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How regional business cycles diffuse across space and time: evidence from a Bayesian Markov switching panel of GDP and unemployment in Poland

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Abstract We investigate the regional business cycles at NUTS-3 granularity in Poland (N=73) using two variables in parallel: GDP dynamics and unemployment. The model allows for both idiosyncratic business cycle fluctuations in a region in the form of 2-state Markov chain, as well as spatial interactions with other regions. The posterior distribution of the parameters is simulated with a Metropolis-within-Gibbs procedure. We find that the regions can be classified into business cycle setters and takers, and this classification exhibits a high degree of overlap with the line of division between metropolitan versus peripheral regions. We also find that, under large N, the fixed-effects methods, as proposed in the previous literature, are vulnerable to both identification issues and (MCMC) convergence problems, especially with short T, which is of critical importance in GDP on the considered spatial granularity level.

Keywords: business cycle, spatial autoregression, NUTS-3, Markov switching, Bayesian analysis

JEL: C11, C23, C24, R12

1. Introduction

A relatively recent thread of business cycle literature inspects the cycles and their synchronicity at sub-national regional levels. For a number of reasons, sub-national cycles can diverge from the national economy pattern (see Warżała (2014)), posing challenges to both regional and national policymakers, not to mention the supra-national entities pursuing cohesion policies (such as the European Union). These reasons include, broadly speaking, the diverse potential and endowment of regions, resulting in increasing concentration of activity in the most economically attractive geographical areas and their consequential sectoral specialization.

Diversified regional production was found to be a leading source of regional cycle assymetries (Fatas (1997)). A vast literature of economic geography identifies facilitation of trade and economic cooperation as an underlying process. Krugman (1990) claims that the reduction of transaction costs leads to increased specialization of regions and thus reduces their cycle convergence. Fatas (1997) notes that highly specialized regions experience sectoral shocks as asymmetric local shocks.

However, Hamilton and Owyang (2012) point to asymmetric local shocks as another possible source of regional divergence, regardless of the sectoral structure of a region's economy. One example thereof are diversified regional economic policies as such (Fatas (1997)). Yet another mechanism of business cycle decorrelation across regions are asymmetric reactions to symmetric shocks, as a consequence of structural heterogeneity. For instance, the literature identifies varying reactions to changes in monetary policy (see Owyang and Wall (2006)) or fiscal policy (see Wall (2007)). Warżała (2011) confirms this asymmetry and finds a significant relation between the level of the given region's development and the degree of its sensitivity to the state of the national economy. More economically advanced regions with more diverse production structure exhibit more stability and crisis-resistance.

On the other hand, regional business cycles are exposed to sychronizing forces as well. One of those is the adherence to a common currency area, hypothesized as an endogenously fulfilled optimum currency area criterion (see Frankel and Rose (1998); Marelli (2007)). On top of that there are country-wide common fiscal policy, uniform legal order, geographical proximity, common language and culture as well as other factors that intensify interregional trade and investment flows and consequently increase the probability of the spatial spillover of a shock, resulting in a synchronization effect.

The extant literature adopts a number of approaches to the analysis of business cycles in general, and to the regional dimension thereof. Time series econometrics dominates the field, in particular the highly popular Markov models which often require the use of Bayesian inference due to their relatively complex specification as confronted with feasible sample sizes. Time series investigations, including the hidden Markov model, look predominantly at the current business cycle position as a function of past observations.

However, a joint investigation of multiple (homogenous) regions opens up new possibilities. The relevant spatial panel models can account for the fact that business cycle position is carried over not only over time, but also through physical space. To accommodate the frequent case of feedback loops between neighbouring regions' cycles, spatial econometric methods need to be introduced.

This paper adopts such a joint, spatio-temporal perspective by looking at regional business cycle positions jointly with their spatial spillovers. To this aim, at the technical level, we combine three tools, i.e. we perform a Bayesian analysis of a hidden panel Markov model with spatial autoregression. We contributes to the literature by inspecting the regional business cycle synchronization and divergences at relatively fine granularity, i.e. for 73 NUTS-3 regions in Poland. The regional breakdown is therefore far more detailed than in both Kondo (2021) and Torój (2020). For each region, we can conclude on the contribution of local and external business cycles over time.

Fine spatial granularity calls for methods that do not assume observational independence, but also poses some challenges: the need for both more computing power and room for potential considerable reinterpretation of previous results as the spatial spillovers can potentially be stronger with smaller regions under consideration.

The rest of the paper is organized as follows. Section 2 reviews the tools widely used in the (regional) business cycle literature, with a focus on hidden Markov models and regional applications. Section 3 introduces the data and methodology for the empirical analysis that follows in Section 4. Finally, Section 5 concludes the paper and discusses potential extensions of the study.

2. Literature review: from country-level Markov switching to regional spatial models

First introduced by Hamilton (1989), the hidden Markov model quickly spread through the field of business cycle research. It differentiates between two states: fast economic growth (expansion) and slow economic growth (recession). The basic Hamiltonian model measures the growth rate of an economic activity indicator as a sum of a state-dependent average growth rate and a random disturbance. The states of expansion and recession are assumed to be a stochastic, 2-state homogenous Markov chain process. By this assumption, the probability of switching between one state and the other depends only on the state noted in the previous period.

The surge in popularity of the hidden Markov models in business cycle analyses, against a popular alternative of factor models (Stock and Watson (1991)), resulted i.a. from their ability to detect the turning points. Chauvet and Piger (2003) demonstrated that the models can detect regime change more quickly than the Business Cycle Dating Committee NBER.

The growing body of Markov-switching model applications inspired a number of researchers to propose case-specific extensions. To obtain a more comprehensive and robust picture, Chauvet and Piger (2003) investigate two economic indicators instead of one: quarterly growth rate of real GDP and the monthly growth rate of employment in the US.

On the specification level, more profound extensions were proposed by i.a. Hansen (1992) who considered the identification of regime change by switching parameters other than the average growth rates, in particular the variance of the error term and the autoregression parameter. Krolzig (1997), in turn, modified the model by using a multi-dimensional density function that was not represented by a Gaussian process. The density function does not necessarily need to have a normal distribution but, for example, as in the case of Dueker (1997), a t-student distribution with the number of degrees of freedom changing accordingly to the regime change.

In a number of studies, a 3-regime specification has been established by dividing the expansion phase into a normal and a rapid growth phase (see Bernardelli and Dędys (2015); Kim and Murray (2002)). Sims and Zha (2006) showed that the Bayesian approach enables the estimation of a model with multipile regimes. In their case, models with 7 through 10 states exhibit the best fit. Another possible alteration of the Hamiltonian model includes the endogeneity assumption for the regime changes. Kim et al. (2004) claim that macroeconomic shocks can be correlated with the business cycle fluctuations and that measurement errors of an unobservable variable can also lead to endogeneity.

One can also reject the assumption of fixed transition probabilities by allowing them to change according to the fluctuations of the economic activity indicators. For instance, Diebold et al. (1994) decided to model the transition probabilities as parametric logistic functions of exogenous variables. A new form of the Hamiltonian model incorporates time-varying transition probabilities, thus making the anticipation of changes in the economy possible. Filardo and Gordon (1998) argue that the decreasing expected duration of the current cycle phase can be a forecast of an upcoming regime switch.

Numerous authors point to business cycle asymmetry as a non-negligible feature. The cycle phases can vary in the steepness, deepness, sharpness and persistence to shocks (see Clements and Krolzig (1998)). Regime-switching models proved to be able to encapsulate the sharpness asymmetry (asymmetry of business cycle turning points) because they allow for the introduction of different parameters depending on the current regime.

Diebold and Rudebusch (1996) combined two alternative approaches to business cycle research: the Hamiltonian Markov-switching model and the dynamic factor model (see Stock and Watson (1989)). They introduced a model with factors subject to regime changes which enabled them to include two crucial features of the business cycle: non-linearity and coincidence of many macroeconomic indicators. They also included the dynamics in the form of autoregression.

Multiple estimation approaches can be found in the literature for models of this nature. Kim and Nelson (1998) proposed multi-move Gibbs sampling in a Bayesian framework for a dynamic factor model with regime switching. Gibbs sampler draws from the conditional posterior density for each parameter. The authors proved that the sampling results converge to the joint posterior distribution of parameters for a sufficiently large number of iterations.

A relatively rich literature investigating the business cycle synchronization and divergence at the sub-national level exists in the case of United States of America. For instance, Harding and Pagan (2006) measured the strength of two business cycles synchronization through the percentage of time when the two economies found themselves in the same regime. The results showed that various regions can be synchronized with the national economy to a different extent.

Owyang et al. (2005) confirmed the existence of significant differences between the American states in both regimes by applying a regime-switching model to economic time series for each state. There were, however, some similarities found between regions in a given territory and with a common specialization. Another observation was that transition probabilities were usually close to 0 or 1 which implies that the identification of the current business cycle phase was relatively easy. On the other hand, the probabilities of staying in a given regime were higher than the probabilities of changing the regime. The authors also proved the occurrence of distinct differences in the timing and length between the national and regional business cycles. First of all, each region could switch regimes at a different point in time than the whole country. Second of all, a recession in the national economy did not determine the state of all regional economies.

The rising interest in sub-national, regional analyses of business cycles has led to methodological questions related to the joint analysis of multiple geographic areas. A key disadvantage of the original Hamiltonian model consisted in its one-dimensionality, meaning it was only applicable to the national and regional cycles separately. This problem has been addressed by Hamilton and Owyang (2012) by coverting the model into a panel through which the authors were able to analyze the fluctuations for many regions simultaneously. Econometrics of panel data can help to capture both the dynamic and geographical relations between units. With the use of a wide and long panel data set the authors extended the model into a multi-dimensional approach by grouping the regions with similar characteristics into clusters, which then led them to the identification of 3 such clusters for the American states.

Multiple researchers confirmed the significance of the spatial element in the business cycle analysis. Artis et al. (2011) constructed a spatial ARMA model where the spatial dependence between regions was represented by a spatial lag of the dependent variable or the error term. This specification also enables further model development through a spatial weights matrix. To address the problem of numerous regions and a large transition matrix, Fruhwirth-Schnatter and Kaufmann (2008) proposed the idea of aggregating the regions into clusters.

Shibaev (2016) further emphasized that geographical units are rarely independent. The author inspected spatial relations in a Markov-switching model with the use of spatial autoregressive error specification which allows for a spatial autocorrelation of shocks in different regions. This application confirmed the existence of the spatial spillover effect in the US which was also proved by Artis et al. (2011), whereas the phenomenon of regional business cycle synchronization was confirmed by Beraja et al. (2016) and Hamilton and Owyang (2012).

A panel Markov-switching model was also implemented in the works of Kondo (2021) and Torój (2020). Both authors introduced the spatial lag into a panel hidden Markov model in order to investigate the spatial spillover effect. Estimation methods included the multi-move Gibbs sampler and the Metropolis-Hastings algorithm. In terms of methodology, this paper continues that strand of literature by combining Markov chains, panel and spatial econometrics with the Bayesian inference.

The work of Torój (2020) does not fit with the rest of the literature insofar as it investigates the profitability of enterprises, rather than standard business cycle measures, for 16 NUTS-2 regions. There are a few other, more typical applications of the Markov-switching model for Poland at the national level, including Podgórska and Decewicz (2001), where the authors decided to apply the model in the analysis of Polish industry and found that the assumption of two regimes is sufficient to examine periodical occurrences of both downturns and upturns in the economy.

3. Data and methodology

3.1. Data

3.2. Hidden panel Markov model with spatial autoregression

Our model, strongly inspired by the works of Kondo (2021) and Torój (2020), builds on the following equation:

$$\mathbf{y_t} = \rho \mathbf{W} \mathbf{y_t} + \mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t} + \varepsilon_t, \tag{1}$$

where $\mathbf{y_t}$ denotes a vertical vector of the analyzed variable (either GDP dynamics or unemployment change) in $N = 73$ Polish NUTS-3 regions in period t, $\mathbf{s_t}$ – unobservable N-element vector of binary states (1 – expansion, 0 – recession) in period t, $\mathbf{m_0}$ and $\mathbf{m_1}$ are vertical, N-element vectors containing the constant for each region during, respectively, expansion and recession, $\mathbf{W} = [w_{n,n}]$ $(n = 1, ..., N)$ is a strictly exogenous, row-normalized spatial weight matrix based on the inverted distance between regions' centroids, ε_t is N-dimensional vector of errors, independent for individual $t = 1, ..., T$ periods, with identical multivariate normal distributions of zero means and variance-covariance matrix comprising the elements of vector σ^2 on the diagonal and zero elsewhere. For each n, the variable $s_{t,n}$ constitutes a 2-state Markov chain with region-specific 1-period-ahead probabilities of remaining in state 1 denoted as $\mathbf{p_{11}}$, and likewise $\mathbf{p_{00}}$ for state 0 (with an obvious consequence of transition probabilities amounting to $1 - \mathbf{p_{11}}$ from 1 to 0 and $1 - \mathbf{p_{00}}$ from 0 to 1). ρ is a spatial autoregression parameter, $\mathbf{1_N}$ represents a column vector of ones, of length N, and $\odot$ is the Hadamard product.

This specification does not take on board some features proposed in the extant literature, as reviewed in the previous section, including multiple states, endogenous transition probabilities and heavy-tailed distributions. We motivate the choice of a relatively simple specification in a twofold manner: due to relatively large N and small T. The former was our intentional choice, the latter – a practical consequence thereof.

Following Kondo (2021), we set the following prior distributions, independent from one another and across $n = 1, ..., N$:

- inverse gamma for σ_n^2: $IG(\frac{\underline{v}}{2}, \frac{\underline{\delta}}{2})$;
- two-dimensional normal for each pair $[m_{0,n}\ m_{1,n}]^T$: $MVN_2(\underline{\boldsymbol{\mu}}, \underline{\boldsymbol{\Sigma}})$;
- beta for $p_{00,n}$ and $p_{11,n}$: $p_{00,n}\ Beta(\underline{\alpha_{00}}, \underline{\alpha_{01}})$, $p_{11,n}\ Beta(\underline{\alpha_{11}}, \underline{\alpha_{10}})$;
- uniform for ρ: $\rho\ U(\frac{1}{\lambda_{min}(\mathbf{W})}, 1)$, where $\lambda_{min}(\mathbf{W})$ is the lowest (in absolute terms) eigenvalue of $\mathbf{W}$, in line with the typical stationary bounds for the spatial autoregression parameter for a row-normalized $\mathbf{W}$ matrix (see Anselin and Florax (1994)).

The hyperparameters were set as follows for the GDP model: $\underline{v} = 6$, $\underline{\delta} = 100$, $\underline{\boldsymbol{\mu}} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$, $\underline{\boldsymbol{\Sigma}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\underline{\alpha_{00}} = 8$, $\underline{\alpha_{01}} = 2$, $\underline{\alpha_{10}} = 2$, $\underline{\alpha_{11}} = 8$. For unemployment, $\underline{\delta} = 10$, $\underline{\alpha_{10}} = 1$, $\underline{\alpha_{11}} = 9$ and otherwise like for GDP. $\underline{\mu}_0$ ($\underline{\mu}_1$) as an element of $\underline{\boldsymbol{\mu}}$ were set as averages of the mean and first (third) quartile of GDP dynamics series, and inversely in the case of the unemployment rate.

For sampling from the posterior distribution, elements of the empirical strategy by Kim and Nelson (1998, 1999) have been applied, additionally modified by Kondo (2021) for the purpose of spatial analysis. The above references demonstrate that conditional posterior distributions are known in almost all cases, except ρ. This raises the case for the Metropolis-within-Gibbs simulation, with Metropolis-Hastings algorithm applied to sample ρ in the last step of a Gibbs iteration. The single g-th iteration of the Gibbs procedure runs as follows:

1. Set the initial value of $\boldsymbol{\theta}^{(g-1)}$, comprising all individual model parameters (i.e. all elements of $\mathbf{p_{00}}$, $\mathbf{p_{11}}$, $\mathbf{m_0}$, $\mathbf{m_1}$, σ^2, ρ), at arbitrary levels (for $g = 1$) or as values sampled in the previous step (for $g > 1$).
2. For each region $n = 1, ..., N$, sample a path of states $s_n^{(g)} = [s_{1,n} \ldots s_{T,n}]^T$ using the multimove Gibbs sampler, as presented in detail by Kim and Nelson (1998):

(a) For $t = 0$, set the probability of expansion and recession at long-term values consistent with $\mathbf{p_{00}}^{(g-1)}$ and $\mathbf{p_{11}}^{(g-1)}$.

(b) Iterate forward with the Hamilton filter (Chib (2001), Kondo (2021)) to compute the probabilities of state 0 and 1 for each region n and period t, conditional on the respective probabilities at $t-1$, data (from the beginning of the sample until t) and $\boldsymbol{\theta}^{(g-1)}$.

(c) Use the multimove Gibbs sampler to draw states $s_{t,n}$, starting with the last period $t = T$ and then iterating backwards, conditionally on states in subsequent periods, data (for the whole sample) and $\boldsymbol{\theta}^{(g-1)}$.

3. For each region $n = 1, ..., N$, sample $p_{00,n}^{(g)}$ and $p_{11,n}^{(g)}$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2) posterior distributions $Beta(\overline{\alpha}_{00,n}^{(g)}, \overline{\alpha}_{01,n}^{(g)})$ and $Beta(\overline{\alpha}_{11,n}^{(g)}, \overline{\alpha}_{10,n}^{(g)})$, where $\overline{\alpha}_{ij,n}^{(g)}$ denotes in each case the corresponding prior parameter augmented by a number of transitions from state i to j $(i, j \in 0; 1$ in $\mathbf{s_n^{(g)}}$ from point 2.
4. For each region $n = 1, ..., N$, draw $\sigma_n^{2(g)}$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2, and other elements of $\boldsymbol{\theta}^{(g-1)}$) posterior distribution: $IG(\frac{\overline{v}}{2}, \frac{\overline{\delta}_n}{2}$, where $\overline{v} = \underline{v} + T$ and $\overline{\delta}_n = \underline{\delta} + \Sigma_{t=1}^T \varepsilon_{n,t}^2$.
5. For each region $n = 1, ..., N$, draw $\begin{bmatrix} m_{0,n}^{(g)} & m_{1,n}^{(g)} \end{bmatrix}^T$ from the conditional (on $\mathbf{s_n^{(g)}}$ from point 2, $\sigma_n^{2(g)}$ from point 4 and other elements of $\boldsymbol{\theta}^{(g-1)}$ posterior distribution $MVN_2(\overline{\boldsymbol{\mu}}_n, \overline{\boldsymbol{\Sigma}}_n)$, where:

$$\overline{\boldsymbol{\Sigma}}_n = \left(\underline{\boldsymbol{\Sigma}}^{-1} + \sigma_n^{-2(g)} \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix}^T \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix} \right)^{-1}, \tag{2}$$

$$\begin{aligned} \overline{\boldsymbol{\mu}}_n = \; & \overline{\boldsymbol{\Sigma}}_n \Bigg[\underline{\boldsymbol{\Sigma}}^{-1} \underline{\boldsymbol{\mu}} + \sigma_n^{-2(g)} \begin{bmatrix} \mathbf{1_T} - \mathbf{s_n^{(g)}} & \mathbf{s_n^{(g)}} \end{bmatrix}^T \cdot \\ & \cdot \left(\begin{bmatrix} y_{n,1} \\ \vdots \\ y_{n,n} \end{bmatrix} - \rho^{g-1} (\mathbf{1_T} \otimes \begin{bmatrix} w_{n,1} & \dots & w_{n,n} \end{bmatrix}) \begin{bmatrix} y_{n,1} \\ \vdots \\ y_{n,n} \end{bmatrix} \right) \Bigg]. \end{aligned} \tag{3}$$

6. Sample $\rho^{(g)}$ from an unknown posterior distribution, conditional on all elements of $\boldsymbol{\theta}^{(g)}$ drawn in the previous steps. The sampling distribution is simulated with the random walk Metropolis-Hastings algorithm, in which the candidates are generated by a truncated normal distribution with mean $\rho^{(g-1)}$, variance has been adjusted to maintain the mean candidate acceptance probability from 0.2 to 0.4, and the upper and lower bounds correspond to the prior distribution of ρ.

It is straightforward to see from Equation (1) that, for each n, the model decomposes each region's cyclical position into 3 parts: (i) internal: n's own business cycle (Markov switching), (ii) external: the spillover of other regions' business cycles (spatial autoregression), (iii) other determinants (error term).

4. Empirical results for Polish NUTS-3 regions

4.1. MCMC convergence

The posterior distribution has been simulated in two chains. For GDP, the Gibbs sampler of $S = 20000$ iterations has been used (including a burn-in of $S_0 = 4000$), and for each of those, the Metropolis-within-Gibbs procedure counted $S = 10000$ iterations (including $S_0 = 3000$ burn-in). For longer series of unemployment, shorter chains have turned out to be sufficient ($S = 20000$ with $S_0 = 4000$, and $S = 9000$ with $S_0 = 2000$, respectively). For all parameters, potential scale reduction factors were equal to 1.00-1.03 (see Figures 1-2 for Gelman-Rubin plot), except the multidimensional statistic in the unemployment analysis, equal to 1.64. In that case, neither the extension of chains nor of the burn-in contributed to a decrease. We identify the underlying reason as related to parameter covariances under high-dimensional parameter space when $N = 73$, i.e. considerably higher than in the previous, comparable literature. Jointly with the time-consuming evaluation of posterior log-density in every iteration, leading to a generally slow MH computation, as well as the simulation-based evaluation of sampling density that prevented us from using a more efficient Hamilton Monte Carlo scheme, this might be indicative of turning to other model specifications (e.g. random effects across units) in future research.

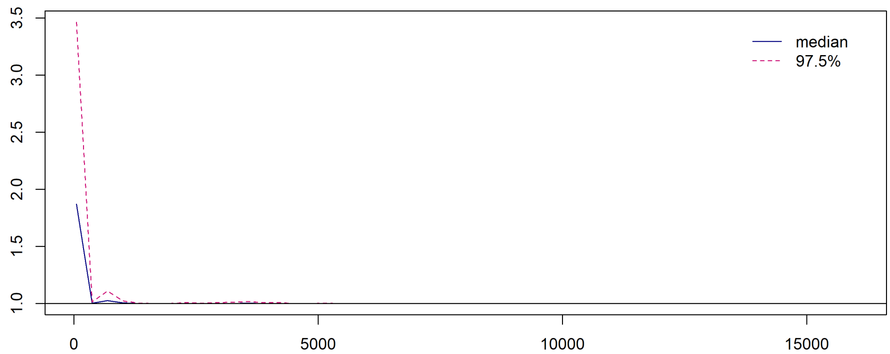

Figure 1: GDP: potential scale reduction factor for ρ

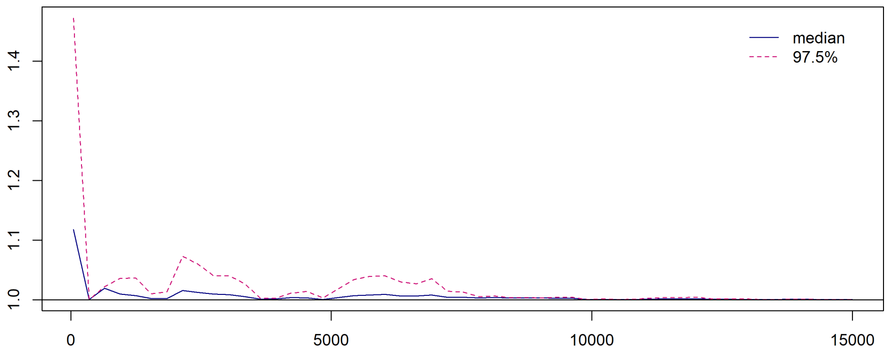

Figure 2: Unemployment: potential scale reduction factor for ρ

For the key parameter ρ, whose distribution has been simulated in an additional nested MH procedure, we take an additional, graphical inspection of the traceplots (Figures 3-6. We treat the patterns of missing trends and regular, normal-like density as reassuring.

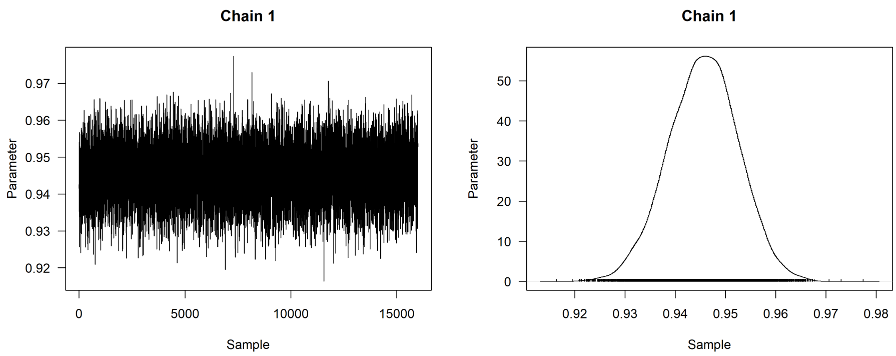

Figure 3: GDP: convergence of chain 1 for ρ

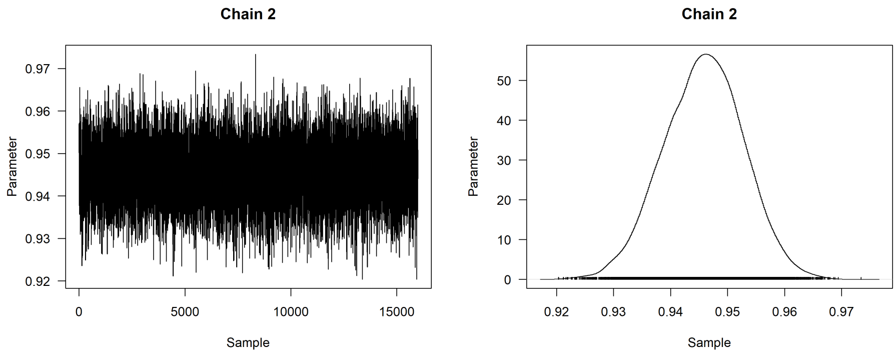

Figure 4: GDP: convergence of chain 2 for ρ

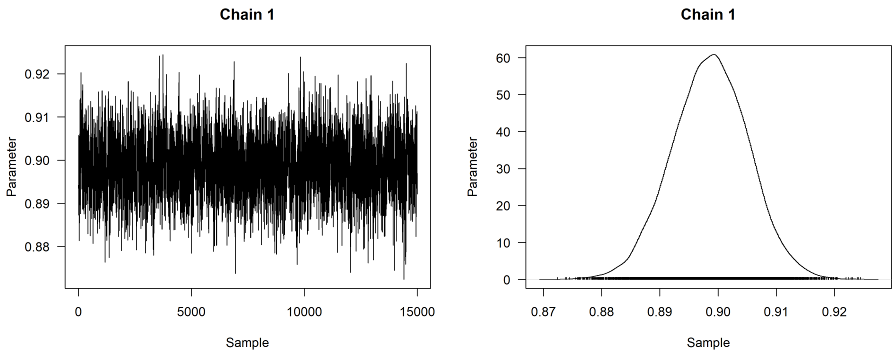

Figure 5: Unemployment: convergence of chain 1 for ρ

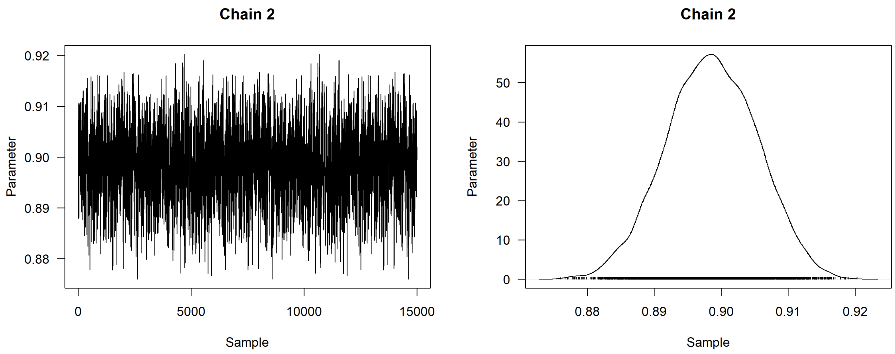

Figure 6: Unemployment: convergence of chain 2 for ρ

Due to a high number of regions in the panel ($N = 73$), we decided to present further results for selected, 8 representative regions, one for every NUTS-1 entity plus the capital city Warsaw (see Table 1 for details). The remaining results are available upon request.

Table 1: Adherence of selected NUTS-3 regions to NUTS-2 and NUTS-1 entities

NUTS-3	**NUTS-2**	**NUTS-1**
Miasto Warszawa (Warsaw)	warszawski stołeczny	woj. mazowieckie
żyrardowski	mazowiecki regionalny	woj. mazowieckie
Miasto Kraków (Cracow)	małopolskie	południowy
poznański	wielkopolskie	północno-zachodni
opolski	opolskie	południowo-zachodni
inowrocławski	kujawsko-pomorskie	północny
kielecki	świętokrzyskie	centralny
białostocki	podlaskie	wschodni

4.2. Amplitudes: expansion and recession means

Figures 7-8 present prior and posterior distributions of μ_0 and μ_1, interpretable as recesssion and expansion means (respectively) of GDP dynamics or unemployment rate, net of spatial interactions (Wy_t) and error terms (ε_t). For GDP growth rate, the averaged posterior mean over regions amounts to 0.33 for recession (ranging from -0.57 to 1.86) and 9.95 for expansion (from 9.55 to 10.51). For unemployment, the analogous averages equal +0.15 p.p. during recessions (max. 0.787 in Warszawa) and -0.39 p.p. during expansions (min. -1.396).

The posterior densities of regime-specific variable means shrink considerably as compared to the priors, for both GDP and unemployment dynamics. This occurred in spite of relatively non-informative priors adopted in the analysis.

For unemployment changes, the difference between expansion and recession mean for an average region is tiny in economic terms, and as compared to GDP dynamics. This should not come as a surprise, given the institutional setup of the Polish labour market. An alternative explanation entails some degree of cross-regional employment mobility across the regions in the occurrence of asymmetric shocks, since the NUTS-3 regions are small enough for agents to avoid the related residence mobility. Interestingly, with less informative priors of μ_0 and μ_1 than in the case of GDP dynamics, posterior denisities for some regions (see Miasto Kraków in Fig. 8, and 11 out of 72 other regions) do not preserve the property $\mu_0 \leq \mu_1$. Since this is a just-identifying condition for the two regimes, their interpretation should be inverse under such circumstances, in spite of the slight difference in prior densities. For this reason, the best (worst) performing regions in unemployment during recession (expansion) exhibit $\mu_0 < 0$ ($\mu_1 > 0$), e.g. region ostrołęcki at -0.401 p.p. (region trójmiejski at +0.821 p.p.)

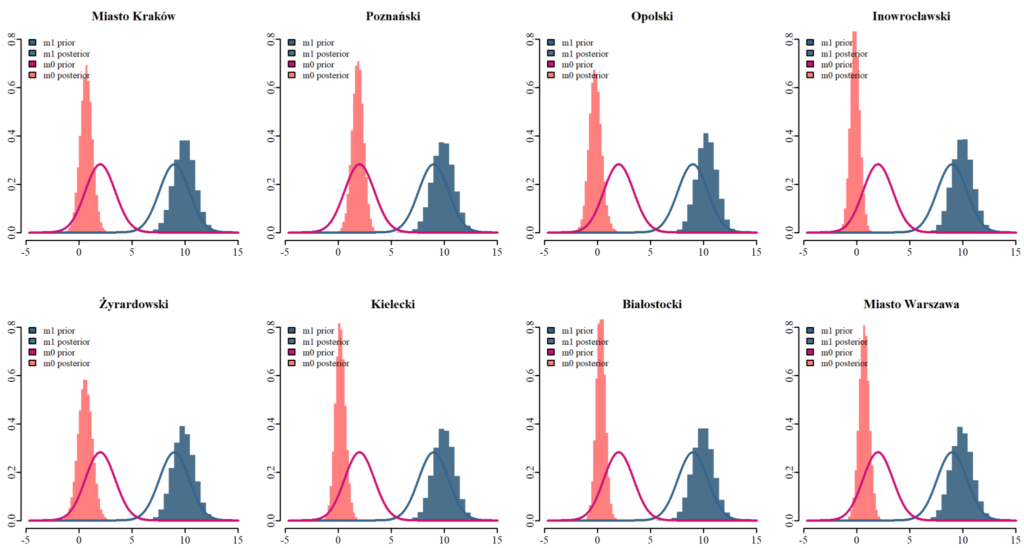

Figure 7: GDP: prior and posterior distributions of μ_0 and μ_1 for selected subregions

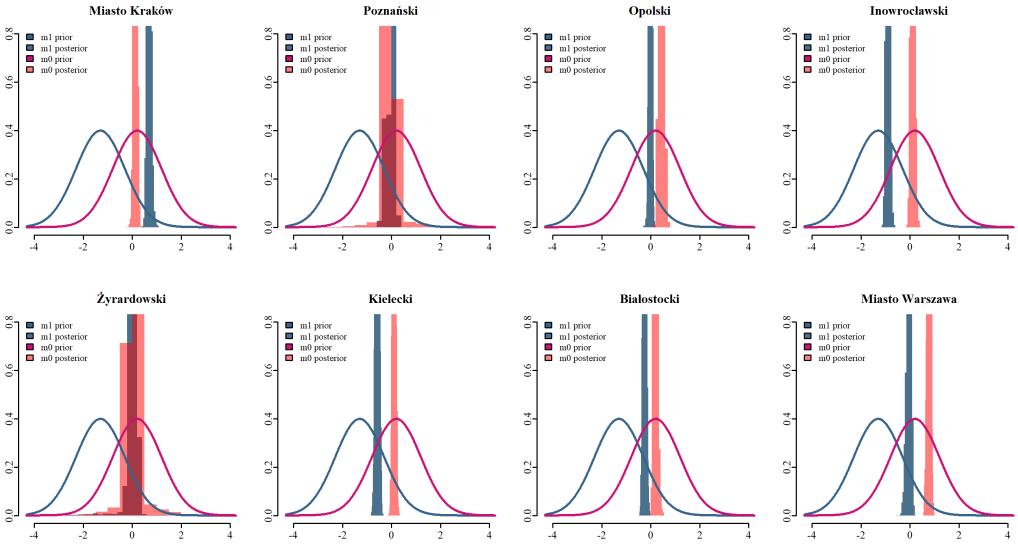

Figure 8: Unemployment: prior and posterior distributions of μ_0 and μ_1 for selected subregions

4.3. Timing: transition probabilities

Figures 9-10 present parameters p_{11} and p_{00}, i.e. the probabilities of remaining in expansion and recession (respectively) one period ahead. Higher values imply a higher long-term frequency of a given state, and obviously a lower chance of transition into the other phase. For GDP, noticeable patterns that emerge in this case are higher posterior means for p_{00} (0.91) than for p_{11} (0.78), suggesting longer NUTS-3 level recessions than expansions. (Note, however, different values of μ_0 and μ_1 over regions that re-define expansion and recession from region to region).

This is no longer the case for unemployment, with $p_{11} = 0.95$ in terms of average regional posterior mean, and $p_{11} = 0.93$ for unemployment, and the posterior density for p_{00} (blue overlines in Figure 10) located slightly to the right of p_{11}. Also note the similarity of posterior distributions for a given variable across all regions.

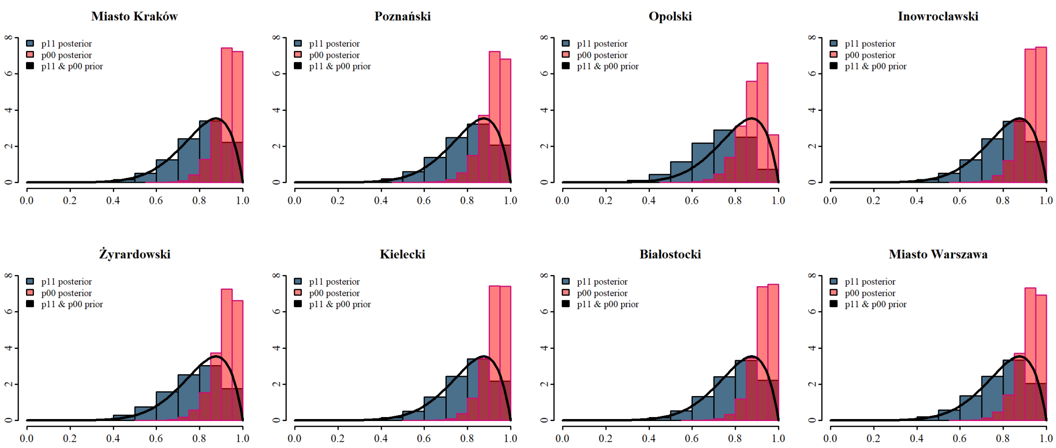

Figure 9: GDP: prior and posterior distributions of p_{11} and p_{00} for selected NUTS-3 regions

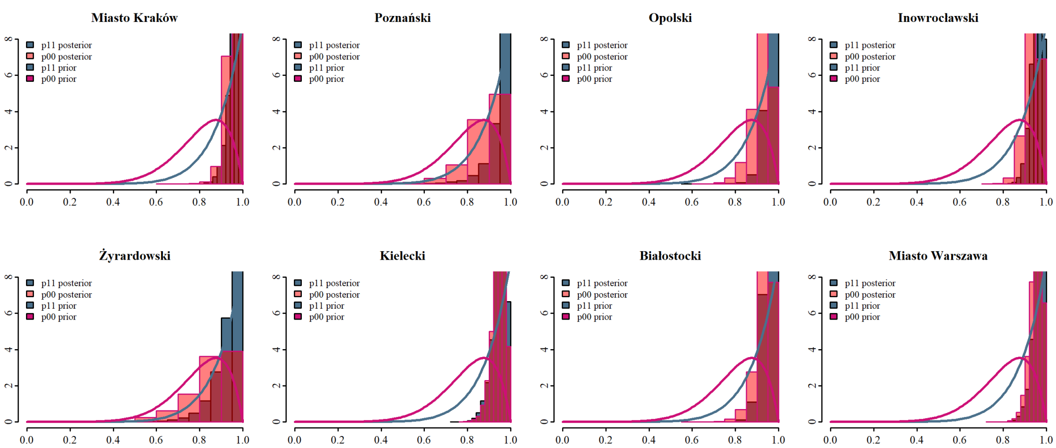

Figure 10: Unemployment: prior and posterior distributions of p_{11} and p_{00} for selected NUTS-3 regions

4.4. Synchronization: business cycles within and between NUTS-2 regions

The probabilities of expansion and recession for a given period, depicted in Figures 11-12, have been computed for every NUTS-3 region using posterior means of parameters. For the sake of tractability, these figures contain only expansion probability (one minus recession probability), whereas the time series for NUTS-3 regions are grouped according to their NUTS-2 adherence.

The model for GDP largely fails to clearly differentiate between own business cycle phases in individual NUTS-3 regions. For most of the time, a number of regions (including almost all from the NUTS-2 areas: lubuskie, świętokrzyskie, warmińsko-mazurskie and zachodniopomorskie) are clearly indicated as in recession. This means that the volatility in GDP dynamics has been interpreted as spatial spillovers from other regions. This largely explains the phenomenon of prior-posterior overlap for expansion means. Also, note that most of the high expansion probabilities have been observed in three voivodships, dominant in economic size and dynamics: mazowieckie, dolnośląskie and śląskie. One might read this as an indication that these voivodships serve as business cycle gravity centers, and other voivodships inherit their business cycles through upstream or downstream connections rather than exhibit their own ones.

The reading of the unemployment analysis results (Figure 12) is completely different: the model is highly conclusive in finding the turning points, as well as both phases, in most NUTS-3 regions. This major difference most likely stems from the different panel dimensionality for the two variables in question. With the same number of regions (N=73), the frequency of the data is annual for GDP and monthly for unemployment, yielding a material difference in T-dimension for both variables. Yet, given the sluggish nature and limited economic meaning of the unemployment rate in Poland, one should not easily reject the results for GDP and focus on unemployment instead.

As regards the business cycle synchronization between regions, some evidence can be found with both variables. First, the GDP analysis suggests some within-voivodship (NUTS-2) synchronicity in the case of three voivodships where autonomous cycles have been detected (say, business cycle setters) in the form of two humps located in the beginning and in the middle of the sample. Since the rest are interpreted by the model as, say, business cycle takers, one can imagine that Pearson correlation analysis will generally exhibit a high degree of correlation, driven both by the idiosyncratic component depicted in Figure 11 and by spatial spillovers. Second, the probabilities in Figure 12 suggest that the unemployment data is too noisy for such conclusions, but some evidence of synchronicity can be found between pomorskie, warmińsko-mazurskie and lubuskie voivodships.

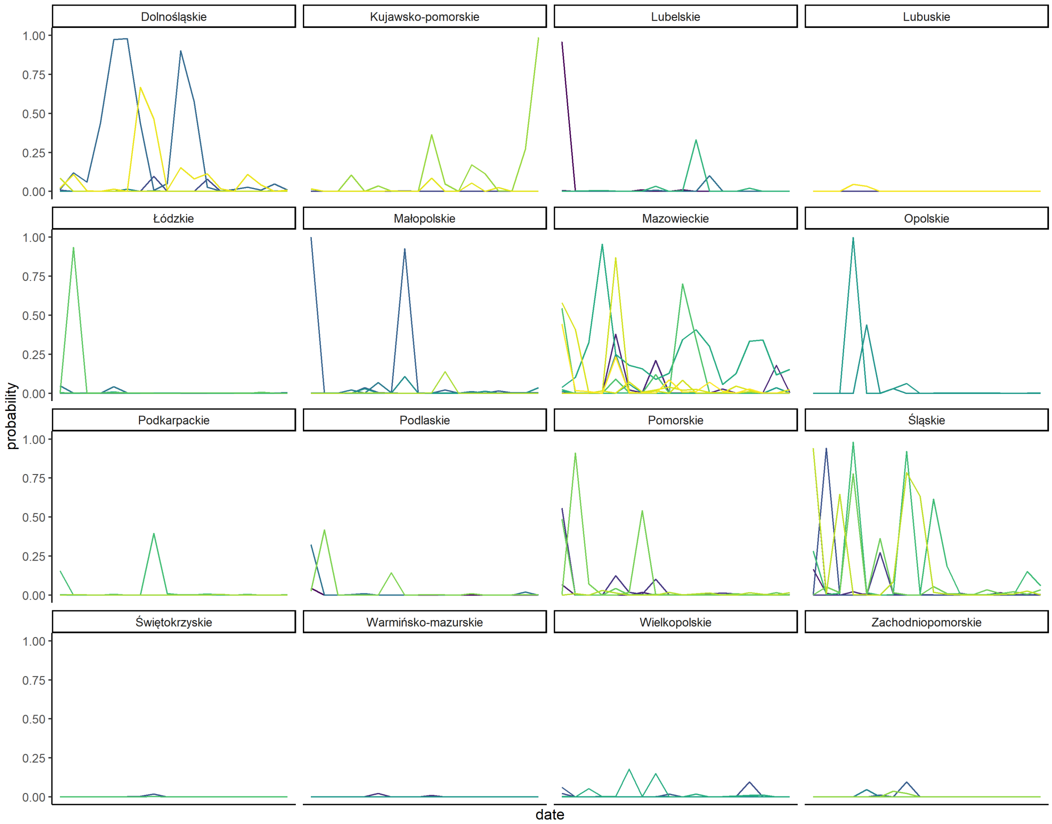

Figure 11: GDP: expansion probabilities computed for posterior means, grouped by NUTS-2

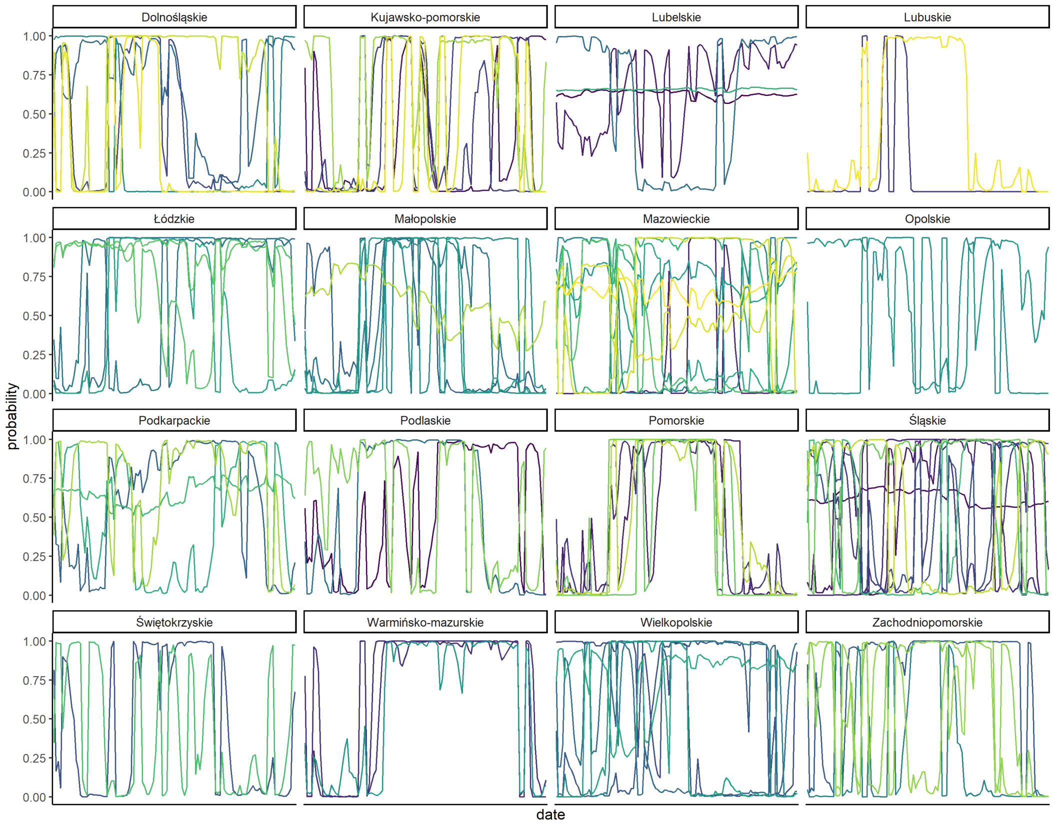

Figure 12: Unemployment: expansion probabilities computed for posterior means, grouped by NUTS-2 regions

4.5. Spatial interactions: impact of other regions' cycles

The posterior distributions of spatial autoregression parameter ρ are presented in Figure 13. Remarkably, the posterior means amount to 0.946 (GDP) and 0.899 (unemployment), and the HPDIs of +/- 2-3 percentage points cover almost entire posterior probability, in spite of a highly uninformative prior distribution. A direct reading of this result suggests that a 1 p.p. rise in the GDP dynamics (unemployment rate) in all other regions translates into a 0.946 p.p. (0.899 p.p.) change in the home region. According to this result, the spatial links across voivodships have turned out to be a material, and sometimes dominant, driver of individual regions' GDP and unemployment rate changes. This should not be viewed as a surprise in a unitary, highly integrated country.

To visualize the differential scope of cross-regional impacts, we simulate the impacts on GDP and unemployment of a transition from recesssion to expansion, in every NUTS-3 region under detailed inspection (note that such transitions have not been found, in some cases). Figures 14-15 present the related spatial multipliers. The results are locally concentrated or spill over through the country, depending (mainly) on the scale of impulse, i.e. the size of the difference between μ_0 and μ_1 and (to a lesser extent) on the slight differences in connectivity of regions,

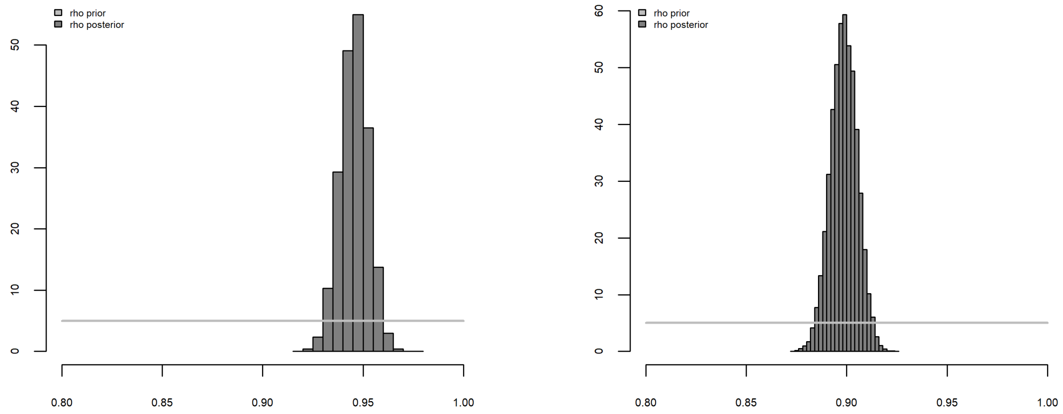

Figure 13: Distribution of ρ parameter

implied by the W matrix.

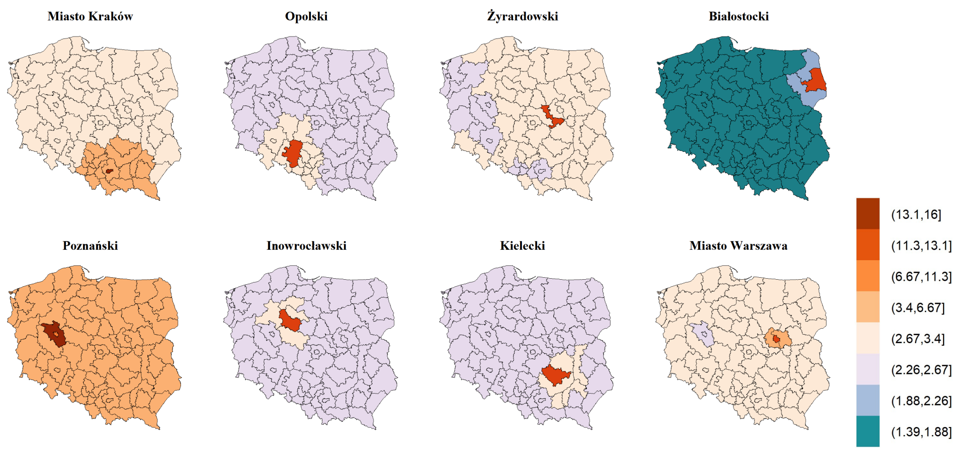

Figure 14: GDP: response of individual regions to a recession-to-expansion transition in a given region

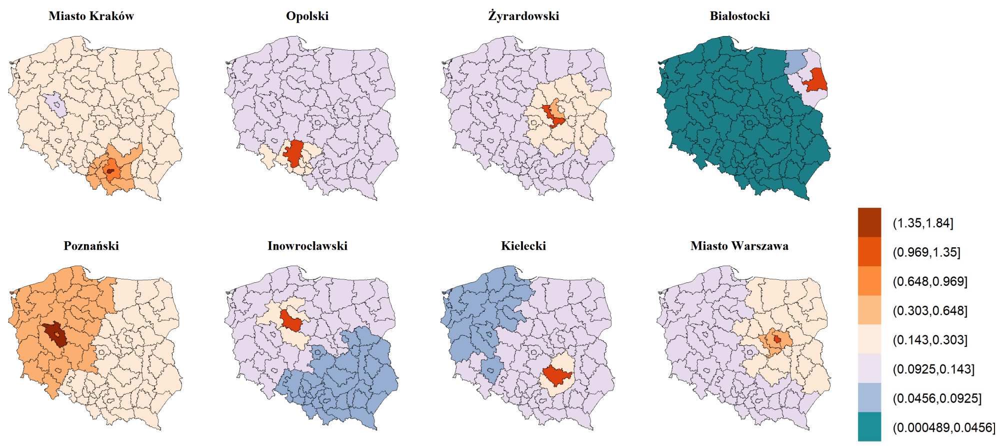

Figure 15: Unemployment: response of individual regions to a recession-to-expansion transition in a given region

Note that both metropolitan areas under inspection (Miasto Warszawa, Miasto Kraków), as well as the ring around a third metropoly (Poznański), impact on the other regions to a higher extent than more peripheral regions (Żyrardowski, Opolski, Kielecki, Inowrocławski and, expecially, Białostocki). This conclusion does not depend on the variable in question, since the positive impact of recession-to-transition probability ranks from highest in region Poznański, Miasto Kraków and Miasto Warszawa to lowest in regions Kielecki, Inowrocławski and Białostocki for both GDP and unemployment dynamics.

The model for unemployment can be further used to decompose the fluctuations for each region into 3 parts: (i) local (related to $\mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t}$), (ii) external (spilled over from other regions via $\rho \mathbf{W} \mathbf{y_t}$) and (iii) residual ($\varepsilon_t$). This decomposition has been effected by assigning state-specific constant to each region (when the probability of a given state exceeds 0.5), then computing the local business cycle as $\mathbf{m_0} \odot (\mathbf{1_N} - \mathbf{s_t}) + \mathbf{m_1} \odot \mathbf{s_t}$, along with the systematic part of the model as $(\mathbf{I} - \rho$

Table 2 presents the top-performing regions in terms of the local business cycle contributions (top cycle setters), whereas Table 3 – the regions representing the most passive behaviour as business cycle takers. In the former case, 6 out of top 12 regions include big metropolies, and 5 of them being these metropolies alone (Miasto Poznań, Miasto Warszawa, Trójmiejski including Gdańsk, Miasto Kraków and Miasto Wrocław).

Turning this table so as to rank the most passive business cycle takers, one obtains Table 3 mostly populated with peripheral regions. It contains one region where a NUTS-2 metropoly is mixed with peripheral areas (Rzeszowski), and the rest are NUTS-3 entities attributable to relatively small cities (Biała Podlaska, Żyrardów, Puławy, Bielsko-Biała, Ostrołęka, Tarnów, Skierniewice) or a ring of perihpery around Poznań (Poznański region, where most of the variance remained unexplained).

Table 2: Decomposition of unemployment fluctuations: NUTS-3 regions with a highest contribution of local fluctuations

NUTS-3	Local	External	Residual
Miasto Poznań	0.746	0.011	0.242
Miasto Warszawa (Warsaw)	0.713	0.010	0.277
Trójmiejski	0.711	0.014	0.275
Starogardzki	0.690	0.006	0.304
Warszawski zachodni	0.680	0.138	0.182
Ełcki	0.679	0.005	0.316
Miasto Kraków	0.671	0.014	0.315
Wałbrzyski	0.667	0.007	0.326
Elbląski	0.644	0.013	0.343
Częstochowski	0.626	0.008	0.367
Miasto Wrocław	0.603	0.007	0.391
Lubelski	0.581	0.019	0.401

Table 3: Decomposition of unemployment fluctuations: NUTS-3 regions with a highest contribution of external fluctuations

NUTS-3	Local	External	Residual
Rzeszowski	0.000	0.874	0.126
Bialski	0.000	0.873	0.127
Żyrardowski	0.000	0.866	0.134
Puławski	0.000	0.862	0.138
Bielski	0.000	0.840	0.160
Ostrołęcki	0.003	0.715	0.282
Tarnowski	0.056	0.650	0.294
Poznański	0.040	0.309	0.652
Skierniewicki	0.368	0.255	0.378

5. Conclusions

We investigate the regional business cycle properties of two variables, GDP dynamics and unemployment, in NUTS-3 regional granulation in Poland. The model includes 3 sources of variation: 2-state Markov switching idiosyncratic business cycle of each region, spatial interaction with other regions, as well as a random component, and we decompose the variance of each variable accordingly. In line with the extant literature, to handle the challenge of statistical identification and constrained parameter space (including probabilities), we use Bayesian methods, simulating the posterior distribution with a Metropolis-within-Gibbs procedure.

The novelty of this study, on top of the geographical extension of the previously proposed methodologies to the Polish data, is threefold. First, we investigate two variables: GDP dynamics and unemployment. The latter data is monthly, and the use of relatively large-T dataset allows to take additional insights into the usability of the model in various data environments. Second, as opposed to the extant literature, we apply the model to a panel abundant in small regions (N=73), which revealed some limitations related to the proposed setup (mainly computational constraints) and suggested the need to develop alternative, robust specifications for such applications. Thirdly, we use the model to put each of these small regions on a continuum from business cycle setters to business cycle takers.

As for the empirical findings, the main one is a high value of spatial autoregression coefficient, amounting to 0.899-0.945 depending on the variable, and suggesting an important role of cross-regional interactions in shaping the regional business cycles on the NUTS-3 level. The systematic part of the idiosyncratic component is of secondary importance in most regions, where no local turning points have been detected in the analysis of GDP dynamics, and the bulk of variability can be regarded as inherited from other regions. As for unemployment, 7 out of 73 regions have been identified as ones where external spillovers (i.e. variability originating in other regions) are responsible for more than 50% of variance, whereby for 52 out of 73 regions, this fraction is higher than 1%.

Unemployment analysis finds turning points in most of the regions. Although the labour market adjustment is more sluggish than the product market adjustment, this result has been possible to achieve since the time series carried far more observations in T dimension than the GDP dataset. However, little can be concluded from the parallel analysis of GDP dynamics, since 18 annual observations are far insufficient to correctly identify the states. The GDP model tended to identify the business cycle switching in no more than 3 voivodships, all top performers in economic development and per capita GDP.

The NUTS-3 level findings have turned out to be more informative than NUTS-2 ones can potentially be, in terms of identifying the role of metropolitan areas. The cities of Warsaw, Poznań, Tricity (including Gdańsk) and Wrocław, along with some NUTS-3 entities in Śląskie voivodship, appear play the role of business cycle setters.

Due to both (i) the low T-dimension of GDP panel and (ii) the limited role of unemployment in business cycle analysis, the results of this study should be interpreted with caution. They could, however, suggest areas where the room for methodological improvement is. One of them is a more robust specification, better fit for large-N problems. Further questions arise around the prior elicitation guidelines of means for the recession and expansion periods, so as to strike the balance between model flexibility and the correct inequality relationship between the posterior means. In line with the spatio-temporal modelling literature, one should also consider enriching the model dynamics beyond the Markov switching process to include some form of temporal autoregression, and handle the econometric sources thereof. Bottom line, the indicative results achieved here suggest that the nation-wide business cycle dynamics covers potentially rich network of structures (which we closed in a single spatial autoregression parameter) whose understanding can be of importance for both regional policy planning, as well as regional and national forecasting.

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