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Hardware and Software over the Course of Long-Run Growth: Theory and Evidence*

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Abstract

Output is generated through purposefully initiated physical action. Production needs energy and information, provided by respective factors: *hardware* ("brawn"), including physical labor and physical capital, and *software* ("brains"), encompassing human cognitive work and pre-programmed software, in particular artificial intelligence (AI). From first principles, hardware and software are essential and complementary in production, whereas their constituent components are mutually substitutable. This framework generalizes the neoclassical model of production with capital and labor, models with capital–skill complementarity and skill-biased technical change, and unified growth theories embracing also the pre-industrial period. Having laid out the theory, we provide an empirical quantification of hardware and software in the US, 1968–2019. We document a rising share of physical capital in *hardware* (mechanization) and digital software in *software* (automation); as a whole software has been growing systematically faster than hardware. Accumulation of digital software was a key contributor to US economic growth.

Keywords: production function, technological progress, complementarity, automation, artificial intelligence.

JEL codes: O30, O40, O41.

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I'm a physicist. We rank things by two parameters: energy and information.

Michio Kaku

1 Introduction

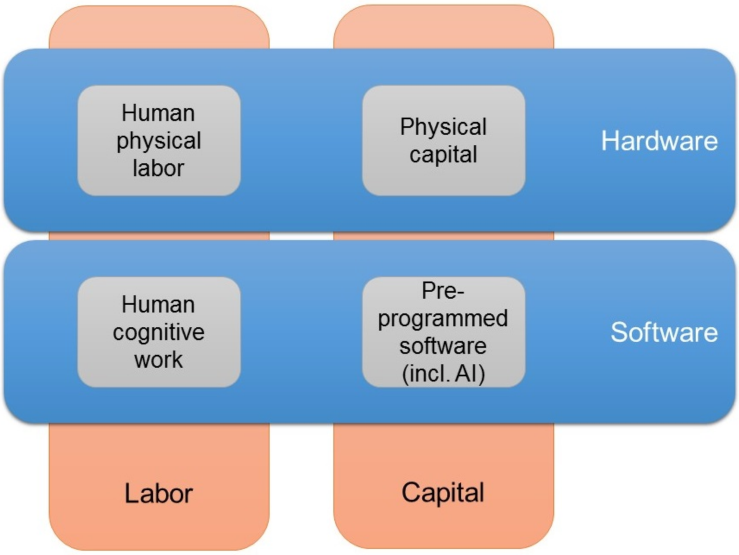
In any conceivable technological process, output is generated through physical action requiring energy. It is a local reduction of entropy, and as such it does not occur by chance but is purposefully initiated. In other words, producing output requires both some physical *action* and some *code*, a set of instructions describing and purposefully initiating the action. Therefore at the highest level of aggregation the two essential and complementary factors of production are physical *hardware* ("brawn"), performing the action, and disembodied *software* ("brains"), providing information on what should be done and how.

This basic observation has profound consequences. It underscores that the fundamental complementarity between factors of production, derived from first principles of physics, is cross-cutting the classical divide between capital and labor. From the physical perspective, it matters whether it's energy or information, not if it's human or machine (Figure 1). For any task at hand, physical capital and human physical labor are fundamentally substitutable inputs, contributing to hardware: they are both means of performing physical action. Analogously, human cognitive work and pre-programmed software are also substitutes, making up the software factor: they are alternative sources of instructions for the performed action. It is hardware and software, not capital and labor, that are fundamentally essential and mutually complementary.

Based on this observation the current paper develops a new macroeconomic framework for modelling aggregate production and long-run economic growth. We then demonstrate how it squares with historical data for the US in 1968–2019.

Unfortunately, in data the fundamental distinction between hardware and software is obscured by the fact that the human body has double duty: it contains both muscles which perform physical action and a brain that stores and processes information. When performing any task, we make use of both energy and information, with varying intensity. The same can be said for modern digital devices, such as computers, smartphones and robots, which also feature both hardware and software. Prior to digital computers, though, all instructions were coming from the human brain, making "software" synonymous with human cognitive work. Therefore, while providing an overarching theoretical frame capable of guiding the narrative across all human history (Growiec, 2022a), the advantages of the hardware–software framework are most clearly seen in the case of the currently unveiling digital era where

Figure 1: Factors of production in the hardware–software framework.



information processing, communication and storage are increasingly detached from the human brain.

The hardware–software framework has a number of distinctive advantages. From the economic modelling perspective, it is a convenient tool for discussing global long-run growth processes because, while rooted in first principles from physics, it nests the following conventional models as special cases:

(i) a standard treatment of an industrial economy producing with capital and labor and respecting Kaldor's facts (this case is obtained by assuming that all physical action is performed by machines and all information processing is done by people),

(ii) a model of capital–skill complementarity and skill-biased technical change (assuming that all information processing is done by people),

(iii) a unified growth theory addressing the period of Industrial Revolution (following the arrival of first machines with an external source of energy, able to perform physical action on their own),

(iv) a theory of inception and further development of the digital era (following the arrival of programmable hardware and pre-programmed software).

In the policy perspective, the hardware–software framework can inform the debate on the future of global economic growth – whether we should expect secular stagnation (Jones, 2002; Gordon, 2016; Gomułka, 2023), balanced growth with limited automation – "race against the machine" (Acemoglu and Restrepo, 2018), accelerated growth with disruptive automation (Brynjolfsson and McAfee, 2014; Brynjolfsson, Rock, and Syverson, 2019) or technological singularity (Kurzweil, 2005;

Bostrom, 2014). In the baseline scenario of the hardware–software framework we expect an acceleration of economic growth at a later stage of the digital era, driven by wide-ranging AI-driven automation and rapid accumulation of programmable hardware (Growiec, 2023). The *software* factor should gradually decouple from human cognitive work and become proportional to programmable hardware instead because pre-programmed software can be virtually costlessly copied and thus can easily scale up to the level of available programmable hardware. Under constant returns to scale and in the absence of further technological revolutions[1], this would gradually reduce the role of technical change augmenting human cognitive work and eventually generate long-run endogenous growth by hardware accumulation alone. In the limit, all production will be automated.

Having laid out the theory, we quantify its predictions empirically, using US data for 1968–2019. The empirical approach of the current study is to construct time series for *hardware*, consisting of human physical labor and physical capital, and *software*, consisting of human cognitive work and digital software. To this end we decompose labor into its physical (manual) and cognitive components, as well as isolate the hardware and software parts of capital investment. Our calculations assume an exogenous rate of technological progress which, in line with the theoretical setup, takes place in the domain of information and therefore is *software-augmenting*.

We find a rising share of physical capital in *hardware* (mechanization) and digital software in *software* (automation) throughout the period 1968–2019. On top of that, as a whole software has been growing systematically faster than hardware. Using a nested CES production function specification, we also perform a growth accounting exercise which suggests that the key contributor to GDP growth in the US has been the accumulation of digital software, followed by the accumulation of human capital.

This paper is related to at least five strands of literature. First, the literature on production function specification and estimation, in particular with capital–skill complementarity, unbalanced growth, as well as investment-specific and skill-biased technical change.[2] Second, the literature preoccupied with accounting for the accumulation of information and communication technologies (ICT) and their broad growth-enhancing role as a general purpose technology.[3] Third, studies focusing on

[1]Given the observed pace of growth in computing power and AI capabilities, further technological revolutions are actually quite likely, though.

[2]Including among others Gordon (1990); Jorgenson (1995); Greenwood, Hercowitz, and Krusell (1997); Hercowitz (1998); Kumar and Russell (2002); Koop, Osiewalski, and Steel (1999, 2000); Krusell, Ohanian, Ríos-Rull, and Violante (2000); Henderson and Russell (2005); Caselli and Coleman (2006); Klump, McAdam, and Willman (2007, 2012); Mućk (2017); McAdam and Willman (2018).

[3]Including among others Bresnahan and Trajtenberg (1995); Timmer and van Ark (2005); Jorgenson (2005); Brynjolfsson and McAfee (2014); Gordon (2016); Brynjolfsson, Rock, and Syverson (2019); Aum, Lee, and Shin (2018); Jones and Tonetti (2020); Farboodi and Veldkamp (2019); Nordhaus (2021).

automation and its impacts on productivity, employment, wages and factor shares.[4] Fourth, the nascent literature on macroeconomic implications of development of AI and autonomous robots.[5] Last but not least, the voluminous literature on R&D based endogenous growth.[6]

The remainder of the paper is structured as follows. Section 2 provides motivation for the current study. Section 3 defines the factors of production of the hardware–software framework and discusses the conceptual underpinnings of the aggregate production function. Section 4 provides the empirical evidence. Section 5 concludes.

2 Motivation

2.1 New Trends of the Digital Era

The world economy has changed a lot since the 1980s. Pre-existing long-run trends in economic development like Kaldor's "stylized facts" (Kaldor, 1961) and the seemingly eternal constancy of "great ratios" (Klein and Kosobud, 1961) have been overturned, and new ones emerged (Jones and Romer, 2010). Among the new tendencies, during the last 40 years the world has been witnessing (even if only recently documenting) systematically declining labor shares (Arpaia, Pérez, and Pichelmann, 2009; Elsby, Hobijn, and Sahin, 2013; Karabarbounis and Neiman, 2014), increasing profit shares (Barkai, 2020), increasing markups and market power (De Loecker, Eeckhout, and Unger, 2020; De Loecker and Eeckhout, 2018; Diez, Leigh, and Tambunlertchai, 2018), increasing market concentration (Autor, Dorn, Katz, Patterson, and Van Reenen, 2020) and increasing within-country income inequality (Piketty, 2014; Piketty and Zucman, 2014; Milanović, 2016). All this was accompanied by a tendency of skill polarization, gradual elimination of routine jobs (Acemoglu and Autor, 2011; Autor and Dorn, 2013), an increasing variety of jobs becoming susceptible to automation (Frey and Osborne, 2017; Arntz, Gregory, and Zierahn, 2016; Eloundou, Manning, Mishkin, and Rock, 2023), and a slowdown in total factor

[4]Including among others Zeira (1998); Acemoglu and Autor (2011); Autor and Dorn (2013); Graetz and Michaels (2018); Acemoglu and Restrepo (2018, 2019a,b); Andrews, Criscuolo, and Gal (2016); Arntz, Gregory, and Zierahn (2016); Frey and Osborne (2017); Barkai (2020); Autor, Dorn, Katz, Patterson, and Van Reenen (2020); Jones and Kim (2018); Hemous and Olsen (2018); Benzell and Brynjolfsson (2019).

[5]Including among others Yudkowsky (2013); Graetz and Michaels (2018); Sachs, Benzell, and LaGarda (2015); Benzell, Kotlikoff, LaGarda, and Sachs (2015); DeCanio (2016); Acemoglu and Restrepo (2018); Aghion, Jones, and Jones (2019); Berg, Buffie, and Zanna (2018); Korinek and Stiglitz (2019); Eloundou, Manning, Mishkin, and Rock (2023).

[6]Including among others Romer (1990); Jones and Manuelli (1990); Aghion and Howitt (1992); Jones (1995); Acemoglu (2003); Ha and Howitt (2007); Madsen (2008); Bloom, Jones, Van Reenen, and Webb (2020); Kruse-Andersen (2023).

productivity growth (Jones, 2002; Gordon, 2016).

By contrast, established economic growth models (see e.g. Barro and Sala-i-Martin, 2003; Jones, 2005a; Acemoglu, 2009) tend to imply stable factor shares, markups and market concentration over the long run, stationary income inequality, a fixed steady-state job structure, and a stable growth rate. They are therefore unable to reconcile the pre-1980 growth experience with the emerging new regularities. Also unified growth theories (Galor and Weil, 2000; Galor, 2005, 2011), despite successfully dissecting the mechanisms of transition from a relatively stagnant agricultural to a fast growing industrial economy during the Industrial Revolution, tend to fail at capturing the unveiling new tendencies.

A likely reason for the apparent mismatch between data and theory is that except for a few forerunners (such as Acemoglu and Restrepo, 2018; Benzell, Kotlikoff, LaGarda, and Sachs, 2015; Berg, Buffie, and Zanna, 2018; Aghion, Jones, and Jones, 2019; Korinek and Stiglitz, 2019), growth models developed thus far have been either rooted entirely in the industrial era, or focused on even earlier eras. They generally do not acknowledge that since the 1980s the Digital Revolution is transforming the world before our eyes in a comparably profound way to what the Industrial Revolution was doing two centuries ago. The computer age – to kindly paraphrase Robert Solow – is now seen everywhere, even in productivity statistics. Since the 1980s personal computers have been permeating firms and households, and digitization gained massive momentum in the 2000s with the spread of the Internet. Quantitatively, since the 1980s "general-purpose computing capacity grew at an annual rate of 58%. The world's capacity for bidirectional telecommunication grew at 28% per year, closely followed by the increase in globally stored information (23%)" (Hilbert and López, 2011). The costs of a standard computation have been declining by 53% per year on average since 1940 (Nordhaus, 2021). Hence, growth in the digital sphere is now an order of magnitude faster than growth in the global capital stock and GDP: data volume, processing power and bandwidth double every 2–3 years, whereas global GDP doubles every 20–30 years. The processing, storage, and communication of information has decoupled from the cognitive capacities of the human brain; "less than one percent of information was in digital format in the mid-1980s, growing to more than 99% today" (Gillings, Hilbert, and Kemp, 2016). Preliminary evidence also suggests that since the 1980s the efficiency of computer algorithms has been improving at a pace that is of the same order of magnitude as accumulation of digital hardware (Grace, 2013; Hernandez and Brown, 2020). Corroborating this finding, in the recent decade we have witnessed a surge in AI breakthroughs based on the methodology of *deep neural networks* (Tegmark, 2017): increasingly autonomous vehicles, high-quality language interpretation, understanding, rephrasing, summarizing and producing human-like text (OpenAI's GPT-4, OpenAI, 2023), generative visual art (Stable Diffusion), self-taught superhuman performance at chess and Go

(AlphaZero, Silver, Hubert, Schrittwieser, et al., 2018), or accurate prediction of protein structures (AlphaFold, Jumper, Evans, Pritzel, et al., 2021). We are also observing that ever since Bill Gates first topped the list of World's Billionaires in 1995, biggest fortunes are made in the computer software business.

2.2 Mechanization, Automation and AI

As a first step in resolving the mismatch between data and theory, the hardware–software framework allows to conceptually disentangle mechanization from automation:

- *Mechanization* of production consists in replacing human physical labor with machines within hardware. Large-scale mechanization is observed since the Industrial Revolution (≈1800 CE onwards). Mechanization applies to physical actions but not the instructions defining them.

- *Automation* of production consists in replacing humans with pre-programmed software in providing instructions, i.e., within software. But for early forerunners, automation is observed since the Digital Revolution (≈1980 CE onwards) when information technologies first came into use as general purpose technologies (Bresnahan and Trajtenberg, 1995). Automation pertains to cases where a task, previously involving human thought and decisions, is carried out entirely by machines without any human intervention. Routine tasks (both physical and cognitive) are typically among the first to be automated (Autor and Dorn, 2013).

The distinction between mechanization and automation is instrumental in addressing questions like "will humans go the way of horses?" (Brynjolfsson and McAfee, 2014), which is supposed to mean whether human work will be eventually fully replaced by machines. The answer is: as far as physical labor is concerned, we have long gone the way of horses; for cognitive tasks (for which horses are of no use) this has not been the case yet, but it may happen in the future in the scenario of full (AI-driven) automation of production. By the same token, it is false comfort to say that "the history of the Industrial Revolution teaches us that when jobs are destroyed, new ones are bound to emerge": the history only tells us that when physical labor is mechanized, additional workers will be demanded in cognitive occupations, but it tells us nothing about cognitive occupations being automated.

The hardware–software framework also helps disentangle the concepts of automation and *artificial intelligence (AI)*. AI algorithms are a special type of software that has the ability to improve its performance based on experience and data. This happens even under a static architecture of AI algorithms, though it is conceivable that AI may also modify its own architecture while heading towards self-improvement.

In principle automation does not need AI, and indeed has historically begun prior to AI. However the development of AI can actually strongly accelerate automation by substituting human cognitive work in non-routine tasks (Brynjolfsson, Rock, and Syverson, 2019; Eloundou, Manning, Mishkin, and Rock, 2023). According to Agrawal, Gans, and Goldfarb (2017), while computers drastically lowered the costs of computing (arithmetic), AI drastically lowers the costs of *prediction*. All in all, AI algorithms provide drastic improvements in the applicability, efficiency, and versatility of software, but do not constitute a qualitative change in its function as means of providing instructions to programmable hardware. Hence, the framework does not envisage a separate "AI revolution", and rather sees AI development as a massive boost to the Digital Revolution which already began with the early computer hardware and software. In our view, AI is to the digital era what the development of electricity and internal combustion engines was to the industrial era: a second wave of key breakthroughs, forcefully accelerating the impact of the initial revolutionary technological ideas on the economy and society, but not a separate technological revolution (Gordon, 2016).

3 The Hardware–Software Framework

We postulate that output is generated through (i) purposefully initiated (ii) physical action. Based on this premise we posit that at the highest level of aggregation any production function should feature *hardware* X, performing the physical action using energy, and *software* S, providing the instructions using information. This leads to a general form of a production function:

$$Output = \mathcal{F}(X, S), \tag{1}$$

where $\mathcal{F} : \mathbb{R}_+^2 \to \mathbb{R}_+$ is increasing and concave in both factors and such that hardware X and software S are essential (i.e., $\mathcal{F}(0, S) = \mathcal{F}(X, 0) = 0$) and mutually complementary. The degree of their complementarity is an open question; the plausible range spans from perfect complementarity (Leontief form) if just one method of producing output exists, to imperfect complementarity if producers are allowed to choose their preferred technology from a technology menu (Jones, 2005b; Growiec, 2013, 2018). Intuitively, a little substitutability is likely because the same outcome can sometimes be generated with more resources (larger X) but less efficient code (smaller S), or vice versa, but the fundamental complementarity should nevertheless prevail. One natural way to instantiate this assumption is to take a CES specification with an elasticity of substitution $\sigma \in (0, 1)$, cf. Klump, McAdam, and Willman (2007, 2012). The particular CES form of the $\mathcal{F}$ function is however not necessary for the results.[7]

The specification (1) abstracts from raw materials, energy resources and data sets which are being used up in the production process. It works as if we assumed that they were given for free and in infinite supply, or at least that they were sufficiently cheap and abundant that they would never become a bottleneck (think, e.g., of the supply of solar energy). Relaxing this simplifying assumption is left for further research.

3.1 Factors of Production

Hardware X includes physical actions performed by both humans and machines. Hence, X encompasses both the services of physical capital K and human physical labor L, where the latter variable excludes any know-how or skill of the worker.

Software S, in turn, encompasses all useful instructions which stem from the available information, in particular the practical implementation of state-of-the-art technologies. Hence, it includes the skills and technological knowledge employed in human cognitive work, H, as well as pre-programmed software Ψ providing instructions to be performed by the associated programmable hardware.[8] Pre-programmed software Ψ may in particular include artificial intelligence (AI) algorithms, able to learn from data as well as potentially self-improve and self-replicate. It is assumed that there are no physical obstacles precluding pre-programmed software from performing any cognitive task available to a human (Yudkowsky, 2013; Dennett, 2017).

Within hardware, agents of physical action are substitutable. The extreme case of perfect substitutability reflects the idea that whatever it is that performs a given task, if the set of actions is the same then the outcome should be the same, too. The same logic applies to software: regardless of whether a set of instructions comes from a human brain or a digital information processing unit, if the actual information content of instructions is the same, then the outcome should be the same, too. Therefore, at the level of sufficiently disaggregated tasks all forms of software can also be considered perfectly substitutable.

However, this intuitive property will not always smoothly aggregate to the macro level. To see this, it helps to view the specification (1) as a reduced form of a richer framework where hardware and software are used in performing heterogeneous tasks, and the overall supply of hardware and software is computed by aggregating over these tasks (Acemoglu and Restrepo, 2018, 2019a,b; Growiec, 2022b). In such a

[7]For example, Growiec and Mućk (2020) propose a more flexible parametric framework that also allows the modeler to control whether the factors are gross substitutes or gross complements.

[8]Contemporary programmable hardware consists typically of computers, robots, and other devices embodying digital chips. In principle, it does not have be silicon-based, though. In fact the first pieces of non-biological programmable hardware were mechanical devices such as the Jacquard loom using punchcards, first invented in 1804.

scenario imperfect substitutability between human and machine contributions to factors of production may ensue from heterogeneity and mutual complementarity of the tasks. A particularly important caveat in this regard is that the baseline hardware–software framework excludes *essential non-automatable* cognitive tasks and sub-tasks – which cannot be circumvented and for which human cognitive work is necessary. For example, if a cognitive task consists of two consecutive steps, the first of which can be performed by a computer algorithm but the latter only by a human, then pre-programmed software and human cognitive work will turn out complementary at the level of the whole task even if they are perfectly substitutable within the two sub-tasks. This apparent complementarity disappears, however, once the task becomes fully automatable.[9]

In line with this discussion we write the general form of a production function as:[10]

$$Output = \mathcal{F}(X, S) = \mathcal{F}(L + K, H + \Psi). \tag{2}$$

Each of the four factors L, K, H, Ψ has its unique properties (Table 1).

- *Human physical labor* L is rivalrous and given in fixed supply per worker and unit of time, $L = \zeta N$ where $\zeta \in [0, \bar{\zeta}]$ denotes the supply of physical labor per worker in a unit of time, expressed in physical capital units, and N is the total number of workers.

- *Physical capital* K is rivalrous but can be unboundedly accumulated per capita. Physical capital K may be non-programmable or programmable. The share of programmable hardware in total physical capital is denoted by χ (so that $\chi \in [0, 1]$).

- *Human cognitive work* H consists of three components, technological knowledge A, the average skill level h, and the number of workers N, as in $H = AhN$.

[9]Note that in the established task-based automation literature (Zeira, 1998; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018; Aghion, Jones, and Jones, 2019) the default situation is that tasks can be only partially automated, whereas in the hardware–software framework in principle tasks can be automated fully. Growiec (2022b) demonstrates that a shift from partial to full automatability of complex tasks is disruptive for the economy – the contribution of human cognitive work switches from essential and scarce to inessential and replaceable – and argues that in the future we may see more and more tasks fully automated with the advancement of AI.

[10]At the cost of less transparent notation, one can generalize equation (2) to accomodate imperfect substitutability between people and machines in both hardware and software (Growiec, 2023): $Output = \mathcal{F}(G_1(L, K), G_2(H, \Psi))$, with gross substitutability of factors within G_1 and G_2. A particularly tractable case to consider is the one where $\mathcal{F}, G_1$ and G_2 are CES. Furthermore, the partial automatability scenario – where some essential tasks will never be automated – can be accomodated by assuming gross complementarity between human and machine inputs in G_2 (Growiec, 2022b). This is the specification we use in the empirical section of our study, covering a historical period during which the potential for automation was limited.

Technological knowledge A, or the size of the "repository of codes" is non-rivalrous (Romer, 1986, 1990) and accumulable.[11] Per-capita skill levels h are rivalrous and bounded above, theoretically by the optimal code for performing a given task, but in practice by a much lower number $\bar{h} > 0$ due to the human inability to rewire our brains in order to perform cognitive tasks more efficiently (Yudkowsky, 2013) as well as more down-to-earth reasons like human mortality and decreasing returns to education.

- *Pre-programmed software* Ψ also consists of three components, technological knowledge A, algorithmic skill level ψ which captures the degree to which pre-programmed software is able to perform the tasks collected in A, and the stock of programmable hardware χK on which the software is run, as in $\Psi = A\psi\chi K$. Technological knowledge A is the same as above.[12] The algorithmic skill level ψ is assumed to be bounded above by the optimal code for performing a given task (i.e., perfect accuracy), though there may be in fact a much lower upper bound $\bar{\psi}$ (Hanson and Yudkowsky, 2013).[13] Because software can be virtually costlessly copied, it is assumed that it can scale up to the level of all available programmable hardware χK.[14]

Table 1: Factors of Production and R&D

Hardware X	Human physical labor	$L = \zeta N$
	Non-programmable physical capital	$(1-\chi)K$
	Programmable physical capital	χK
Software S	Human cognitive work	$H = AhN$
	Pre-programmed software†	$\Psi = A\psi\chi K$

Note: † includes AI algorithms.

3.2 Technological Progress

Following Romer (1986, 1990), the hardware–software framework envisages technological progress (growth in A) as expansion of the "repository of codes", i.e., as the development of new, better instructions allowing to produce higher output with a

[11] Depending on the institutional setup (e.g., intellectual property rights), technological knowledge A may be characterized by varying levels of excludability.

[12] If in reality the sets of codes available to humans and pre-programmed software are different, the discrepancy between the measures of both sets can be captured by the ratio ψ/h.

[13] Depending on the institutional setup (e.g., proprietary code vs. open source), the algorithmic skill level ψ may be characterized by varying levels of excludability.

[14] Which implies that, in its basic form, the framework abstracts from economic and legal constraints on the diffusion of software, such as the protection of intellectual property rights.

given amount of hardware. Whether these new instructions take the form of new abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items, they are all *information* and not actual *objects* or *actions*, and it is precisely this informational character that makes technologies non-rivalrous and a source of increasing returns to scale (Romer, 1990). What is novel here in comparison to Paul Romer's seminal contributions, though, is that these instructions can be applied to the tasks at hand both by humans and machines.[15]

The informational character of technological ideas also naturally classifies them to the domain of software, or "brains". Technological ideas do not enter into hardware because the purpose of hardware is to perform physical action, and *work* in the physical (mechanical) sense cannot be better or worse, there can only be more or less of it. Thus, developing a machine able to, for example, transport a bigger load in the same time and using the same amount of fuel, or to perform more digital computations per second using the same amount of energy, translates into *accumulation*, not *augmentation* of capital K. In turn, better targeted physical action achieved thanks to, say, a more precise tool or a better organized production stream indicates not an improvement in hardware, but software – instructions initiating the physical actions. In line with this argumentation, all technological progress is modeled as *software-augmenting* here. In the hardware–software framework, in contrast to the capital–labor one, there is no room for discussion on the direction of technical change – a parsimonious property that is highly valuable from a reductionist point of view.

3.3 The Aggregate Production Function

Since Solow (1956, 1957) it has become commonplace to take capital K and labor L as the key inputs of the aggregate production function. Furthermore, it has become a very frequent, if not default, practice to assume purely labor-augmenting (Harrod-neutral) technical change, as in $Y = F(K, AL)$. Of course, like any other aggregate

[15] In the growth literature, the technology level A is frequently interpreted as mass of product designs (in *increasing variety* models) or an aggregate quality index of produced goods (in *quality ladder* models), Barro and Sala-i-Martin (2003). Note also the difference between technological ideas and data: "Ideas and data are types of information. Following Romer (1990), an idea is a piece of information that is a set of instructions for making an economic good, which may include other ideas. Data denotes the remaining forms of information. It includes things like driving data, medical records, and location data that are not themselves instructions for making a good but that may still be useful in the production process, including in producing new ideas." (Jones and Tonetti, 2020, p. 2821) In contrast to Jones and Tonetti (2020) and Farboodi and Veldkamp (2019) the hardware–software framework does not include data as a factor in the production function. Instead data, like energy, is tentatively assumed to be sufficiently cheap and abundant that it will never become a bottleneck in production.

production function specification, this is a simplification that disregards the fact that K and L are amalgamates of heterogeneous components (Temple, 2006). The key question is, though, whether this simplified form is sufficient for capturing the key macroeconomic facts. Evidence is mounting that it is no longer the case. From the literature[16] it is becoming clear that the capital–labor framework, while sufficient to model the classic Kaldor (1961) facts, fails at capturing the new phenomena specific to the digital era, present in macro data since the 1980s.

In contrast, the hardware–software production function (2) specifies the production factors in accordance with the physical divide between energy and information, "brawn" and "brains". Using the concepts introduced above, the aggregate production function F is formalized as:

$$Y = F(X, S) = F(\zeta N + K, A(hN + \psi\chi K)), \tag{3}$$

where Y is aggregate value added (or GDP). The function F is increasing and concave in both its arguments, and hardware X and software S are essential and complementary. The standard replication argument applied to this production function specification implies constant returns to scale with respect to the rivalrous factors X and $S/A = hN + \psi\chi K$. With respect to X, S/A and A, though, returns to scale are increasing (Romer, 1986, 1990).

From the laws of thermodynamics, implying in particular that performing physical action requires expediting energy, it is expected that an essential fraction of GDP must consist of material outputs, serving – at the very least – to sustain the hardware (including human bodies) and allow it to work (Georgescu-Roegen, 1971, 1975). This observation reinforces the assumption that hardware X must be essential in the production process.

Pre-programmed software can be deployed in production processes only if the technology allows for the existence of programmable hardware ($\chi > 0$). Once it is introduced, though, there is no upper bound for its capacity relative to the cognitive capacity of the human brain. It may even one day come to exhibit superhuman cognitive performance.[17] This is because (i) the human brain has fixed computational capacity whereas pre-programmed software (including AI) can be run on programmable hardware with any level of computing power, (ii) AI algorithms have the ability to learn from data and potentially self-improve their architecture. Nevertheless, even without superhuman AI performance all cognitive tasks are amenable to automation with sufficient computing power χK. The only pre-condition for

[16] Such as Gordon (1990); Greenwood, Hercowitz, and Krusell (1997); Krusell, Ohanian, Ríos-Rull, and Violante (2000); Caselli and Coleman (2006); Klump, McAdam, and Willman (2007); Jones and Romer (2010); McAdam and Willman (2018).

[17] See Chollet (2019) for an excellent review of definitions of *intelligence* (cognitive performance, cognitive capabilities, etc.) of non-human agents.

this outcome is that in the full model the possibility of accumulating the requisite computing power is not precluded by, e.g., preferences or institutions.[18]

It is instructive to consider four special cases of the framework, representing four distinct conventional models.

Industrial economy producing with capital and labor. Under the assumption that all physical work is done by machines ($\zeta = 0$) and all cognitive work is done by humans ($\chi = 0$), the production function (3) reduces to the conventional capital–labor specification with purely labor-augmenting technichal change, $Y = F(K, AhN)$. Capital and labor are then naturally gross complements, as suggested by bulk of the recent empirical literature (Klump, McAdam, and Willman, 2007, 2012; Mućk, 2017).

Capital–skill complementarity and skill-biased technical change. Under the assumption that all cognitive work is done by humans ($\chi = 0$), the production function (3) reduces to the specification with capital-skill complementarity (Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Caselli and Coleman, 2006; McAdam and Willman, 2018) and skill-biased (or more precisely, cognitive labor-augmenting) technical change, $Y = F(\zeta N + K, AhN)$. Gross complementarity between hardware and software implies that physical capital is complementary to cognitive ($\approx$ skilled) labor H but substitutable with physical ($\approx$ unskilled) labor L, in line with findings of this literature.

Industrial Revolution. The hardware–software framework represents the Industrial Revolution as an episode where physical capital begins to be accumulated after the initial restriction $K \approx 0$ is lifted.[19] In result human physical labor is gradually replaced with machines within hardware in a process of *mechanization* of production.

Digital Revolution. The framework represents the Digital Revolution as an episode where pre-programmed software begins to be accumulated after the initial restriction $\chi = 0$ (and thus $\Psi = 0$) is lifted. In result human cognitive work is gradually replaced with machine code within software in a process of *automation* of production.

3.4 Production Function For Ideas

Consistently with the hardware–software framework, research and development (R&D) processes can also be viewed as a function of hardware X and software S. Hardware includes R&D capital alongside human physical labor (Growiec, 2022c; Growiec,

[18] However, in a more general model with complex, multi-step tasks, human cognitive work can become essential for generating output if at least one step of at least one essential task is not automatable (Growiec, 2022b). Essentiality implies that there is no way around this particular step and no possibility of substituting out the entire task.

[19] The initial restriction $K \approx 0$ can be understood as the absence of machines with their own energy source (e.g., engine), able to perform physical action without energy inputs from the human.

McAdam, and Mućk, 2023). Software encompasses all the ideas supplied by scientists and technical personnel, as well as code encapsulated in pre-programmed software. Formally the idea production function obeys the general equation (2), specializing into:

$$\dot{A} = \Phi(X, S) = \Phi(\zeta N + K, A(hN + \psi\chi K)), \tag{4}$$

where $\dot{A}$ represents the flow of new technological ideas. It is assumed that the idea production function Φ is increasing and concave in both factors, X and S. The characterization of returns to scale is uncertain, however, as there may be important spillover effects and duplication externalities in R&D, the magnitude of which is subject to dispute (Jones, 1999; Ha and Howitt, 2007; Madsen, 2008; Bloom, Jones, Van Reenen, and Webb, 2020; Kruse-Andersen, 2023).[20]

3.5 Stages of Economic Development

Let us now trace how the hardware–software framework squares with the key properties of production processes across the human history (Growiec, 2022a). In this regard it must be noted that the framework itself does not explain the causes of technological revolutions which push the economy to the next stage of development, other than speculating that in certain circumstances, given the relative supply of aggregate hardware vs. software, such a shift would be particularly demanded. However, the framework does predict the secular trends emerging after each technological revolution has exogenously occurred.

At this stage it is helpful to invoke the following asymptotic result:

$$F(1, \infty) = \lim_{y \to \infty} F(1, y) = a_X \in (0, +\infty). \tag{5}$$

Following from the assumptions of (i) constant returns to scale, and (ii) gross complementarity between hardware X and software S, the limit in (5) exists and is finite. One cannot achieve unbounded output growth unless both hardware and software grow unboundedly as well.

Stage 1. Pre-industrial production. In a pre-industrial economy, output was produced primarily in farming. Our civilization could only access the energy transformed in natural processes, such as photosynthesis and human metabolism. Without machines fueled with external energy sources, there was no significant accumulation of productive capital K. Output was produced with a technology that used only human physical labor for performing the physical actions and required

[20] A more detailed discussion of equation (4) and an empirical assessment of the importance of R&D capital in the idea production function is provided by Growiec, McAdam, and Mućk (2023).

also the services of land, a vital but essentially fixed[21] factor of agricultural production. There was also no pre-programmed software Ψ. Setting a constant $K = \tilde{K}$, representing land, and $\chi = 0$ in equation (3) yields the following simple formula:

$$Y = F(X, S) = F(\zeta N + \tilde{K}, AhN) \approx N \cdot F(\zeta, Ah), \tag{6}$$

where the last approximation follows from the assumption that $\tilde{K}$ is fixed and small relative to ζN. Hence, under gross complementarity of hardware and software, pre-industrial output per worker was bounded above ($Y/N \leq \zeta a_X$) due to the insurmountable scarcity of hardware (land and human physical labor), even with an abundance of technological ideas A.

Stage 2. Industrial production. Following the Industrial Revolution (≈1800 CE onwards) human physical labor was gradually replaced with steam-, oil-, and electricity-powered machines in a process of *mechanization* of production. The stock of physical capital per worker K/N began to grow exponentially. Productive physical actions were, however, still dependent solely on the instructions produced through human cognitive work; there was no programmable hardware and no pre-programmed software yet. As hardware was accumulated faster than software, the latter eventually became relatively scarce, at which point demand for human cognitive skills began to grow, setting up a secular upward trend in wages (Galor, 2005). Setting $\chi = 0$ in (3) yields:

$$Y = F(X, S) = F(\zeta N + K, AhN). \tag{7}$$

The hypothetical limit of full mechanization and skill satiation, but with no automation, $K \to \infty$ and $h \to \bar{h}$, where $\bar{h}$ is the upper limit of human capital (skill) accumulation, implies $Y = F(K, A\bar{h}N)$. Hence for a mature industrial economy we obtain the standard balanced growth path result (Uzawa, 1961; Acemoglu, 2003). Under gross complementarity of capital and labor (really: hardware and software) and "labor-augmenting" (really: software-augmenting) technical change, the industrial economy tends to a balanced growth path where capital per worker K/N and output per worker Y/N grow at the same rate as technological knowledge A. Technological progress, generated through R&D, is the unique engine of long-run growth (Romer, 1990).

Stage 3. Digital production. Following the Digital Revolution (≈1980 CE onwards) we are observing gradual *automation* of production. Human cognitive skills which scale with the working population N are gradually replaced with pre-programmed

[21]By making this assumption we concentrate on a mature agricultural economy and exclude the periods of transition from hunting and gathering to sedentary agriculture or conquests of new agricultural land.

software which scales with programmable hardware χK that grows faster. Consequently, software-augmenting technical change no longer affects only the efficiency of human cognitive work, but also to an increasing degree the capacities of pre-programmed software. As automation progresses, skill-biased technical change gradually morphs into routine-biased technical change (Acemoglu and Autor, 2011; Autor and Dorn, 2013). This is the world in which we live now.

At a later stage of the digital era, however, pre-programmed software will likely consist largely of sophisticated, general-use AI algorithms, allowing for multiple-fold increases in the algorithmic skill level ψ (Agrawal, Gans, and Goldfarb, 2017; Berg, Buffie, and Zanna, 2018) and thus fortifying the emerging upward trend in the contribution of the non-human component to software.

The limit of $K \to \infty$ with full automation implies

$$Y = F(\zeta N + K, A(hN + \psi\chi K)) \approx K \cdot F(1, A\bar{\psi}\bar{\chi}), \tag{8}$$

where $\bar{\psi}$ is the upper limit of algorithmic skill accumulation and $\bar{\chi} \in (0, 1]$ is the limiting share of programmable hardware in all physical capital as $K \to \infty$. Full automation of the production process in the limit means that the human contribution to output will gradually fall to zero.[22]

Equation (8) delivers an AK-type implication: in contrast to the industrial economy, long-run growth of the digital economy is driven not by technological progress but by the accumulation of (programmable) hardware (Jones and Manuelli, 1990; Barro and Sala-i-Martin, 2003). If $A \to \infty$ then $Y/K \to a_X$. This striking result is driven by two forces: (i) that pre-programmed software expands proportionally with programmable hardware, and (ii) that hardware and software are gross complements, and thus in the long run the pace of accumulation of hardware – the scarce factor – determines the pace of economic growth. The constancy of the output growth rate over the long run follows in turn from the assumption of constant returns to scale in production, making F asymptotically linear in K (Jones, 2005a; Growiec, 2007).

Although asymptotically constant, the pace of hardware accumulation and output growth may be nevertheless stupefying, potentially with doubling times of the order of 2–3 years, which are currently observed for digital processing power, data volume and bandwidth (Hanson, 2001; Hilbert and López, 2011).[23]

Hypothetical stage 4. Post-digital production. Under high to full automation of production processes, programmable hardware χK will gradually become the

[22]Full automation does not necessary mean that human work will one day become *useless* for the economy (Harari, 2017), though. The decline in human productivity relative to machines will surely be reflected in sub-par growth in wages, but the extent of technological unemployment will eventually depend also on the elasticity of labor supply. See also Korinek and Juelfs (2022).

[23]Long-run growth in the alternative scenario where some essential tasks will never be automated is investigated in Growiec (2023).

bottleneck of further development, the key factor constraining its pace. This will increase the incentives to invest in R&D directed towards radical innovations holding the promise to eliminate this bottleneck. Such breakthrough technology would have to tap an entirely new source of energy, fundamentally increase energy efficiency, or otherwise massively improve unit productivity of programmable hardware.[24]

Formally, such an episode of "new mechanization" may be modelled by introducing an additional component to the hardware amalgamate, as in:

$$X = \zeta N + K + \omega M, \tag{9}$$

where M denotes the new form of hardware, and $\omega \gg 1$ captures its unit productivity relative to K. This form of hardware must be programmable, so that AI could scale with M and avoid becoming a growth bottleneck itself.

Long-run implications include gradual replacement of K-type hardware with M and a permanent acceleration in growth. Indeed this additional acceleration in hardware X accumulation may eventually lead to a new growth regime "with a doubling time measured in days, not years" (Hanson, 2000).

3.6 Factor Shares

The assumption of gross complementarity of hardware and software provides a clear-cut implication for factor shares: factor income will be disproportionately directed towards the scarce factor. The hardware–software framework delivers the following (empirically testable and intuitively explicable) predictions.

Stage 1. Pre-industrial production. In a mature pre-industrial economy able to achieve systematic technological progress (growth in A), increasing scarcity of human physical labor and agricultural land ($\zeta N + \tilde{K}$) relative to human cognitive work (AhN) implies that an ever increasing portion of value added is directed to hardware at the expense of software. The counterfactual limit of $A \to \infty$ without an industrial revolution (with a fixed $K = \tilde{K}$) implies a zero software share of output as virtually all revenues are directed towards agricultural (physical) workers and owners of agricultural land.

Stage 2. Industrial production. The first stage of development of an industrial economy features gradual *mechanization* of production: physical capital accumulation systematically reduces the role of human physical labor in hardware. Given the substitutability between capital K and physical labor ζN, the physical labor

[24] Among the probable scenarios, one could envision the arrival of quantum computing (in which case the Google AI Quantum team has already achieved a major breakthrough, Arute, Arya, Babbush, et al. (2019)), disruptive nanotechnology, massively improved solar power cells, fusion power, or perhaps something yet unimagined.

share goes down whereas the capital share goes up – a trend which was most clearly seen in the early 19th century; Karl Marx called it "the exploitation of the working class".

However, as the pace of capital accumulation in a growing industrial economy outruns technical change (growth in A), this secular trend is accompanied also by an increasing output share accruing to software (i.e., human cognitive work) at the expense of hardware ($\zeta N + K$, gradually dominated by K). Hence, during the second stage of development of an industrial economy, human cognitive work becomes increasingly scarce and thus increasingly well remunerated, raising the returns to education and the skill premium, and setting up a secular upward trend in wages. Such trend was observed in reality in developed countries from the late 19th and through most of the 20th century.[25] In the counterfactual limit of $A \to \infty$, $K \to \infty$ and $h \to \bar{h}$ without a digital revolution, the industrial economy tends to a balanced growth path, along which $Y = F(K, A\bar{h}N)$, the hardware (=capital) share stabilizes around some intermediate value $\bar{\pi}_X \in (0, 1)$, and the economy respects Kaldor's facts (Kaldor, 1961).

Stage 3. Digital production. The first stage of development of a digital economy features gradual *automation* of production: accumulation of pre-programmed software Ψ gradually reduces the role of human cognitive work H in software. Given the substitutability of these two factors, the cognitive labor share goes down whereas the pre-programmed software share goes up. (And if data and software rents are not separately accounted, also firms' profit shares and measured markups go up, as documented e.g. by Barkai (2020); De Loecker and Eeckhout (2018).) This is the world of today, where disruptive digital technologies fuel the "rise of the global 1%".

The hardware-software framework predicts a change in this secular trend in the future, though. It expects that due to exponential technological progress in A, systematic improvements in algorithmic skill ψ and progressing automation, hardware will gradually become the bottleneck of global development, a key factor constraining the pace of further economic growth.[26] Consequently the revenues will be increasingly redirected from software towards programmable hardware, and the software share π_S will set on a secular downward trend. In the hypothetical limit of $K \to \infty, \chi \to \bar{\chi}, \psi \to \bar{\psi}$, assuming the absence of a next technological revolution, the hardware share will tend to unity. At that point in time, though, only a negligible fraction of total remuneration will be going to human workers.

[25] As Galor and Moav (2006) put it, "The accumulation of physical capital in the early stages of industrialization enhanced the importance of human capital in the production process and generated an incentive for the capitalists to support the provision of public education for the masses, planting the seeds for the demise of the existing class structure".

[26] This is a robust prediction which fails only if full automation is not possible (then human cognitive work remains the growth bottleneck forever) or if there is also hardware-augmenting technical change (which leads to super-exponential, explosive growth), cf. Growiec (2023).

4 Empirical Evidence

Mapping the theoretical concepts of L, K, H and Ψ to real-world data is a challenge. In the data there is no direct split of workers' time and remuneration between their physical labor and cognitive work; each worker in some proportion does both. Similarly, programmable hardware also has double duty as means of performing physical action and as a device that stores and runs its code; measured capital investment and returns conflate both. It is not even clear in the accounting whether a certain investment helps accumulate programmable or non-programmable capital. Finally, if not for intellectual property rights pre-programmed software can be virtually costlessly copied to a multiplicity of devices, making it notoriously difficult to price it and calculate its marginal productivity.

In this section we provide a first attempt at quantifying hardware and software, using US data for the years 1968–2019. We construct the relevant time series and plug them in a growth accounting exercise. We use four data sources: (i) the O*NET Content Model database, providing detailed information on work characteristics and equipment used in almost 1,000 occupational groups; (ii) microdata from the CPI IPUMS (Flood, King, Rodgers, Ruggles, Warren, and Westberry, 2022) on hours worked by occupation in the US from 1968 to 2019; (iii) tables on US investment in fixed assets from the US Bureau of Economic Analysis; (iv) aggregate US statistics from Penn World Table 10.0.

4.1 Decomposing Labor: Manual vs. Cognitive Tasks

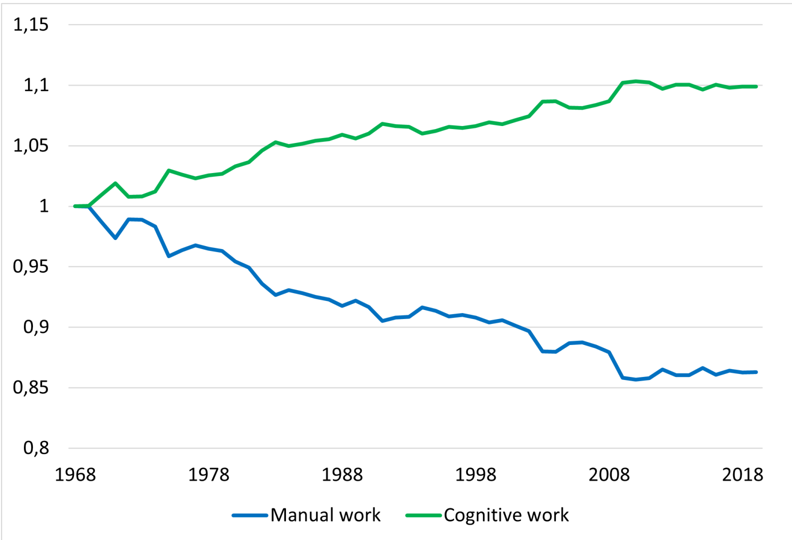
Our first step is to isolate the hardware and software component within labor (L and H, respectively). To this end we decompose work tasks in individual professions into manual and cognitive tasks using the method proposed by Autor, Levy, and Murnane (2003); Acemoglu and Autor (2011). However, while these seminal papers and the subsequent task-based literature (e.g., Autor and Handel, 2013; Autor, Dorn, and Hanson, 2015; Lewandowski, Park, Hardy, Du, and Wu, 2022) focused on the split between routine and non-routine task categories, we identify the *manual* vs. *cognitive* content of jobs. We merge raw O*NET (v.25.3) files on Work Activities, Work Context, Abilities and Skills and identify manual tasks using a specific list of selected Work Activities and Work Context Importance scales.[27] For each occupation we measure the share of manual work as the average importance of manual tasks, while the share of cognitive work is obtained as a residual. In so doing we

[27]Routine manual: 4.C.3.d.3 Pace determined by speed of equipment; 4.A.3.a.3 Controlling machines and processes; 4.C.2.d.1.i Spend time making repetitive motions. Non-routine manual, physical adaptability: 4.A.3.a.4 Operating vehicles, mechanized devices, or equipment; 4.C.2.d.1.g Spend time using hands to handle, control or feel objects, tools or controls; 1.A.2.a.2 Manual dexterity; 1.A.1.f.1 Spatial orientation.

follow O*NET procedure of standardisation into 0-100 scores (because in raw data, each separate task descriptor in O*NET is associated with a different scale).

These shares are then matched with occupation-level employment data. The shares of individual occupations in overall hours worked in the US economy are extracted from the Current Population Survey (CPS) IPUMS database (Flood, King, Rodgers, Ruggles, Warren, and Westberry, 2022), containing microdata from the monthly US labour force survey. The classification system used in the IPUMS database covers more than 450 occupations. We include observations of persons who were professionally active, had a specific occupation and disclosed the number of hours worked. To map the $\sim$ 1000 occupations in O*NET with $\sim$ 450 occupations in CPS IPUMS, we use the crosswalk O*NET-SOC 2019 to 2018 SOC from the O*NET Resource Centre. Upon aggregation we obtain the split of labor between manual and cognitive work in the US in the period 1968–2019 (Figure 2).[28]

Figure 2: Dynamics of the share of manual and cognitive work in the US (1968=1)



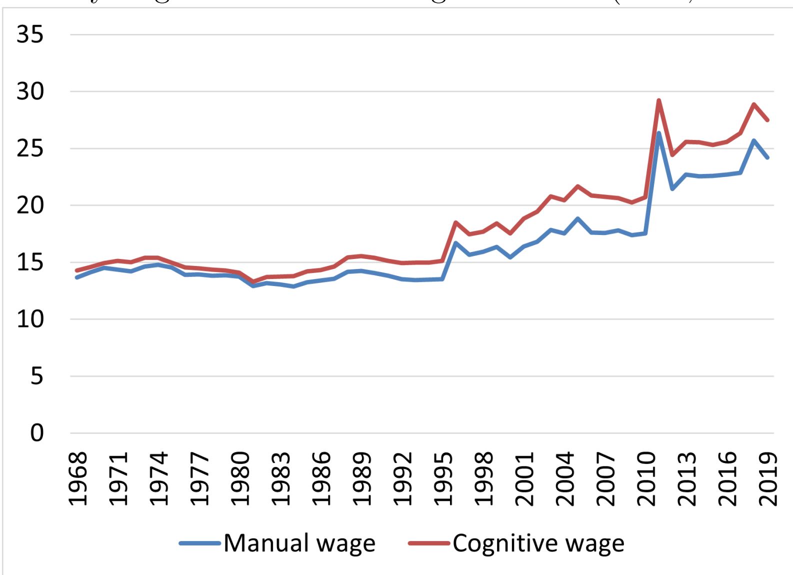
Finally, we obtain our final time series corresponding to physical labor L (that enters hardware) and cognitive labor H (that enters software) by multiplying the shares of manual and cognitive work by total hours worked in the US economy in a given year.

We also construct time series of average real wages in manual and cognitive tasks, using remunderation data from CPS IPUMS. We calculate a weighted average of

[28]Our method for splitting total hours worked into manual and cognitive tasks yields conservative estimates, with relatively little growth in the ratio of cognitive to manual work. This is partly because, due to data limitations, we identify the manual vs. cognitive content of jobs at only one point in time, and all the measured temporal variation comes from changes in the occupational structure of employment. However, in reality the task content of jobs has also been gradually evolving. Our intuition is that particularly manual tasks at any given job have been subject to mechanization. If that were the case, it would amplify the measured growth of cognitive work relative to physical labor.

hourly wages across occupations, using the total manual and cognitive hours worked in each occupation as weights (Figure 3). The measured difference between hourly wages for performing physical labor and cognitive work is rather low (cognitive work pays about 10% more on average, with a slow increase in the premium over time), mirroring our conservative estimates of the split between manual vs. cognitive tasks within jobs.

Figure 3: Hourly wage in manual and cognitive work (USD, constant prices)



4.2 Decomposing Capital: Physical Capital vs. Digital Software

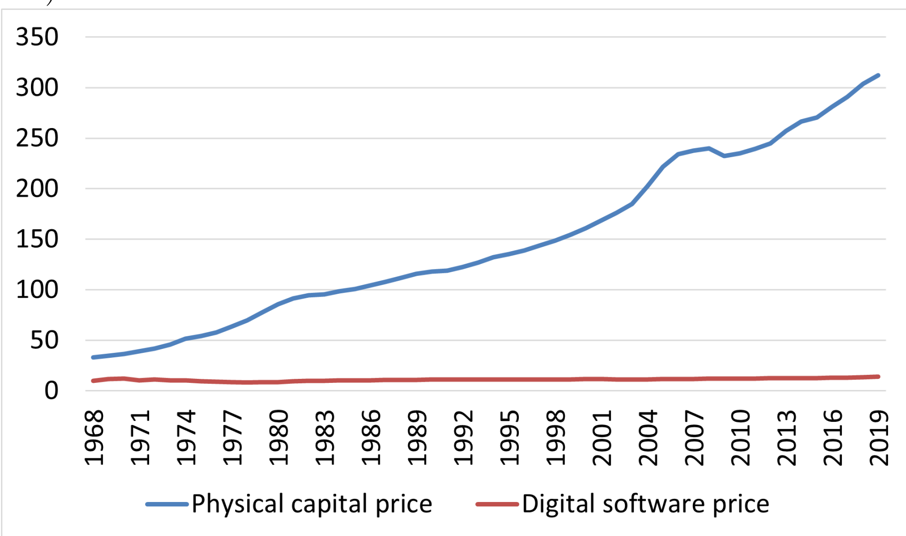
The process of breaking down total capital into physical capital K (that enters hardware) and digital software Ψ is analogous. First, we take US Bureau of Economic Analysis data which allows us to divide investment into structures, intellectual property products (IPPs) and 25 categories of equipment.[29] We assume that investment in structures contributes 100% to hardware, while investment in IPPs contributes 100% to software. The challenge, however, is to determine to what extent investments in the specific types of equipment affect the hardware and software stock. In an attempt to solve this problem via proxy, we use O*NET data which provides

[29] Private fixed assets; Computers and peripheral equipment; Communication equipment; Medical equipment and instruments; Nonmedical instruments; Photocopy and related equipment; Office and accounting equipment; Fabricated metal products; Engines and turbines; Metalworking machinery; Special industry machinery, n.e.c.; General industrial, including materials handling, equipment; Electrical transmission, distribution, and industrial apparatus; Trucks, buses, and truck trailers; Autos; Aircraft; Ships and boats; Railroad equipment; Furniture and fixtures; Agricultural machinery; Construction machinery; Mining and oilfield machinery; Service industry machinery; Electrical equipment, n.e.c.; Other nonresidential equipment; Residential equipment.

information on what type of equipment is used in the day-to-day work in various professions. We assume that the more manual the job is, the more hardware-intensive equipment the worker uses. In contrast, highly cognitive occupations are assumed to be more likely to use equipment containing mostly digital software.[30] We attribute to each type of equipment its specific proportion of hardware and software by merging the Tools Used by Occupation dataset from O*NET and the occupation-level manual-cognitive split discussed above. Using the O*NET-SOC 2019 codes we merge the Tools Used by Occupation dataset with BEA dataset on investment by category.

As a result of these steps, we obtain time series on investments in physical capital (hardware) and digital software. Next, we use the standard perpetual inventory method to build up the stocks of physical capital (hardware) and digital software. We apply asset-specific depreciation rates based on Fraumeni (1997). These rates range from 0.026 per annum (structures) to 0.315 (computers and peripheral equipment).

Figure 4: Unit price of physical capital (hardware) and digital software (USD, constant prices)



To transform both nominal series into real ones, we construct separate price deflators for physical capital (hardware) and digital software. To this end we use price deflators by asset category from US BEA. We calculate a weighted average of asset-specific deflators, using the total hardware and software stock in each asset category as weights (Figure 4). The result is striking: hardware prices were rapidly growing throughout the entire time frame 1968–2019, whereas software prices were roughly constant. This finding is in line with the plentiful past evidence that the

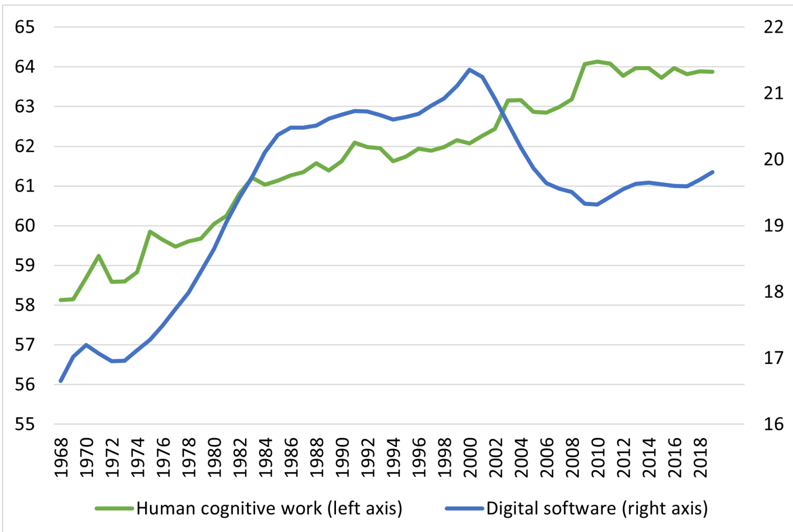
[30]This is a tentative assumption that calls for refinement in the future. Anecdotal evidence suggests that it is not always the case that cognitive tasks are performed with “smart” devices, and manual work – with simple tools. However we do not have sufficient data to verify this.

price of equipment relative to structures (which we count as 100% hardware) exhibits a secular downward trend (e.g., Greenwood, Hercowitz, and Krusell, 1997; Gordon, 2016). Specifically the relative prices of ICT equipment have been falling most precipitously (Timmer and van Ark, 2005), and accordingly in US BEA data *computers and peripheral equipment* are the category which witnessed most extreme price declines, not just in relative but also in absolute terms.

4.3 Constructed Time Series

Over the period 1968–2019 there was a clear increase in the share of cognitive work in labor and of digital software in capital (Figure 5). Significant differences in software intensity have persisted between capital and labor, though. Despite all the Digital Revolution, digital software still accounted for only about 20% of total capital in the US in 2019 (in current prices), up by only 3.5pp. since 1968. At the same time, human cognitive work (software) constituted about 64% of total labor input, up by 6pp. since 1968.

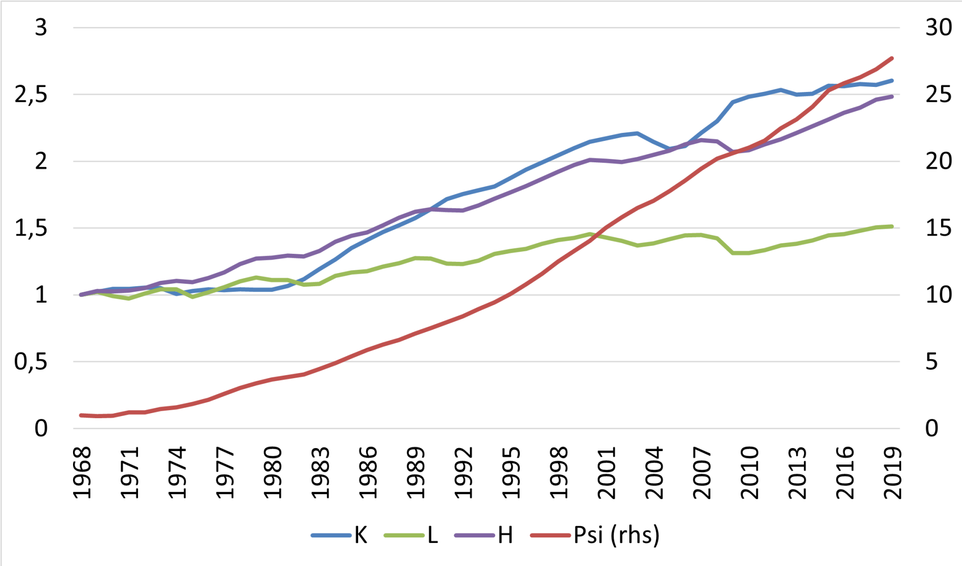
Figure 5: Share of human cognitive work in labor and digital software in capital (in %)



At this point we must also posit a functional form for exogenous software-augmenting technical change $A(t)$ feeding into H and Ψ. As this is the first attempt to quantify hardware and software, we opt to keep things as simple as possible. Therefore we postulate exponential technological progress at a constant rate $g > 0$, i.e. $A(t) = e^{gt}$. In the baseline calibration (see below) we assume $g = 0.5\%$ per annum.

With this in hand we find that while the stocks of all four factors of production (L, K, H, Ψ) have been growing over time, in line with US population growth and fixed asset formation, their growth rates were rather disparate (Figure 6). Human

Figure 6: Dynamics of physical capital K, digital software Ψ, physical labor L and cognitive work H (1968=1)



physical labor was growing at 0.8% per annum on average, cognitive work – at 1.8%, real physical capital K – at 1.9%, and digital software – at 6.7% (compared to the average real GDP growth rate of 2.7% per annum). Clearly, even without specifying the relative productivity of L vs. K in hardware and H vs. Ψ in software, we already see that as a whole software has been growing systematically faster than hardware.

4.4 Calibration of the Aggregate Production Function

We now combine all four factors of production in a modified version of aggregate production function (3). This aggregate production function will later be used in a growth accounting exercise.

We use the nested normalized CES production function specification, with hardware and software being gross complements:

$$Y = Y_0 \left(\alpha \left(\frac{X}{X_0} \right)^{\theta} + (1-\alpha) \left(\frac{S}{S_0} \right)^{\theta} \right)^{\frac{1}{\theta}}, \qquad \theta < 0, \alpha \in (0,1). \tag{10}$$

In contrast to (3), we now also use normalized CES aggregates for hardware and software, thereby relaxing the assumption of perfect substitutability between people and machines within hardware and within software. We do so because in reality there is a multiplicity of tasks to be performed, both in terms of physical action and information processing; even if people and machines are perfectly substitutable in performing each task, the tasks themselves may be complementary and many tasks certainly have not been fully automatable in the considered time period (Growiec, 2022b). Hence we write:

$$X = X_0 \left(\gamma \left(\frac{L}{L_0} \right)^{\mu} + (1-\gamma) \left(\frac{K}{K_0} \right)^{\mu} \right)^{\frac{1}{\mu}}, \qquad \mu \leq 1, \gamma \in (0,1), \tag{11}$$

$$S = S_0 \left(\beta \left(\frac{H}{H_0} \right)^{\omega} + (1-\beta) \left(\frac{\Psi}{\Psi_0} \right)^{\omega} \right)^{\frac{1}{\omega}}, \qquad \omega \leq 1, \beta \in (0,1). \tag{12}$$

In line with usual practices in the normalization literature (Klump, McAdam, and Willman, 2012), the normalization points with subscript 0 are taken as (geometric) sample means.

However, as the hardware–software framework is a new theoretical setup, there is no evidence in the literature on the values of distribution parameters α, β, γ, and elasticity parameters θ, μ, ω. We set them so as to roughly match the (i) the average GDP growth rate (2.7% in data), (ii) average labor share (0.61 in data), (iii) the cognitive wage premium (in data, an average hour of cognitive work is worth $\sim 10\%$ more than an hour of manual work), and (iv) the share of digital software in overall capital (19.4% on average), and exclude parametrizations with very strong variation in the labor share or the cognitive wage premium.

Specifically, to compare the predictions of the hardware–software framework with observations on the US labor share and cognitive wage premium, we need to derive their model-based counterparts. Postulating the normalized CES specification (10) and assuming that factors are priced at their respective marginal products (subject to a possible constant markup), we obtain:

$$\pi_X = \alpha \left(\frac{X}{X_0} \frac{Y_0}{Y} \right)^{\theta}, \qquad \pi_S = (1-\alpha) \left(\frac{S}{S_0} \frac{Y_0}{Y} \right)^{\theta}, \tag{13}$$

$$\pi_L = \gamma \left(\frac{L}{L_0} \frac{X_0}{X} \right)^{\mu}, \qquad \pi_H = \beta \left(\frac{H}{H_0} \frac{S_0}{S} \right)^{\omega}. \tag{14}$$

The labor share and the cognitive wage premium are derived as follows:

$$\pi_{Labor} = \pi_X \pi_L + \pi_S \pi_H, \tag{15}$$

and

$$\frac{w_H}{w_L} = \frac{\pi_S \pi_H}{\pi_X \pi_L} \frac{L}{H}. \tag{16}$$

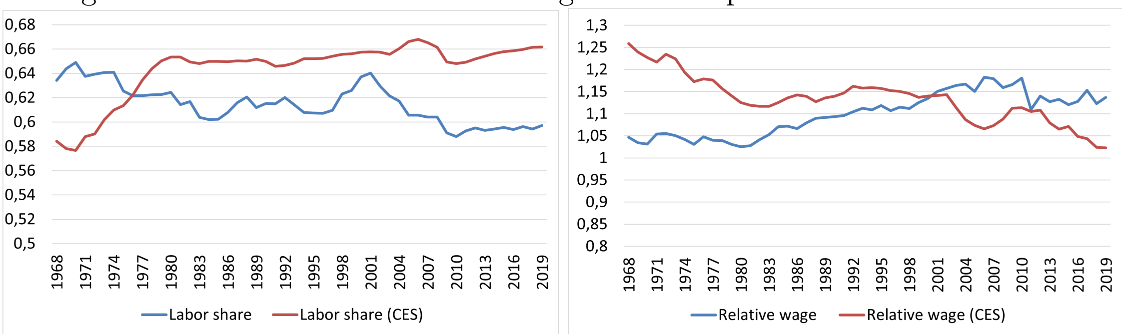
The selected baseline parameters approximately achieving the aforementioned objectives are listed in Table 2.

Table 2: Baseline parameterization of the nested CES production function

Output		Hardware		Software		Tech
α	θ	γ	μ	β	ω	g
0.40	-0.2	0.64	1	0.65	-0.37	0.005

In this parameterization, the elasticity of substitution between hardware and software is $\sigma_{X,S} = \frac{1}{1-\theta} = 0.83$ – somewhat above the usual estimate of the elasticity of substitution between aggregate capital and labor from the literature, $\sigma = 0.6$ (Klump, McAdam, and Willman, 2012), but in the theoretically postulated domain of gross complementarity. In turn, physical capital and human physical labor are perfectly substitutable, whereas human cognitive work and digital software are gross complements, with an elasticity of substitution of $\sigma_{H,\Psi} = \frac{1}{1-\omega} = 0.73$, in line with the partial automation scenario ($\sigma_{H,\Psi} < 1$). These parameter choices also imply that at the point of normalization, cognitive work earns about 13% more than manual work, whereas the labor share is 0.64. However, while roughly matching the average labor share and skill premium in the data, and the extent of their temporal variation, we cannot reproduce the downward trend in the labor share and an upward trend in the cognitive wage premium (Figure 7).

Figure 7: The labor share and the cognitive skill premium: model vs. data



Our main results (Figure 8) suggest that growth in software (3.7% per annum) systematically outruns that of hardware (1.2% per annum). We also find that the extents of both mechanization (share of machines in hardware) and automation (share of machines in software) are upward trending, but with very different slopes: over the considered time frame mechanization progressed by about 42% (or 0.7% per annum), and automation (in real terms) by as much 441% (or 3.0% per annum).

4.5 Growth Accounting

Log-differentiating equation (10) with respect to time, one obtains the following Solow-type decomposition of economic growth:

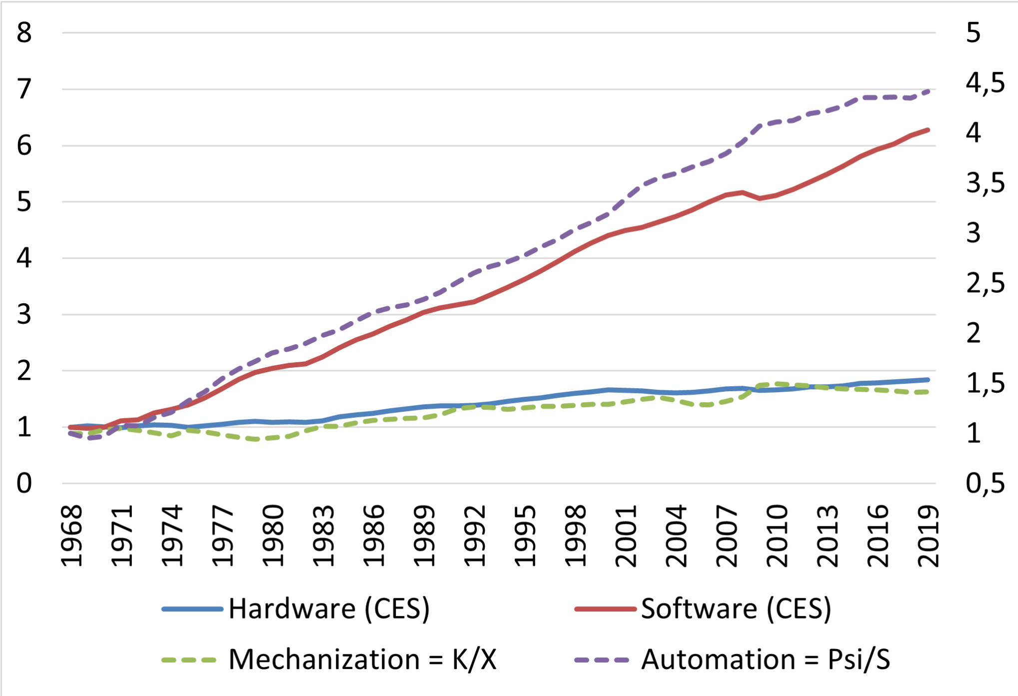
$$g_Y = \pi_X g_X + \pi_S g_S, \tag{17}$$

where $\pi_X = \frac{\partial Y}{\partial X}\frac{X}{Y}$ is the hardware share of output, and analogously $\pi_S = \frac{\partial Y}{\partial S}\frac{S}{Y}$ is the software share. Due to constant returns to scale with respect to rivalrous inputs and purely software-augmenting technical change, $\pi_X + \pi_S = 1$.

Decomposing (10) further,

$$g_Y = \pi_X \pi_L g_L + \pi_X \pi_K g_K + \pi_S \pi_H g_H + \pi_S \pi_\Psi g_\Psi, \tag{18}$$

Figure 8: Hardware, software (1968=1) and the extent of mechanization and automation (1968=1, right axis) under the baseline calibration



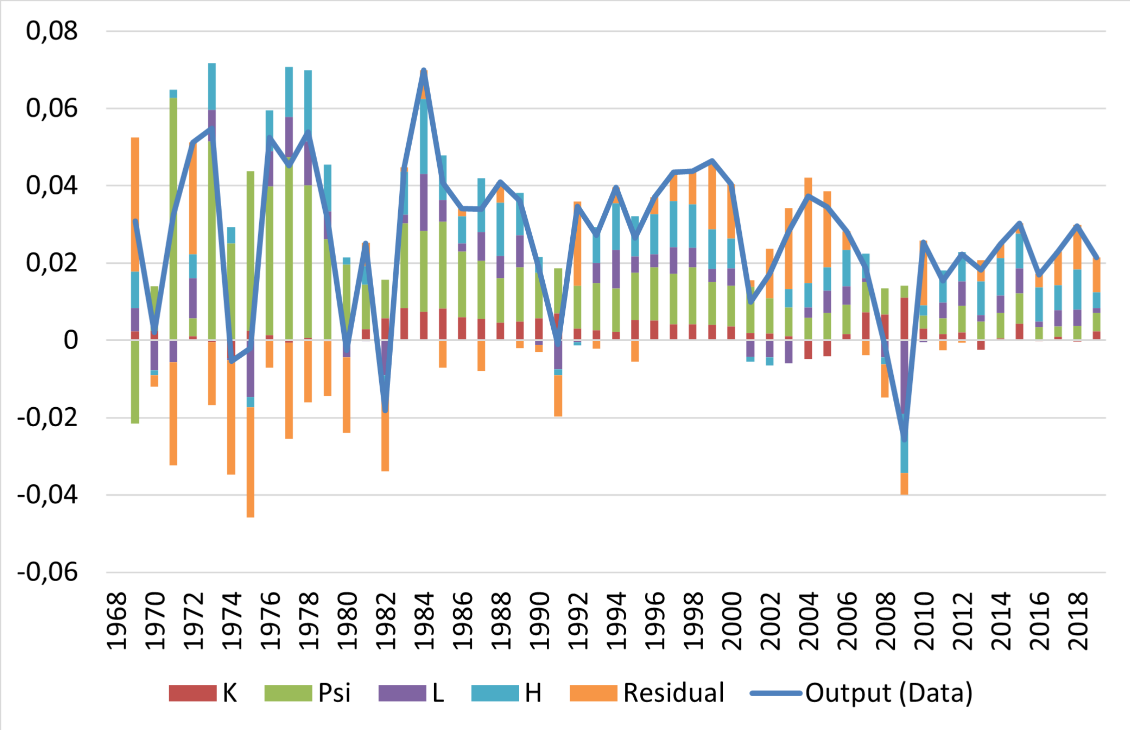
where $\pi_L = 1 - \pi_K = \frac{\partial X}{\partial L}\frac{L}{X}$ is the human physical labor share within hardware, and $\pi_H = 1 - \pi_\Psi = \frac{\partial S}{\partial H}\frac{H}{S}$ is the human cognitive labor share within software.

Under the baseline calibration we find that the key contributor to GDP growth in the US in 1968–2019 was the accumulation of digital software, followed by the accumulation of human capital (Table 3, Figure 9). Furthermore, while the contribution of human capital was roughly steady throughout the studied time period, the contribution of digital software was particularly strong in the 1970s–1990s and generally declining over time, particularly after the dotcom bubble which burst in 2000. Our speculative hypothesis, to be verified as new data come along, is that this slowdown period may constitute an interlude before the next upcoming wave of AI-driven automation in the coming years (Brynjolfsson, Rock, and Syverson, 2019).

Table 3: Contributions to annual GDP growth, 1968–2019 (pp.)

	GDP	K	Ψ	L	H	Residual
pp.	2.71	0.27	1.45	0.22	0.68	0.09
% of total		10.3%	54.8%	8.1%	25.8%	3.3%

Figure 9: GDP growth decomposition, 1968–2019



5 Conclusion

In this paper we have put forward the hardware–software framework – a new conceptual framework of production and long-run growth, based on first principles and emphasizing the role of energy and information in the growth process. Nevertheless it remains closely linked with the existing economic literature. It nests four conventional macro models as special cases, and can be used to inform the debate on the future of global economic growth.

As an empirical application of the theory, we have constructed time series of physical capital K, digital software Ψ, physical labor L and cognitive work H for the US in 1968–2019. We have then plugged these series in a growth accounting exercise. Our results suggest that the key contributor to GDP growth in the US in 1968–2019 was the accumulation of digital software, followed by the accumulation of human capital. This is consistent with the interpretation (Growiec, 2022a) that we are still at an early stage of the digital era, and more profound economic transformations should be expected as AI-driven automation gains steam and more and more production processes are fully automated, thereby reducing the contribution of human cognitive work towards zero (Brynjolfsson, Rock, and Syverson, 2019; Korinek and Stiglitz, 2019; Growiec, 2022b; Korinek and Juelfs, 2022; Eloundou, Manning, Mishkin, and Rock, 2023).

Our results can be extended in a number of directions. First, one can build formal macroeconomic models based on the hardware–software framework, with a variety of applications. For example, Growiec (2023) applied the hardware–software framework to build scenarios for the future and address the question: what will drive global economic growth in the digital age? Second, using certain identifying assumptions one can construct time series for hardware and software stretching further back in time, thus quantifying the role of these fundamental factors of production over the very long run, including for example the period of Industrial Revolution. This is needed to ascertain usefulness of the framework as a building block for a unified growth theory (Kremer, 1993; Galor, 2005, 2011). Third, one can add more detail to the model, such as heterogeneous tasks with varying extents of automatability (Growiec, 2022b). This would improve the fit of the model to the data and make it better suited to producing quantitative predictions of economic growth at later stages of the digital era.

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