1}, ..., \boldsymbol{\theta^S}$. Additionally, I define $\boldsymbol{\Sigma}$ in such a way that, for each sector s, there variance of the error is equal to σ_s^2. Extending the framework of Torój (2016), I let errors to be correlated across sectors s, v in the same region with covariance $\sigma_{s,v}$. This information can be summarized in the following, symmetric, semi-positive-definite

$S \times S$ matrix $\boldsymbol{\Omega} \equiv \left[\begin{array}{cccc} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1S} \\ . & \sigma_2^2 & \cdots & \sigma_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ . & . & \cdots & \sigma_S^2 \end{array} \right]$ and

$$\boldsymbol{\Sigma} = \boldsymbol{\Omega} \otimes \mathbf{I_R}. \tag{18}$$

11

Given (18), it is computationally efficient to notice that

$$-\frac{1}{2}\ln|\mathbf{\Sigma}| = -\frac{1}{2}\ln|\mathbf{\Omega}\otimes\mathbf{I_R}| = -\frac{1}{2}\ln|\mathbf{\Omega}|^R = -\frac{R}{2}\ln|\mathbf{\Omega}|$$

Hence, collecting the unknown parameters into the vector $\boldsymbol{\theta}^T = \begin{bmatrix} \boldsymbol{\theta^1}^T & ... & \boldsymbol{\theta^S}^T & vec(\mathbf{\Omega}) \end{bmatrix}$, the econometric problem can be represented as:

$$
\begin{aligned}
\hat{\boldsymbol{\theta}} &= \arg\max_{\boldsymbol{\theta}} \ln L\left(\boldsymbol{\theta}|\boldsymbol{\beta},\boldsymbol{\beta_0},\mathbf{W}^*,\mathbf{va},\ \mathbf{y}\ \right) = \\
&= \arg\max_{\boldsymbol{\theta}} \left[-\frac{S\cdot R}{2}\ln(2\pi) + \ln|\mathbf{A}(\boldsymbol{\theta})| - \frac{R}{2}\ln|\mathbf{\Omega}(\boldsymbol{\theta})| - \frac{1}{2}\boldsymbol{\varepsilon}(\boldsymbol{\theta})^T\left(\mathbf{\Omega}(\boldsymbol{\theta})^{-1}\otimes\mathbf{I_R}\right)\boldsymbol{\varepsilon}(\boldsymbol{\theta}) \right].
\end{aligned}
\tag{19}
$$

An additional Monte Carlo study conducted with a data generating process of the form (16) and the true parameters equal to all three sets of point estimates reported in Section 4 has confirmed that the estimator (19) is unbiased and consistent[4].

The standard errors of estimation are derived from the variance-covariance matrix calculate according to the delta formula (cf. Cameron and Trivedi, 2005, p. 156):

$$Var\left(\hat{\boldsymbol{\theta}}\right) = \mathbf{G}^T\mathbf{\Sigma}^{-1}\mathbf{G}, \tag{20}$$

with $G = \frac{\partial\mathbf{g}(\hat{\boldsymbol{\theta}})}{\partial(\hat{\boldsymbol{\theta}})}|_{\hat{\boldsymbol{\theta}}}$, $\mathbf{g}()$ being the right-hand side of (15).

The problem (19) can be numerically complicated, for a few reasons. Firstly, especially under the functional form (14), various combinations of shape and scale parameter can lead to a similar profile of the decay in distance. Secondly, the introduction of non-diagonal $\mathbf{\Omega}$ considerably adds to the dimensionality of the problem, also by strengthening the cross-equation dependency of parameters. Thirdly, given (14), some specific sets of starting values imply a sharp decrease of $w_{r,p}$ around some distance threshold. This can lead to the situation in which pairs of regions whose distance is far away from this threshold (on either side) cannot contribute to the local sensitivity of the likelihood to parameter values. For these reasons, the following algorithm of searching for the starting values has been followed for versions (11), (12) and (13) to ensure an appropriate convergence:

---

[4]See Appendix A for details. It needs perhaps to be stressed that this study has also confirmed the same as regards the results obtained by Torój (2016) where the elements of matrices $\mathbf{W^s}$ were defined as: $w_{rp}^s = \frac{w_{distance}^s(r,p)\cdot w_{supply}^s(r,p)}{\sum\limits_r w_{distance}^s(r,p)\cdot w_{supply}^s(r,p)}$, i.e. a rescaled product of two factors, whereby $w_{distance}^s(r,p)$ was defined as (11)-(14) in this study and $w_{supply}^s(r,p) = \left(\frac{va_r^s}{\sum\limits_i va_i^s}\right)^{\gamma^s}$. The idea behind the use of the latter factor was to take account of the fact that, other things being equal (e.g. distance), the producers from a given region tend to order from regions where the supply of intermediate goods is higher, for some structural reason. However, a risk arises when the estimation does not start from the true values of $\boldsymbol{\theta}$ and γ^s. In such a situation, the likelihood function has a degenerate global minimum for $\gamma^s = 1$ and $\boldsymbol{\theta^1},...,\boldsymbol{\theta^S}$ indicating independence of distance (e.g. $\theta^s = 0$ in equation (11)). For this reason, I decided to calibrate γ^s for the purpose of simulation in Section 5 and leave the adequate extension of the empirical problem for future research.

1. Start with near-zero sensitivity to distance, i.e. $\theta^s = -0.0001$ for the power variant, $\theta^s = -0.0001$ for the triangle variant and $\theta^s = 0.99$ for the interval-wise variant, for all $s = 1, ..., S$. Set $\sigma_{v;s} = 0$ for all $v \neq s$. Conditionally upon that, find the likelihood-maximising values of σ_s^2 for all s.

2. Starting with $\sigma_{v;s} = 0$ for all $v \neq s$, σ_s^2 found in step 1 and keeping θ^s unchanged at levels from step 1, find the optimum $\mathbf{\Omega}$ under the condition of semi-positive-definiteness.

3. Conditionally upon the entire $\mathbf{\Omega}$ found it step 2, find the optimum θ^s for all s.

4. **Starting with values of $\mathbf{\Omega}$ (from steps 1-2) and θ^s (from step 3), solve the problem (19) with respect to all these parameters.**

Then, for version (14):

1. For each s, find a combination of θ_1^s, θ_2^s that provides the best least-squares fit to the values of $w_{interval}^s (r, p)$ for all region pairs. This is accomplished by (i) looking at a grid of values from 0.01 to 2000 and then (ii) locally minimising the sum of squares for each sector separately, starting with the grid minimum. Fitting to the interval-wise variant (rather than power or triangle) is motivated by the fact that this is the most flexible form (though relatively generously paramterized).

2. Compute $\mathbf{\Omega}$ as the empirical variance-covariance matrix corresponding with θ_1^s, θ_2^s for $s = 1, ..., S$ established in step 1.

3. For each s, conditionally upon $\mathbf{\Omega}$ from step 2 and θ_1^v, θ_2^v (for all $v \neq s$) from step 3, maximize $\ln L$ with respect to θ_1^s, θ_2^s.

4. **Use $\mathbf{\Omega}$ from step 2 and θ_1^s, θ_2^s ($s = 1, ..., S$) from step 3 as initial values in the optimisation problem (19).**

In all of the above cases, the local method of Nelder and Mead (1965) has been used first and the global method of simulated annealing (Belisle, 1992) as second.

4 Spatial modelling results

The source of data on **va** is Eurostat's regional accounts. The value added is available in breakdown into NUTS-3 regions (402 for Germany, 96 for France and 72 for Poland) and sectors (7 groups of NACE 2.0 sections) – see Figure 2 for the graphical representation as of 2011. The additional source of data on manufacturing for Poland is the Local Data Bank maintained by the Central Statistical Office in Poland. Final output data,**y**, is only available from the I-O tables in sectorial breakdown, but not in territorial breakdown. It can be further decomposed between consumption (including government consumption), capital formation and exports. As of 2010, in all three countries under investigation, the consumption (both government and private) has accounted for more than a half of the final demand.

This is why I decided to interpolate the final output, sector by sector, in proportion to the local populations. I thereby implicitly assume that consumption volumes and tastes per head do not differ significantly region by region.

The calibration of β and β_0 was based on two sources. Firstly, the cost structure coefficients $a^{s;v}$ and sectorial global output x^s were derived from World Input-Output Tables (WIOT) as of 2011. Secondly, sectorial data on the value added, va^s, has been derived as partial sums of **va** over regions, for each s and for each year, separately. This is to ensure that no need for a constant term arises in equation (15)

Note that two alternatives strategies of using data are available. Firstly, instead of using NUTS-3 level data for 7 groups of NACE 2.0 sections, one could in principle use NUTS-2 level data for 20 NACE 2.0 sections. Hence there is a trade-off between the level of aggregation in both dimensions in question. As the focus here is on the role of distance here, I prefer to use the data of lower granularity in the spatial dimension. However, one can think of strategies of using the two sources jointly (e.g. with Bayesian techniques). Secondly, rather than using value added (**va**), one could in principle think of using global output data structured in the same sectorial-regional way. This entails an adequate modification in calibrating both β and β_0.

Problem (19) can easily be extended to a panel setup for multiple years. I use the time period from 2009 through 2013 because it was centered around the year 2011, for which β and β_0 were calibrated. Central statistical offices frequently update input-output tables in 5-year intervals. Due to relatively short time dimension of the panel, I do not take additional account of serial correlation (nor do I extend heteroskedasticity and cross-sectional dependence beyond what equation (18) implies), and hence log-likelihood in (19) can be treated as additive for individual years.

The estimation results (see Table 1) confirm the role of physical distance in the construction of $\mathbf{W^s}$ matrices. The role of spatial decay is generally confirmed as significant when we take into account the standard errors of the estimates (in the power, triangle or interval-wise specification) and the difference between the point estimates and hypothetical values implying non-responsiveness to the physical distance. Additionally, the results obtained without any role attributed to the distance (i.e. with equal weights to all regions and $\hat{\mathbf{\Omega}}$ being the only estimated parameters) can be treated as a restricted model, and hence one can apply the likelihood ratio test. In all 3 countries and for all 4 functional forms, the hypothesis that the distance has no impact can be rejected at any significance level. Most of the correlations between residuals from different sectors (as implied by $\hat{\mathbf{\Omega}}$) are statistically significant and material (see Appendix B).

Some similarities differentials between countries and sectors can also be observed. In all three analysed countries, manufacturing appears to be relatively distance-tolerant, within a range of 100-200 km. In Poland and Germany, a similar profile emerges for advanced services (e.g. financial, professional, administrative), whereby this tolerance is higher in Poland. According to the estimates obtained, distant supplies are of relatively material size in Poland, France and Germany, though the distance profiles vary considerably. It is also noteworthy that French regions, in general, appears to be more oriented towards local supplies that the regions in Poland and Germany (with a notable exception

Figure 2: Value added in PL, FR, DE across NUTS-3 regions in sectorial breakdown (2011)
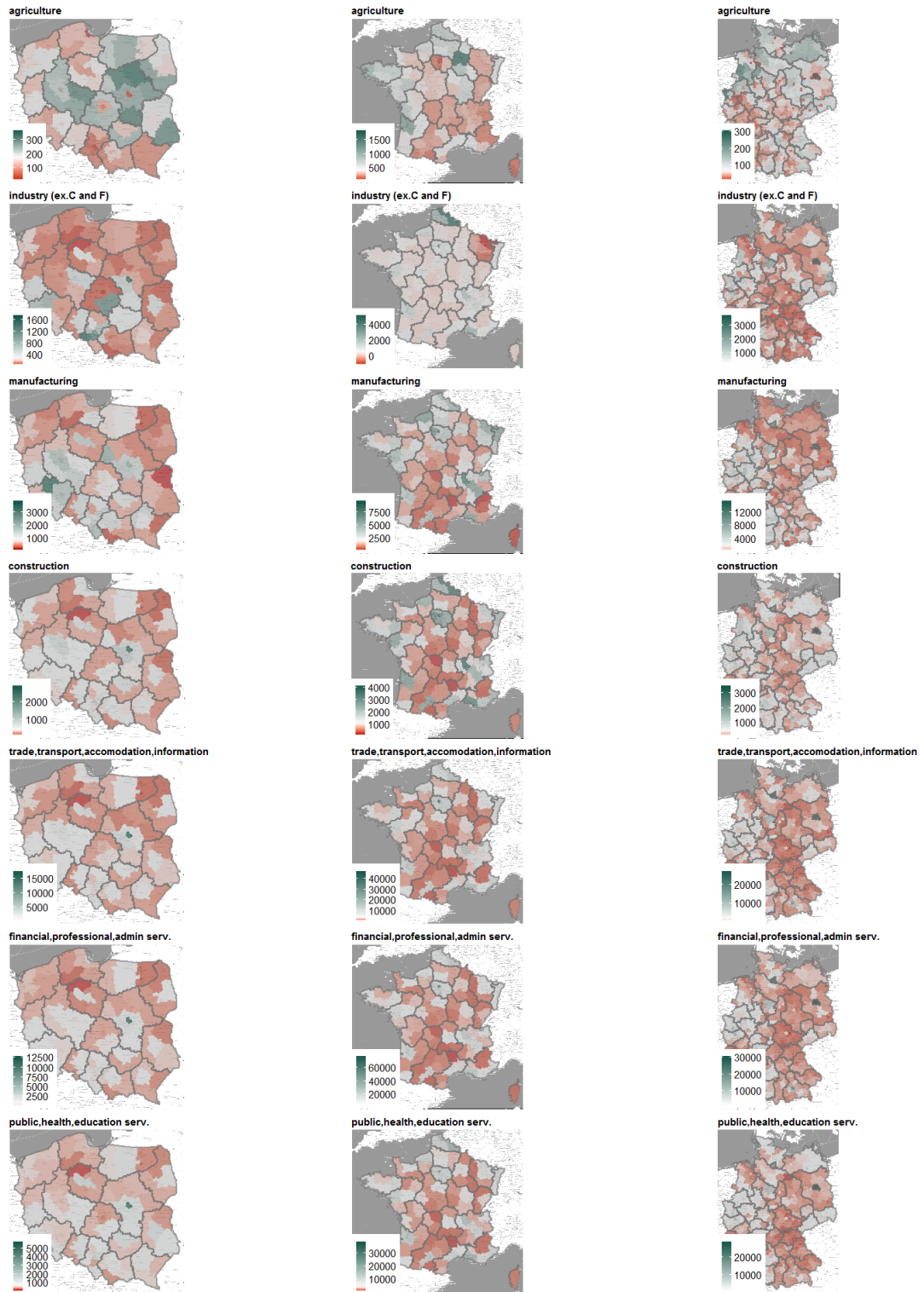
Source: Eurostat, Stamen Maps, own elaboration.

of agriculture). In all analysed economies, an intuitive, strong bias in administration, health and educational services has been confirmed.

Figure 3 compares different profiles of spatial decay for different sectors and countries, along with a weighted average. The weights were derived according to formulae (11) and (14) (Akaike version) in Buckland et al. (1997), i.e. the Akaike information criterion for individual models $k = 1, ..., 4$ was computed as $I_k = -2 \ln L_k + 2p_k$ (p − number of estimated parameters) , while weights were derived as $w_k = \frac{\exp(-I_k/2)}{\sum_{k=1}^{4} \exp(-I_k/2)}$. One can conclude that the difference in likelihood value at maximum points is sufficient for the gamma specification to dominate in terms of the best likelihood value (although, in terms of the AIC-weighted average, it appears to be equivalent to the power specification due to more parsimonious specification of the latter). All in all, in further exposition, I shall concentrate on this functional form only.

One cannot directly conclude from the results reported in Table 1 whether the distance is statistically significant in the gamma model, because it depends on two parameters. Their individual standard errors of estimation appear to be relatively high, but their respective covariance of estimate shall also be taken into account as, to some extent, the parameters of the gamma-pdf trade off the profile of the function between each other. In this case, the uncertainty around the profile of spatial decay in weights has been assessed with a parametric bootstrap method. Assuming the normality of residual distribution, and using the variance-covariance matrix of the estimates evaluated as inverse Hessian matrix at the maximum, I draw a matrix of 10000 distance profiles and then, on a 1 kilometer grid, I look at the interval of middle 50% and 90% draws (see Figure 4). One can conclude that the resulting estimates of the distance functions appear to be relatively precise, at least for some sectors, although the assumption of residual normality is clearly violated (cf. Figure 5).

The skewness of the above illustrated histograms is largely related to the spatial distribution of the dependent variable between big cities, urban and rural areas. One potential reason for this phenomenon is the imperfection in approximation of the regional distribution of $\mathbf{y}$ (by ignoring any regional proxies of exports and investment, or by assuming identical sectorial composition of households' and general government's consumption across regions). Another source of this phenomenon can be the omission of other determinants, including a proxy of the sectorial supply-side potential across regions. Regardless of the origins of non-normality (which affects both the boostrap intervals in Figure 4 and the point estimates obtained from the likelihood function derived under the assumption of normality), one could either attempt to tackle this issue by looking for better (or additional) proxy variables or apply the Bayesian methods to explicitly take account of non-normality in the estimation and statistical inference. I leave this issue for future research.

Table 1: Estimation results for subregions (power, triangle, gamma specifications)

|	sector (s)	power			triangle			gamma				
functional form		$\hat{\theta}^s$	$SE(\hat{\theta}^s)$	$\hat{\sigma}^2_s$	$\hat{\theta}^s$	$SE(\hat{\theta}^s)$	$\hat{\sigma}^2_s$	$\hat{\theta}^s_1$	$SE(\hat{\theta}^s_1)$	$\hat{\theta}^s_2$	$SE(\hat{\theta}^s_2)$	$\hat{\sigma}^2_s$
PL	agriculture, forestry, fishery (A)	-0.42	0.12	9889.1	-0.0017	0.0002	8537.7	0.0085	0.0545	155.5668	630.8563	11241.3
	industry except manufacturing (B,D,E)	-0.74	0.06	106300.4	-0.0019	0.0004	126667.3	351.1222	656.7935	0.2698	0.5058	115605.9
	manufacturing (C)	-0.71	0.05	235058.3	-0.0018	0.0002	281621.0	1916.4274	3394.2585	0.0655	0.1158	245431.6
	construction (F)	-0.91	0.02	77067.2	-0.0013	0.0003	116862.9	0.2627	0.0570	86.7874	11.6934	73088.0
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	-1.89	0.67	2937869.4	-0.0014	0.0005	4328406.1	0.0309	0.0091	828.7027	338.4496	3962756.2
	finance, insurance, real estate activities (K,L)	-1.16	0.04	1718306.1	-0.0016	0.0005	2087287.4	85.9043	97.4661	5.2023	5.8758	2401884.9
	other services (M,N,O,P,Q,R,S,T)	-1.11	0.03	272623.6	-0.0009	0.0004	444005.3	0.0025	0.0005	5524.7792	3071.7860	224639.4
FR	agriculture, forestry, fishery (A)	-0.8011	0.0352	181184.3	-0.0003	0.0003	60054.8	1952.9518	37136.0103	0.4496	8.5463	59128.9
	industry except manufacturing (B,D,E)	-6.9873	674899.3110	3308894.2	-0.0009	0.0002	506833.1	1798.4948	5089.7916	0.0626	0.1771	461249.6
	manufacturing (C)	-1.0736	0.0333	1658618.5	-0.0011	0.0001	3350599.4	1199.2790	1596.0677	0.1060	0.1408	2808964.3
	construction (F)	-1.1175	0.0296	444189.5	-0.0007	0.0001	937630.3	909.0661	24089.9874	0.0292	0.7590	41211.8
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	-1.3010	0.0471	74022050.2	-0.0013	0.0002	46076529.2	1623.5021	1548.2492	0.0652	0.0623	39123419.6
	finance, insurance, real estate activities (K,L)	-5.9612	5449.7380	112033979.6	-0.0013	0.0002	106647120.7	963.6262	266792159	0.0103	7846.5300	31303748.9
	other services (M,N,O,P,Q,R,S,T)	-5.6730	5315.2642	15274112.2	-0.0006	0.0002	26543245.6	1884.3801	2019762	0.0075	7.7846	3927307.8
DE	agriculture, forestry, fishery (A)	-0.2034	0.1018	2698.6	0.0000	0.0004	2054.7	0.0037	0.0026	387.8263	238.8606	3030.1
	industry except manufacturing (B,D,E)	-1.3139	0.0258	106938.3	-0.0006	0.0005	144370.6	78.6504	917.6242	0.0276	0.2943	54888.6
	manufacturing (C)	-1.1542	0.0143	1756637.8	-0.0011	0.0002	2578597.5	1588.7457	3572.3649	0.1133	0.2547	2540556.7
	construction (F)	-1.1592	0.0065	45827.9	0.0000	0.0004	713944.1	0.0056	0.0013	274.0123	46.9223	12727.7
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	-1.9429	0.1011	3399370.8	-0.0003	0.0006	5490701.7	8.6203	2.4626	18.5703	4.8017	5459003.6
	finance, insurance, real estate activities (K,L)	-2.8715	1.3266	4217002.6	-0.0006	0.0005	9296509.0	1686.4955	2461.0429	0.1118	0.1632	9297385.9
	other services (M,N,O,P,Q,R,S,T)	-1.7305	0.0416	1727938.6	0.0000	0.0006	4181081.7	74.1840	53.8870	0.1012	0.0715	431141.2

Source: own elaboration.

17

Table 2: Estimation results for subregions (interval-wise specification)

| functional form	sector (s)	interval								
		$\hat{\theta}_1^s$	$SE\left(\hat{\theta}_1^s\right)$	$\hat{\theta}_2^s$	$SE\left(\hat{\theta}_2^s\right)$	$\hat{\theta}_3^s$	$SE\left(\hat{\theta}_3^s\right)$	$\hat{\theta}_4^s$	$SE\left(\hat{\theta}_4^s\right)$	$\hat{\sigma}_s^2$
PL	agriculture, forestry, fishery (A)	1.00	0.67	0.00	0.29	1.00	0.82	0.05	0.11	8111.9
	industry except manufacturing (B,D,E)	0.07	0.13	0.05	0.14	0.10	0.18	0.07	0.08	114917.3
	manufacturing (C)	0.00	0.18	0.78	0.37	1.00	0.36	0.00	0.07	230406.0
	construction (F)	0.22	0.13	0.18	0.18	0.45	0.17	0.17	0.06	110225.3
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	0.29	0.26	0.25	0.23	1.00	0.44	0.13	0.09	4155057.2
	finance, insurance, real estate activities (K,L)	0.06	0.20	0.00	0.18	0.86	0.35		0.08	1998620.1
	other services (M,N,O,P,Q,R,S,T)	0.95	0.36	0.38	0.26	0.88	0.39	0.54	0.16	435370.3
FR	agriculture, forestry, fishery (A)	1.00	0.40	0.00	0.23	0.87	0.39	0.49	0.18	57546.6
	industry except manufacturing (B,D,E)	0.53	0.28	0.00	0.20	0.26	0.28	0.41	0.17	487079.6
	manufacturing (C)	0.00	0.09	0.31	0.10	0.14	0.11	0.14	0.05	28258882.6
	construction (F)	0.06	0.10	0.23	0.10	0.03	0.12	0.30	0.06	792901.7
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	0.05	0.12	0.19	0.12	0.00	0.13	0.18	0.07	403732890.4
	finance, insurance, real estate activities (K,L)	0.10	0.16	0.24	0.15	0.16	0.17	0.23	0.11	97699025.3
	other services (M,N,O,P,Q,R,S,T)	0.17	0.15	0.27	0.15	0.10	0.17	0.40	0.10	238059962.0
DE	agriculture, forestry, fishery (A)	0.00	0.20	0.00	0.20	1.00	0.31	1.00	0.27	1947.3
	industry except manufacturing (B,D,E)	0.00	0.08	0.00	0.06	0.00	0.07	0.04	0.02	136011.4
	manufacturing (C)	0.11	0.09	0.00	0.07	0.10	0.08	0.11	0.03	24223305.3
	construction (F)	0.00	0.10	0.00	0.09	0.48	0.10	0.32	0.05	68730.2
	trade, repair, transport, storage, accommodation, restaurants, communications (G,H,I,J)	0.02	0.13	0.18	0.12	0.04	0.11	0.16	0.05	53750092.9
	finance, insurance, real estate activities (K,L)	0.22	0.16	0.15	0.13	0.01	0.12	0.19	0.07	9127825.2
	other services (M,N,O,P,Q,R,S,T)	0.04	0.14	0.24	0.13	0.14	0.12	0.25	0.06	4115939.8

Source: own elaboration.
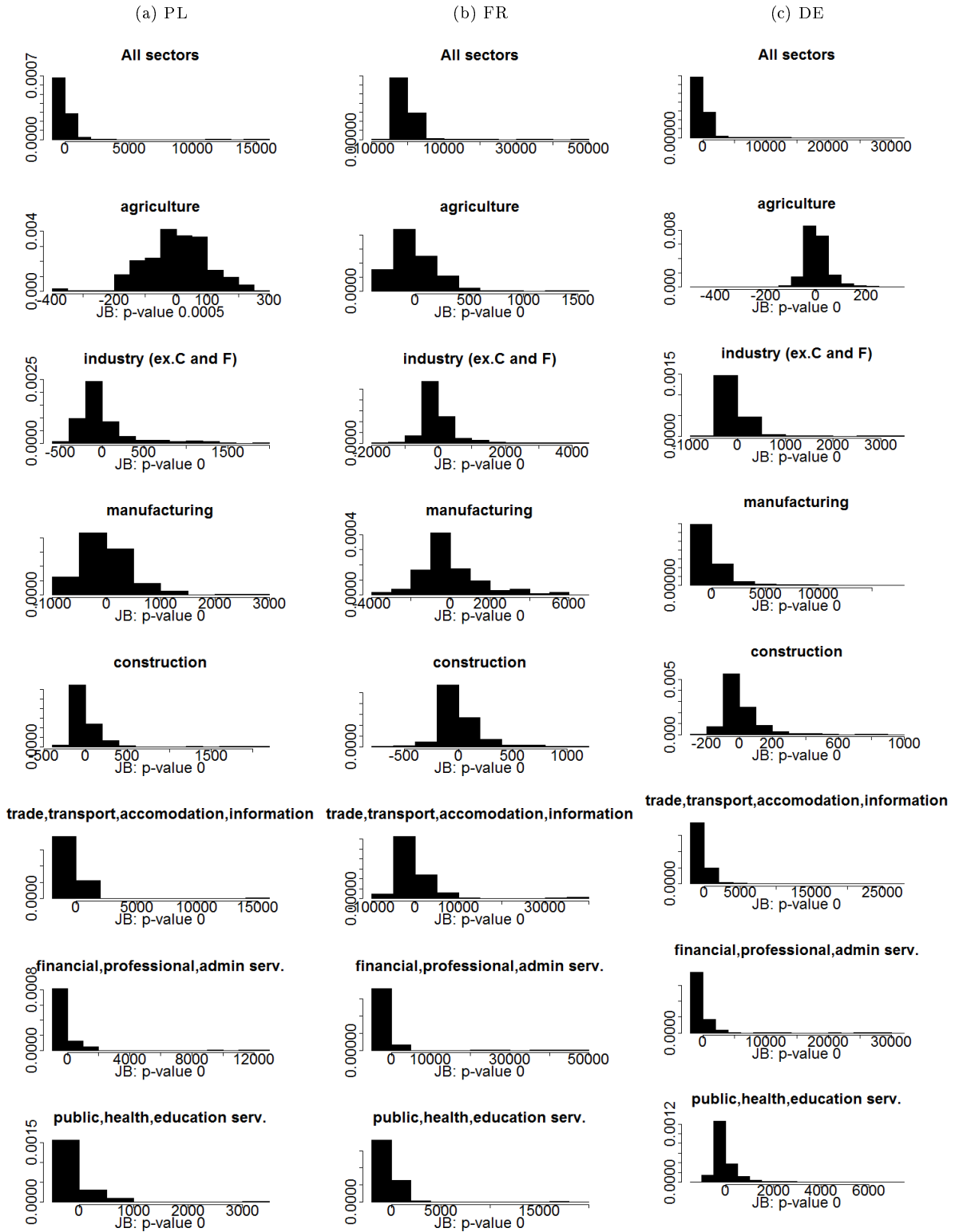
18

Figure 3: Distance functions and AIC-based comparison

(b) FR   (c) DE
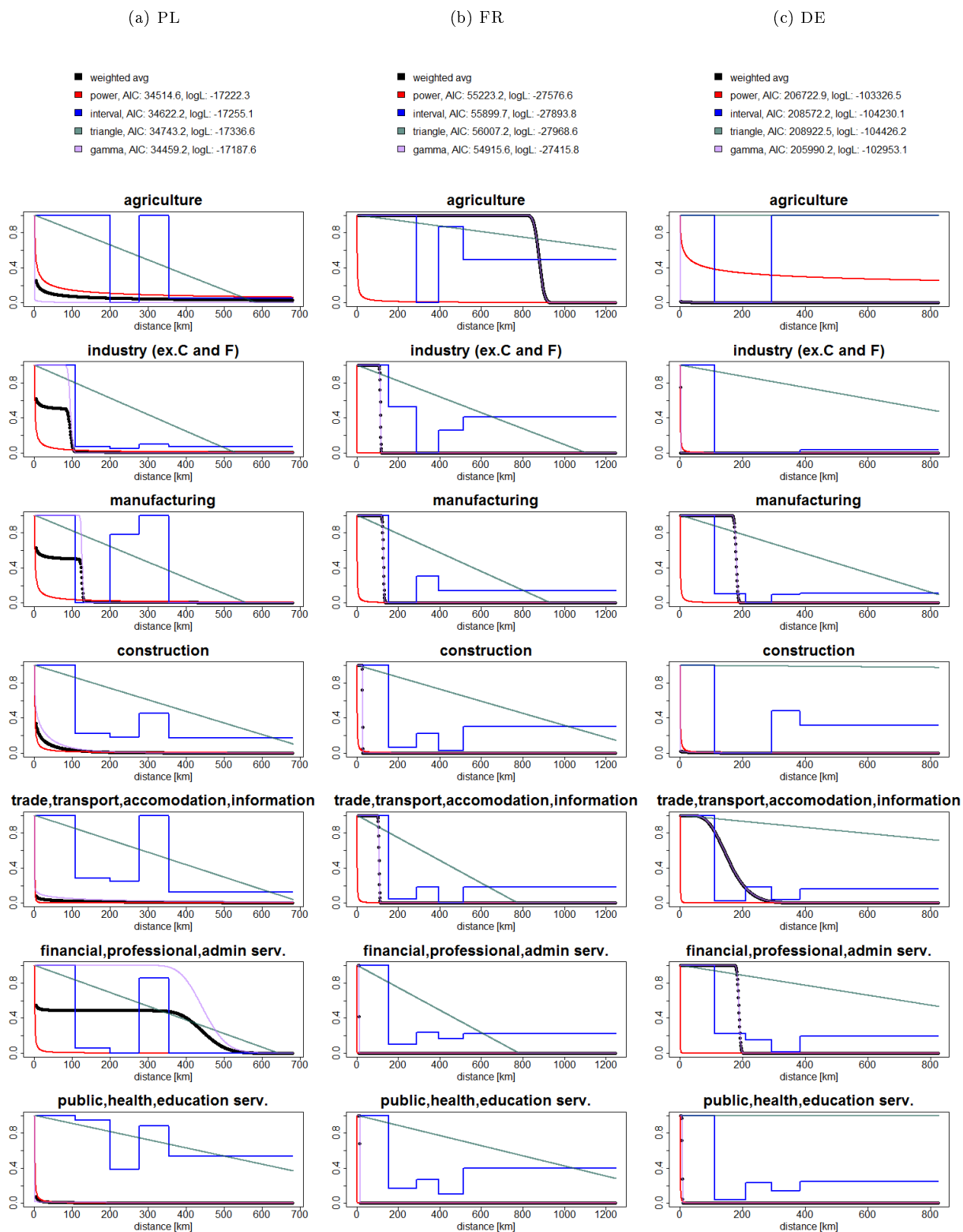
Source: own elaboration.   19

Figure 4: Uncertainty around gamma distance function values

(a) PL　　　　　　　　　　(b) FR　　　　　　　　　　(c) DE
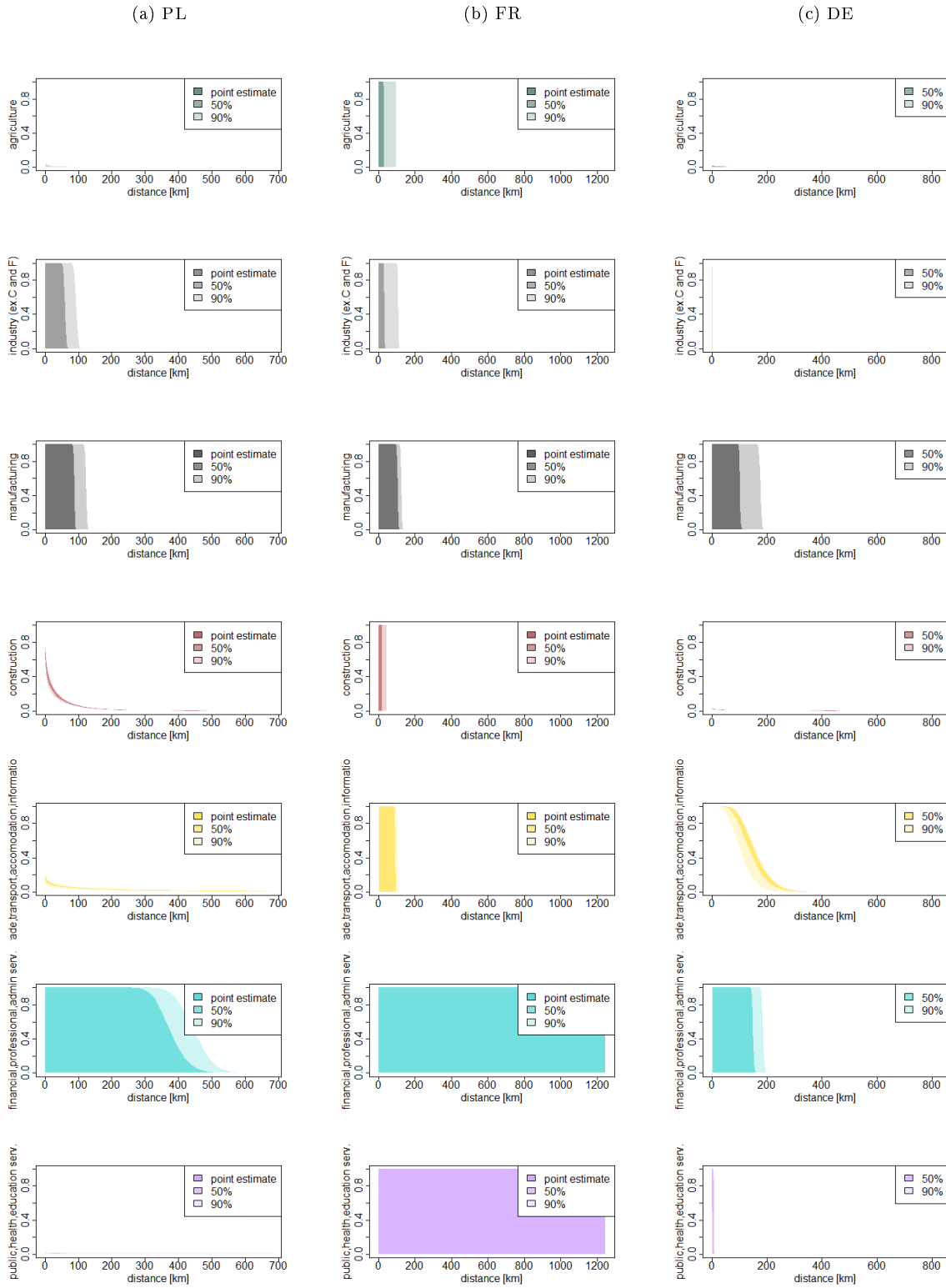
Figure 5: Distribution of residuals

(a) PL

(b) FR

(c) DE

5 Spatial propagation of economic effects: illustrative simulations for France, Germany and Poland

The estimation of $\mathbf{W^s}$ matrices in the previous section is the key step in the construction of a simulation-ready multi-region I-O matrix. However, two additional steps have to be taken.

Firstly, the data availability at the NUTS-3 level only allows for $S = 7$ in the empirical analysis, while the WIOT tables at the sectorial level encompass 56 sectors. Therefore, an adequate mapping of $\hat{\boldsymbol{\theta}}^s$ needs to be performed and it is straightforward, as Eurostat's regional data is classified into sectors as *groups of NACE sections*, while sectors in WIOD also correspond to *NACE (sub)sections* (Timmer, 2012).

Secondly, in the empirical analysis in Sections 3 and 4, attention was paid to both demand-side considerations (cf. equation (3)) and distance as a determinant of linkage between each pair of regions (finding $\mathbf{W^s}$ by solving problem (19)). This allows to build a valid multi-region, multi-sector flow matrix that fulfils the balancing restrictions implied by the country-wide input-output matrix (e.g. it is ensured that an impulse yields the same impact on the national economy, regardless of its location). However, by using this matrix for simulations, one assumes that the system is entirely demand-driven.

While it is convenient to abstain from supply-side considerations in the empirical problem (for reasons mentioned in Section 3), one cannot ignore the perils in the simulation analysis. If an enterprise or investment is expected to increase intermediate demand in some sectors, one cannot always assume that these sectors will expand in the proximity, especially when they are scarce in the area before the impulse occurs. For instance, the intermediate demand for financial services is likely to create value added in big cities, and the supply of coal or energy is likely to originate at locations where these resources are already available.

To take account of this, I additionally expand the simulation analysis by redefining the region weights

$$w_{rp}^s = \frac{w_{gamma}^s(r,p) \cdot w_{supply}^s(r,p)}{\sum_r w_{gamma}^s(r,p) \cdot w_{supply}^s(r,p)}, \text{ whereby } w_{supply}^s(r,p) = \left(\frac{va_r^s}{\sum_i va_i^s}\right)^{\gamma^s} . \text{ Note that } \gamma^s = 0 \text{ assumes}$$

away the supply-side considerations across regions, and implies that every region on the same circle around the demanding location has the same probability of being chosen as the region of supply. On the contrary, setting $\gamma^s = 1$ implies that the demanding company is placing its orders in other regions in such a way that, *ceteris paribus* (i.e. for all regions whose centroids are located at the same radius), it is drawing the supplier from the probability mass proportional to the existing supply in the sector s. It is therefore reasonable to set a value of $\gamma^s \in [0; 1]$ and, in the simulation that follows, the middle point of this interval is used, i.e. $\gamma^s = 0.5$ for $s = 1, ..., S$.

The modified $\mathbf{W^s}$ matrices ($s = 1, ..., 56$) lead to the final version of the multisector-multiregion I-O matrix $\mathbf{X^{RS \times RS}}$. Using the vector of global output (by regions and sectors, arranged accordingly), one obtains the cost structure matrix $\mathbf{A^{RS \times RS}}$. Two further extensions to this matrix shall be applied to analyse an enterprise, and to take account of both indirect and induced effects.

Firstly, additional R columns and rows are supplied to take account of the separate household sector that receives wages and creates part of the final demand. In the additional rows, the employment cost (derived as a fraction of global output from WIOT) has been entirely allocated to the region where the output arises (and thus cross-regional commuting has been ignored; one can treat it as equivalent to assuming that the effects of commuting cancel out across regions). In the additional columns, it has been assumed that the regional distribution of demand for a given sector's products is equivalent to the estimated structure in the business-to-business relationships. The only exception is the retail trade, in which only retail margins are treated as the final output in the national accounts, and hence these only contribute to the local consumption.

Secondly, I include one additional row and column to account for the enterprise in question. The row only contains zeros, and the column describes the cost structure of the company.

The final cost structure matrix will be denoted as $\mathbf{A}^{[R(S+1)+1] \times [R(S+1)+1]}$, and the corresponding Leontief matrix $\mathbf{L}^{[R(S+1)+1] \times [R(S+1)+1]} = \mathbf{I}_{R(S+1)+1} - \mathbf{A}^{[R(S+1)+1] \times [R(S+1)+1]}$. The simulation formula is standard and reads:

$$\triangle \mathbf{x}^{R \cdot (S+1)+1} = \left\{ \mathbf{L}^{[R \cdot (S+1)+1] \times [R \cdot (S+1)+1]} \right\}^{-1} \cdot \triangle \mathbf{y}^{R(S+1)+1}, \tag{21}$$

whereby $\triangle \mathbf{y}^{R(S+1)+1}$ is the vertical vector of final output (containing the output of the analysed enterprise as the last element) and $\triangle \mathbf{x}^{R(S+1)+1}$ – the resulting vertical vector of global output across all sectors (including households) and regions.

In practice, it may be challenging to construct the last column of matrix $\mathbf{A}^{[R(S+1)+1] \times [R(S+1)+1]}$. While it is not unusual to decompose costs according to sectors from which materials were purchased (or, more precisely, the sectors where the purchased intermediate goods were produced), the region of their origin may often remain unknown, even to the purchasing company or household. Note that it is not the place of purchase, nor the formal invoicing address, but the location of the previous value-adder in the chain. With no information available, it might be helpful to use a respective column of the matrix $\mathbf{A}^{RS \times RS}$ as a proxy.

In our illustrative simulation, I compute the economic effects (indirect, induced, and total – including direct) of generic enterprises located in the Southern parts of Poland, France and Germany (see Figure 6):

- for PL: in Katowice (Podregion Katowicki, NUTS-3 region PL22A, $R = 72$);

- for FR: in Marseille (Bouches du Rhônes, NUTS-3 region FR824, $R = 96$);

- for DE: in Stuttgart (Stuttgart Stadtkreis, NUTS-3 region DE111, $R = 402$).

In each case, the enterprise operates in sector *manufacturing of chemicals and chemical products* (part of *manufacturing*, sector 11 in WIOD), has a cost structure representative for its region and sector (according to the $\mathbf{A}^{RS \times RS}$ matrix), the cost level of 100 m EUR and the sales revenue of 200 m EUR.
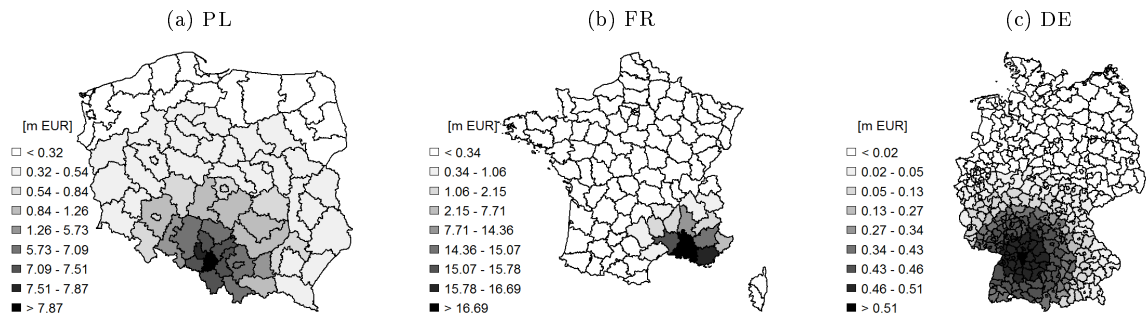
Figure 6: Example simulation: direct effects on global output

(a) PL　　　　　　　　　(b) FR　　　　　　　　　(c) DE

Impulse location(s)
□ Rest of country
■ Location of the enterprise
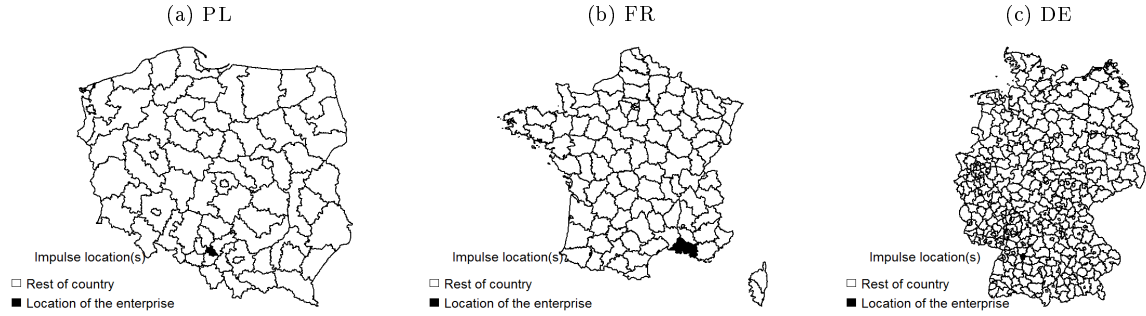
Source: own elaboration.

Figures 7, 8 and 9 represent, respectively, the volume of indirect, induced and total effects expressed in terms of global output, as a total for all sectors.[5] The pace of spatial decay is related to both the gamma function in individual sectors, and the sectorial composition of the impulse (i.e. whether the sectors with the highest weight for nearby locations are part of first-order effects rather than further-order effects, or *vice versa*). The obtained figures are easily convertible into analogous effects in value added and, under additional assumptions, employment and fiscal revenues at the central and local levels.
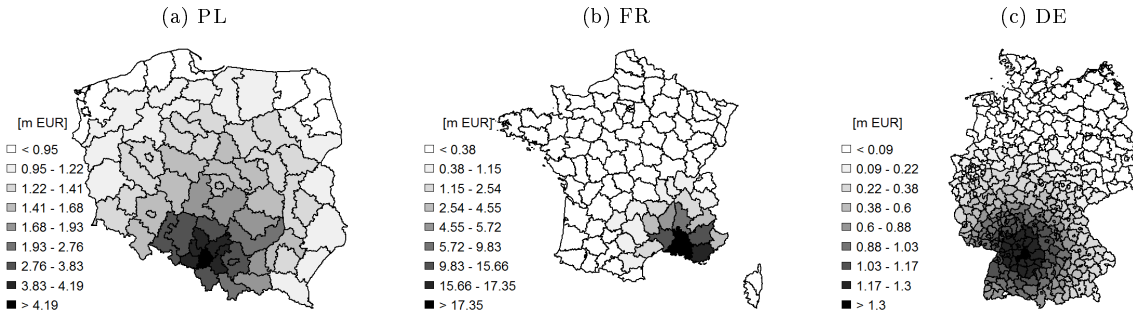
Figure 7: Example simulation: indirect effects on global output

(a) PL　　　　　　　　　(b) FR　　　　　　　　　(c) DE

[m EUR]
□ < 0.32
□ 0.32 - 0.54
□ 0.54 - 0.84
□ 0.84 - 1.26
■ 1.26 - 5.73
■ 5.73 - 7.09
■ 7.09 - 7.51
■ 7.51 - 7.87
■ > 7.87

[m EUR]
□ < 0.34
□ 0.34 - 1.06
□ 1.06 - 2.15
□ 2.15 - 7.71
■ 7.71 - 14.36
■ 14.36 - 15.07
■ 15.07 - 15.78
■ 15.78 - 16.69
■ > 16.69

[m EUR]
□ < 0.02
□ 0.02 - 0.05
□ 0.05 - 0.13
□ 0.13 - 0.27
■ 0.27 - 0.34
■ 0.34 - 0.43
■ 0.43 - 0.46
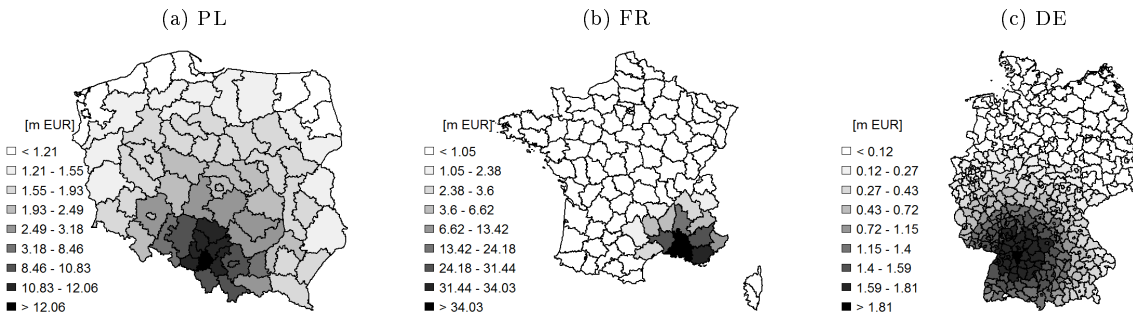■ 0.46 - 0.51
■ > 0.51

Source: own elaboration.

---

[5]Results for individual sectors, in the form of maps for individual sectors, as well as numerical tables, are available upon request.

Figure 8: Example simulation: induced effects on global output

(a) PL   (b) FR   (c) DE

Source: own elaboration.

Figure 9: Example simulation: total effects (direct, indirect, induced) on global output

(a) PL   (b) FR   (c) DE

Source: own elaboration.

6   Conclusions

There is usually no straightforward computational strategy for conducting the economic impact assessment exercises on a high level of spatial disaggregation, or when impact on multiple regions shall be considered. Such tasks involve generating multi-region input-output tables. In this paper, I investigate a promising, but relatively little expored technique of using spatial econometric methods. The presented approach largely builds on the contribution by Torój (2016), i.e. using the framework of multi-equation spatial Durbin model with fixed structural parameters and unknown spatial weight matrices, while extending the analysis in a number of dimensions: alternative functional forms, generalized stochastic properties of the model, inclusion of multiple countries and refinement of numerical algorithms.

The required data input involves (i) I-O table on the sectorial level for a national economy and (ii) regional data on value added in individual sectors, along with some auxiliary inputs (regional data on global output in individual sectors, regional data on population). On the regional level, I focus on the NUTS-3 spatial breakdown (in EU's nomenclature) as the most spatially disaggregated level for which

relevant regional data in EU countres is available. These requirements are fulfilled for almost all EU countries, and I concentrate on three relatively big economies comprised in this set: France, Germany and Poland. In all three cases, the results of illustrative simulations with the multi-region Leontief model have been presented, confirming the intuitive spatial spillover of the indirect and induced effects, mostly into the neighbouring regions.

This study has empirically confirmed the role of distance in determining the geography of supplies, and that this role varies from sector to sector. Out of different functional forms under consideration (linear, power, interval-wise, gamma-pdf-based), the specification based on gamma probability distribution function has turned out to ensure the optimum data fit as it can virtually fit any of the competing formulae at the cost of using only one more parameter per sector. The estimates of the impact of distance are relatively precise, at least for some sectors. Also, the extension of the previous literature by allowing the residuals to be correlated across sectors has turned out to be justified. As regards the cross-country comparative perspective, it appears that the French economy is characterized by more local bias than the Polish and German economy. In all analysed economies, an intuitive, strong bias in administration, health and educational services has been confirmed, as well as relatively high propensity for remote supplies in manufacturing.

The empirical investigation has identified, and left open, a few issues for consideration in future research. Most notably, the inspection of residuals has confirmed significant deviations from normality, i.e. from the assumption underlying both the likelihood function and the computation of uncertainty measures. In my assessment, this is due to the uncaptured traits of value added spatial distribution between big cities and mixed, urban-rural regions. Within the proposed estimation framework, one could explore various directions of search for a better proxy of sectorial-regional final output (than the proxy used here and based on population). Alternatively, one could switch to a Bayesian framework and explicitly take account of the non-normality. By pursuing this strategy, one could also take into consideration the prior knowledge of the role of distance in individual sectors (i.e. a high impact of transport cost on simple products, along with a relatively lower role in specialized services) and combine the obtained results with the ones possible to extract at a higher level of sectorial disaggregation (but at the cost of higher spatial aggregation up to NUTS-2).

In estimating the spatial weight matrices, one could include non-distance compontents into the estimating equation. A question that remains open is whether, and in what form, sectorial proxies for supply-side potential of regions could become part of the empirical problem. Other determinants could also be considered, such as e.g. being an urban region, road distance between region capitals or travelling time between region capitals (sometimes being a more refined, and relevant, variable than the physical distance). Regardless of the exact specification of the weight matrix, it must be emphasized that the proposed approach is relatively demanding on the computational side, like many estimation problems in spatial econometrics.

Finally, the method presented here is a relatively novel approach, declared by the previous literature as promising, but subject to little empirical exploration. Consequently, one should perform additional analyses for countries where multi-region input-output tables exist (whether survey-based or derived

from other sources), and perform a validation analysis. Flegg and Tohmo (2016) present an example
of such an analysis when discussing the FLQ technique.

References

Akita T., Kataoka M. (2002): *Interregional Interdependence and Regional Economic Growth:
An Interregional Input-Output Analysis of the Kyushu Region*, Review of Urban and Regional
Development Studies, 14, 18–40.

Atkinson R.D. (1993): *Defense Spending Cuts and Regional Economic Impact: An Overview*,
Economic Geography, 69(2), 107–122.

Baade R.A., Matheson V.A. (2004): *The Quest for the Cup: Assessing the Economic Impact of
the World Cup*, Regional Studies, 38(4), 343–354.

Belisle C.J.P. (1992): *Convergence theorems for a class of simulated annealing algorithms*, Rd. J.
Applied Probability, 29, 885–895.

Bonfiglio A. (2009): *On the parametrization of techniques for representing regional economic
structures*, Economic Systems Research, 21(2), 115–127.

Buckland S.T., Burnham K.P., Augustin N.H. (1997): *Model Selection: An Integral Part of
Inference*, Biometrics, 53(2), 603–618.

Cameron A.C., Trivedi P.K. (2005): *Microeconometrics. Methods and Applications*, Cambridge
University Press, Cambridge.

Canning P., Wang Z. (2005): *A Flexible Mathematical Programming Model to Estimate
Interregional Input-Output Accounts*, Journal of Regional Science, 45, 539–563.

Central Statistical Office in Poland (2014): *Input-Output Table at Basic Prices in 2010*,
Statistical Information and Elaborations.

Chenery H.B. (1953): *The Structure and Growth of the Italian Economy*, chapter Regional Analysis,
97–129, Rome: US Mutual Security Agency.

Chrzanowski M. (2013): *Wykrycie lokalnych obszarów kooperacji w województwie
lubelskim (analiza output-input)*, Kapitał Intelektualny Lubelszczyzny 2010-2013, URL
http://www.kil.lubelskie.pl/wp-content/uploads/2013/08/Wykrycie-lokalnych-obszar
%C3%B3w-kooperacji-w-wojew%C3%B3dztwie-lubelskim-analiza-output-input-
dr-Marek-Chrzanowski.pdf.

Cicas G., Hendrickson C.T., Horvath A., Matthews H.S. (2007): *A Regional Version of a US Economic Input-Output Life-Cycle Assessment Model*, International Journal of Life Cycle Assessment, 12(6), 365–372.

Cristina de la Rúa, Lechón Y. (2016): *An integrated Multi-Regional Input-Output (MRIO) Analysis of miscanthus biomass production in France: Socio-economic and climate change consequences*, Biomass and Bioenergy, 94, 21–30.

Easterling W.E. (1997): *Why regional studies are neneed in the development of full-scale integrated assessment modelling of global change processes*, Global Environmental Change, 7(4), 337–356.

Flegg A., Tohmo T. (2013): *Regional Input-Output Table and the FLQ Formula: A Case Study for Finland*, Regional Studies, 47(5), 703–721.

Flegg A.T., Tohmo T. (2016): *Estimating Regional Input Coefficients and Multipliers: The Use of FLQ is Not a Gamble*, Regional Studies, 50(2), 310–325.

Flegg A.T., Webber C.D., Elliott M.V. (1995): *On the Appropriate Use of Location Quotients in Generating Regional Input–Output Tables*, Regional Studies, 29, 547–561.

Folmer H., Nijkamp P. (1985): *Methodological aspects of impact analysis of regional economic policy*, Papers of the Regional Science Association, 57(1), 165–181.

Fritz O., Kurzmann R., Pointner W., Streicher G., Zakarias G. (2001): *Modeling and Control of Economic Systems*, chapter Modeling the Regional Economy: A Regional Econometric Input-Output Approach, Elsevier, Oxford.

Funck R., Rembold G. (1975): *A multiregion, multisector forecasting model for the Federal Republic of Germany*, Papers of the Regional Science Association, 34(1), 69–82.

Giesecke J.A., Burns W.J., Berrett A., Bayrak E., Rose A., Slovic P., Suher M. (2012): *Assessment of the Regional Economic IImpact of Catastrrophic Events: CGE Analysis of Resource Loss and Behavioral Effects of an RDD Attack Scenario*, Risk Analysis, 32(4), 583–600.

Gordon I.R. (1976): *Gravity Demand Functions, Accessibility and Regional Trade*, Regional Studies, 10, 25–37.

Hakfoort J., Poot T., Rietveld P. (2001): *The Regional Economic Impact of an Airport: The Case of Amsterdam Schiphol Airport*, Regional Studies, 35(7), 595–604.

Hallegatte S. (2008): *An Adaptive Regional Input-Output Model and its Application to the Assessment of the Economic Cost of Katrina*, Risk Analysis, 28(3), 779–799.

Harris R.I., Liu A. (1998): *Input-Output Modelling of the Urban and Regional Economy: The Importance of External Trade*, Regional Studies, 32(9), 851–862.

Horváth E., Frechtling D.C. (1999): *Estimating the Multiplier Effects of Tourism Expenditures on a Local Economy through a Regional Input-Output Model*, Journal of Travel Research, 37(4), 324–332.

Hulu E., Hewings G.J.D. (1993): *The Development and Use of Interregional Input-Output Models for Indonesia Under Conditions of Limited Information*, Review of Urban and Regional Development Studies, 5, 135–153.

Isard W., Kuenne R.E. (1953): *The Impact of Steel upon the Greater New York-Philadelphia Industrial Region*, Review of Economics and Statistics, 35, 289–301.

Jackson R.W., Schwarm W.R., Okuyama Y., Islam S. (2006): *Annals of Regional Science*, A Method for Constructing Commodity by Industry Flow Matrices, 40, 909–920.

Koutaniemi M., Louhela T. (2006): *Construction of Regional Input-Output Tables in Finland 2002*, 46th Congress of the European Regional SciRegional Studies: "Enlargement, Southern Europe and the Mediterranean", August 30th - September 3rd, 2006, Volos, Greece.

Kowalewski J. (2015): *Regionalization of National Input-Output Tables: Empirical Evidence on the Use of the FLQ Formula*, Regional Studies, 49(2), 240–250.

Kratena K., Zakarias G. (2004): *Input Coefficient Change using Bi-proportional Econometric Adjustment Functions*, Economic Systems Research, 16, 191–203.

Kuhn A. (2010): *Input-Output Rechnung im Überblick*, Statistisches Budnesamt, URL https://www.destatis.de/.

Lamonica G.R., Chelli F.M. (2017): *The performance of non-survey techniques for constructing sub-territorial input-output tables*, Papers in Regional Science, doi:10.1111/pirs.12297.

Leontief W. (1936): *Quantitative Input-Output Relations in the Economic System of the United States*, Review of Economics and Statistics, 18, 105–125.

Leontief W. (1941): *The Structure of American Economy 1919–1939*, New York: Oxford University Press.

Leontief W., Strout A. (1963): *Structural Interdependence and Economic Development*, chapter Multiregional Input-Output Analysis, 119–149, London: Macmillan (St. Martin's Press).

LeSage J.P., Pace R.K. (2009): *Introduction to Spatial Econometrics*, Chapman & Hall.

Lindall S., Olson D., Alward G. (2006): *Deriving Multi-Regional Models Using the IMPLAN National Trade Flows Model*, Journal of Regional Analysis and Policy, 36, 76–83.

Liu L.N., Vilain P. (2004): *Estimating Commodity Inflows to a Substate Region Using Input-Output Data: Commodity Flow Survey Accuracy Tests*, Journal of Transportation and Statistics, 7, 23–37.

Loveridge S. (2004): *A Typology and Assessment of Multi-sector Regional Economic Impact Models*, Regional Studies, 38(3), 305–317.

McCann P. (2007): *Urban and Regional Economics*, Oxford University Press.

Miller R.E., Blair P.D. (2009): *Input-Output Analysis. Foundations and Extensions*, Cambridge University Press, Cambridge.

Morrison W.I., Smith P. (1974): *Nonsurvey input-output techniques at the small area level: an evaluation*, Journal of Regional Science, 14(1), 1–14.

Moses L.N. (1955): *The Stability of Interregional Trading Patterns and Input-Output Analysis*, American Economic Review, 45, 803–832.

Nelder J.A., Mead R. (1965): *A simplex algorithm for function minimization*, Computer Journal, 7, 308–313.

Okamoto N., Ihara T. (eds.) (2005): *Spatial Structure and Regional Development in China. An Interregional Input-Output Approach*, UK: Palgrave Macmillan.

Pak M., Poissonnier A. (2017): *Accounting for technology, trade and final consumption in employment : an Input-Output decomposition*, Insee Documents de Travail, G201611, URL https://www.insee.fr/en/statistiques/2563542.

Polenske K.R. (1970): *An Empirical Test of Interregional Input-Output Models: Estimation of 1963 Japanese Production*, American Economic Review, 60, 76–82.

Polenske K.R. (2004): *Wassily Leontief and Input-Output Economics*, chapter Leontief's 'Magnificent Machine' and Other Contributions to Applied Economics, 269–289, Cambridge University Press, Cambridge.

Rey S.J. (2000): *Integrated regional econometric + input-output modeling: Issues and opportunities*, Papers in Regional Science, 79(3), 271–292.

Rose A., Benavides J., Chang S.E., Szczesniak P., Lim D. (1997): *The regional economic impact of an earthquake: direct and indirect effects of electricity lifeline disruptions*, Journal of Regional Science, 37(3), 437–458.

Rose A., Liao S.Y. (2005): *Modeling regional economic resilience to disasters: a computable general equilibrium analysis of water service disruptions*, Journal of Regional Science, 45(1), 75–112.

Round J.I. (1978): *An interregional input–output approach to the evaluation of nonsurvey methods*, Journal of Regional Science, 18, 179–194.

Santos J.R., May L., Haimar A.E. (2013): *Risk-Based Input-Output Analysis of Influenza Epidemic Consequences on Interdependent Workforce Sectors*, Risk Analysis, 33(9), 1620–1635.

Sonis M., Hewings G.J., Okuyama Y. (**2000**): *Feedback loops analysis of Japanese interregional trade, 1980-85-90*, Journal of Economic Geography, 1.

Steinback S.R. (**1999**): *Regional Economic Impact Assessments of Recreational Fisheries: An Application of the IMPLAN Modeling System to Marine Party and Charter Boat Fishing in Maine*, North American Journal of Fisheries Management, 19, 724–736.

Theil H. (**1967**): *Economics and Information Theory*, New York: American Elsevier.

Timmer M. (**2012**): *The World Input-Output Database (WIOD): Contents, Sources and Methods*, URL www.wiod.org/publications/source_docs/WIOD_sources.pdf.

Tobler W. (**1970**): *A computer movie simulating urban growth in the Detroit region*, Economic Geography, 46(Supplement), 234–240.

Tohmo T. (**2004**): *New Developments in the Use of Location Quotients to Estimate Regional Input-Output Coefficients and Multipliers*, Regional Studies, 38(1), 43–54.

Tomaszewicz ., Trębska J. (**2005**): *Modelling Economies in Transition*, chapter Regional and interregional input-output tables for Poland, Uniwersytet Łódzki.

Torój A. (**2016**): *Regional Economic Impact Assessment with Missing Input-Output Data: A Spatial Econometrics Approach for Poland*, Central European Journal of Economic Modelling and Econometrics, 8, 61–91.

Turner K., Lenzen M., Wiedmann T., Barrett J. (**2007**): *Examining the Global Environmental Impact of Regional Consumption Activities – Part 1: A Technical Note on Combining Input-Output and Ecological Footprint Analysis*, Ecological Economics, 62(1), 37–44.

Uribe P., de Leeuw C., Theil H. (**1966**): *The Information Approach to the Prediction of Interregional Trade Flows*, Review of Economic Studies, (33), 209–220.

van der Linden J., Oosterhaven J. (**1995**): *European Community Intercountry Input-Output Relations: Construction Method and Main Results for 1965–85*, Economic Systems Research, 7(3), 249–269.

Wang Y., Geschke A., Lenzen M. (**2017**): *Constructing a Time Series of Nested Multiregion Input–Output Tables*, International Regional Science Review, 40(5).

Welfe W., Świeczewska I., Florczak W., Karp P. (**2008**): *Rozwój regionalny - ujęcie ekonometryczne*, Społeczna Wyższa Szkoła Przedsiębiorczości i Zarządzania w Łodzi, URL http://www.eedri.pl/pdf/116.pdf.

West G.R. (**1990**): *Regional Trade Estimation: A Hybrid Approach*, International Regional Science Review, 13, 103–118.

Wiedmann T., Lenzen M., Turner K., Barrett J. (2006): *Examining the global environmental impact of regional consumption activities - Part 2: Review of input-output models for the assessment of environmental impact embodied in trade*, Ecological Economics, doi:10.1016/j.ecolecon.2006.12.003.

Wiedmann T., Wilting H.C., Lenzen M., Lutter S., Palm V. (2011): *Quo Vadis MRIO? Methodological, data and institutional requirements for multi-region input–output analysis*, Ecological Economics, 70, 1937–1945.

Zawalińska K., Rok J. (2017): *Wojewódzkie tablice przepływów międzygałęziowych dla Polski: konstrukcja i interpretacja*, Studia Regionalne i Lokalne, 3(69), 29–53.

Appendix A: Properties of the estimation strategy under gamma-pdf based distance function – Monte Carlo study

In order to verify the properties of the proposed estimation strategy, a Monte Carlo study has been designed. Using the data generating process (16), the values of $\mathbf{y}$ and the map of Poland ($R = 72$), as well as the estimated parameter values in the true data generating process (including $\mathbf{\Omega}$), two types of datasets have been generated: 5-year sample (as in the data) and 1-year sample (to verify whether the estimator is T-consistent). In every case, 100 replications have been considered. The distributions of the estimated parameters are illustrated in Figure 10. The distributions for all shape and scale estimates are concentrated around the true value, and become more precise when T is growing. Some asymmetry has also been found, especially for extremely low or extremely high values (due to the trade-off between scale and shape parameters).

Figure 10: Estimator distribution in the Monte Carlo study (DGP as estimated for Poland, $T = 5$ and $T = 1$, 100 replications)
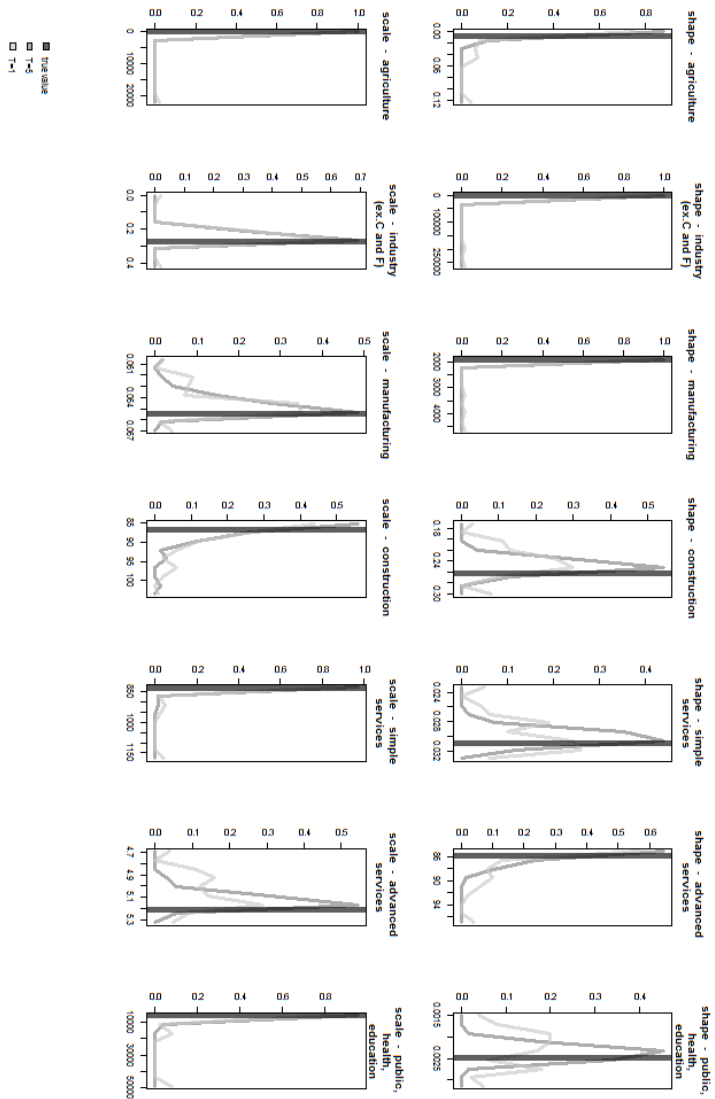
Appendix B

Table 3: Error correlations between sectors 1 to 7 (corresponding to non-diagonal $\hat{\Omega}$ elements)

	PL				FR				DE			
	pow	int	tri	gam	pow	int	tri	gam	pow	int	tri	gam
1,2	-0.446	-0.392	-0.394	-0.585	-0.708	0.015	0.022	-0.004	0.01	-0.055	-0.033	-0.066
1,3	-0.311	-0.322	-0.311	-0.636	-0.615	0.156	0.124	0.086	-0.004	-0.032	-0.028	-0.515
2,3	0.313	0.406	0.436	0.457	0.526	0.667	0.679	0.665	0.218	0.451	0.479	-0.096
1,4	-0.51	-0.363	-0.373	-0.642	-0.813	0.071	0.051	-0.229	0.327	0.143	0.16	0.026
2,4	0.56	0.601	0.631	0.593	0.725	0.759	0.768	0.203	0.532	0.667	0.681	0.024
3,4	0.339	0.481	0.518	0.497	0.791	0.866	0.88	0.411	0.573	0.713	0.722	0.505
1,5	-0.453	-0.331	-0.33	-0.592	-0.851	-0.13	-0.126	-0.156	0.013	-0.063	-0.048	-0.531
2,5	0.534	0.592	0.611	0.591	0.82	0.788	0.794	0.788	0.549	0.641	0.652	0.055
3,5	0.173	0.386	0.429	0.424	0.605	0.644	0.694	0.628	0.483	0.702	0.713	0.709
4,5	0.945	0.96	0.96	0.956	0.805	0.739	0.78	0.305	0.774	0.855	0.855	0.487
1,6	-0.444	-0.316	-0.315	-0.589	-0.818	-0.13	-0.127	-0.237	-0.005	-0.076	-0.06	-0.555
2,6	0.547	0.582	0.602	0.594	0.798	0.75	0.761	0.537	0.539	0.642	0.651	0.044
3,6	0.211	0.344	0.393	0.417	0.579	0.59	0.639	0.276	0.508	0.715	0.723	0.722
4,6	0.942	0.943	0.944	0.949	0.738	0.644	0.689	0.165	0.793	0.873	0.873	0.491
5,6	0.994	0.994	0.994	0.996	0.987	0.976	0.977	0.863	0.959	0.97	0.971	0.97
1,7	-0.512	-0.366	-0.366	-0.62	-0.723	0.041	0.03	-0.164	-0.074	-0.09	-0.068	-0.553
2,7	0.537	0.598	0.613	0.576	0.684	0.739	0.753	0.5	0.576	0.689	0.7	0.066
3,7	0.202	0.41	0.434	0.386	0.394	0.676	0.711	0.245	0.442	0.651	0.661	0.397
4,7	0.944	0.966	0.963	0.948	0.581	0.79	0.817	-0.02	0.738	0.887	0.888	0.153
5,7	0.96	0.969	0.968	0.958	0.901	0.873	0.889	0.799	0.867	0.887	0.889	0.674
6,7	0.962	0.962	0.96	0.959	0.93	0.867	0.88	0.892	0.898	0.913	0.914	0.706