



**COLLEGIUM OF ECONOMIC ANALYSIS  
WORKING PAPER SERIES**

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Economy with Application to CEE Countries

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# Natural Rate of Interest in a Small Open Economy with Application to CEE Countries

Maciej Stefański

SGH Warsaw School of Economics<sup>1</sup>

First version: December 2018

This version August 2023

## Abstract

This paper extends the Laubach-Williams (2003) framework, which is widely used to estimate the natural rate of interest, to make it more suitable for studying small open economies. The model is augmented with consumer inflation expectations, foreign output gap, the exchange rate, energy prices and a lending spread. It also uses survey data to improve the accuracy of output gap and potential growth estimates. This model is subsequently applied to CEE countries (Poland, Czechia and Hungary) and the euro area. The natural interest rate is found to be relatively volatile and pro-cyclical; it fell following the global financial crisis, but rebounded after 2012; however, while as of 2017 it remained lower than before the crisis, it was positive for all analysed economies. The model gives more precise and robust estimates than the standard Laubach-Williams framework, but ex-post revisions remain substantial.

**Keywords:** Natural interest rate, small open economy, CEE, Kalman filter.

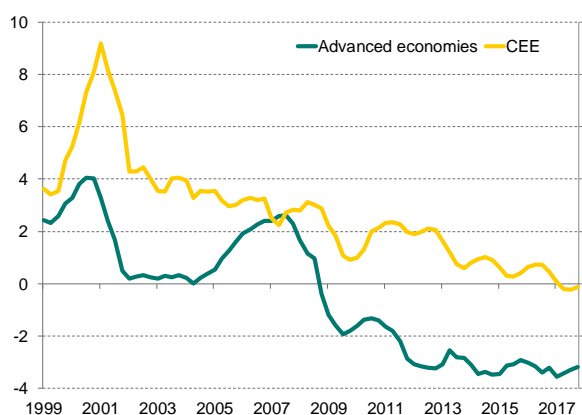
**JEL:** E43, E52, C32.

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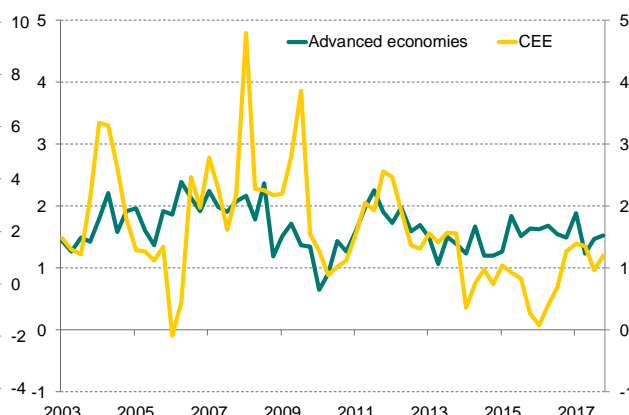
<sup>1</sup> E-mail address: [maciej.stefanski@doktorant.sgh.waw.pl](mailto:maciej.stefanski@doktorant.sgh.waw.pl)

# 1. Introduction

Following the global financial crisis, monetary policy in advanced economies was loosened in an unprecedented manner – central banks lowered their interest rates all the way down to zero or even entered a negative territory, and when it was not enough, unconventional policies, such as asset purchases (quantitative easing) and lending-support schemes, were launched. However, the downward trend in ex-post real interest rates<sup>2</sup> can be extended back to the pre-crisis period (Figure 1). Standard monetary theory – the New Keynesian model – suggests that such a decrease in real interest rates should stimulate output growth and ultimately lead to a rise in inflation. This did not happen – inflation remained relatively low, especially after 2013, and at some point, fears of falling into ‘deflationary trap’ emerged. The lack of inflation’s response to monetary policy was even more pronounced in Central and Eastern European (CEE) countries, where price growth had been in a downward trend ever since the crisis (Figure 2).



**Figure 1** Real (shadow) interest rates (%)



**Figure 2** Quarterly core inflation SAAR (%)

In principle, real interest rate is a difference between central bank benchmark interest rate and annual core inflation. For the US, euro area and the UK, the benchmark rate is replaced with the Wu-Xia shadow rate. For Hungary, 3M interbank offer rate is used. GDP PPP-weighted indices. Advanced economies: Canada, Euro Area, US, UK. CEE: Czechia, Hungary, Poland.  
Source: Own calculations based on OECD, Wu-Xia (2017) and Bloomberg data.

There are several potential explanations for this phenomenon. Firstly, it is often argued that inflation has become more dependent on external factors, such as oil prices and inflation and output gap abroad, as a result of globalisation (i.a. Auer et al., 2017). Secondly, falling inflation expectations, especially those of consumers, could have played a role (Coibion and Gorodnichenko, 2015). Thirdly, the relationship between interest rates, output and inflation

<sup>2</sup> Wu-Xia shadow rates, which take unconventional policies into account, are used post-crisis.

could have weakened (the flattening of the Phillips curve; i.a. Blanchard, 2016). Finally, the natural rate of interest (NRI) could have fallen (Holston et al., 2017).

The aim of this paper is to investigate the latter hypothesis by establishing whether the NRI has decreased over the last decade. However, as most of the literature focuses on advanced economies – the US in particular – and analyses them in a closed economy framework, we investigate CEE countries, which requires opening the economy up.

We use the Laubach-Williams (2003) framework, which is the most popular method of estimating NRI, as a starting point. The augmented model is derived from the Gali-Monacelli (2005) open economy new Keynesian model. Hence, the framework is extended with variables important from an open economy perspective: foreign output gap, the exchange rate and energy prices. As the original model does not take inflation expectations into account (or assumes they are equal to past inflation), we also add survey-measured consumer inflation expectations to make the framework more theoretically appealing in the spirit of Gali-Gertler (1999) hybrid Phillips curve. In this way, the augmented model controls for the majority of alternative explanations for the lack of the link between monetary policy and inflation. Moreover, lending spreads are added to control for financial shocks and differences between central bank and market interest rates. Finally, we use survey data to improve the accuracy of output gap and potential growth estimates – and ultimately NRI estimates, as potential growth is the key determinant of NRI in the model.

The extended model is applied to the euro area and CEE countries. As of 2017, NRI is found to have fallen significantly in Czechia and Poland (from 6-7% in the pre-crisis peak to about 1% in Czechia and 3% in Poland), only slightly in the euro area (from 1.5% to 0.6%) and to have risen in Hungary (from approximately 4% to 7%). The latter result seems suspicious and probably results from mismeasurement of real interest rates that do not take into account a bulk of unconventional policies employed by the Hungarian Central Bank. NRI is relatively volatile and pro-cyclical (especially in Poland). The estimates are quite precise for the euro area (standard errors of 0.6 pp), but less so for the CEE countries. However, standard errors are significantly lower than in the non-augmented model. NRI is to a similar extent driven by fluctuations in potential growth and other factors ( $z_t$  in the Laubach-Williams framework).

The results are largely robust to the use of ex-ante real interest rates and the exclusion of most of additional variables (energy prices, inflation expectations, the exchange rate). Foreign output, lending spread and – especially – the use of survey data to aid the estimation of output gap turn out to be the key additions that improve the model fit and have a significant impact on NRI estimates. As differences between CEE countries are substantial, baseline individual country estimates are preferable over panel estimates. However, alternative specifications of the NRI equation indicate that potential growth might not be as an important driver of the natural interest rate as assumed by the baseline specification. Moreover, ex-post revisions remain substantial for all analysed economies, especially during the global financial crisis and the euro area crisis, when they often substantially exceed 1 percentage point.

There are several ways in which we contribute to the literature on the NRI estimation. Firstly, we develop an open economy framework, which is much more comprehensive than the Laubach-Williams model, but remains relatively simple. Secondly, we demonstrate that using survey data to aid the estimation of output gap improves the model fit and the accuracy of NRI estimates significantly. Thirdly, we show that even a medium-term NRI can be quite volatile and pro-cyclical, calling for a more active monetary policy.

The paper is structured as follows: in section 2, we briefly review the literature on natural interest rates and their estimation and discuss the choice of our starting point model. Section 3 develops the theoretical model that stands behind the NRI estimation, while Section 4 lays out the empirical specification and discusses the estimation method. Section 5 discusses the data. Section 6 presents the main results, while section 7 provides robustness checks. Finally, section 8 discusses the implications and limitations of our results and concludes.

## 2. NRI literature review and the starting point

### 2.1 Literature review

Natural rate of interest is a concept originally developed by Wicksell (1936) who defined it as a real interest rate that does not influence prices and equalises desired savings with desired investment on the loanable funds market. Contemporaneous definition is not substantially different – according to Woodford (2003), NRI is a hypothetical real interest rate that would emerge under elastic prices. Absent of cost-push shocks, it is consistent with zero output gap and inflation on target. At the same time and in line with the Wicksell's definition, NRI is thought to be determined by the desired supply of savings and the desired demand for investment (Borio et al., 2017).

Since NRI is unobservable, it needs to be estimated. The most widely used method of its estimation, developed by Laubach and Williams (2003), focuses on the output-stabilising role of NRI. Hence, Laubach-Williams NRI is a real interest rate that stabilises output gap – and ultimately inflation – absent of shocks. The framework is based on the basic version of the New Keynesian model (e.g. Gali, 2008) and comprises of two main equations: the IS and the Phillips curves. NRI is estimated with Kalman filter and determined by potential growth and agnostic 'other factor' which follows a random walk. In their more recent study, conducted together with Kathryn Holston, Laubach and Williams find that NRI has declined following the global financial crisis from approximately 2.5% to 1.5% in Canada and the UK and from 2% to close to 0% in the US and the euro area (Holston et al., 2017).

As Laubach and Williams put it themselves, their NRI is a medium-run concept - the framework aims at extracting the persistent component of the natural rate (Holston et al., 2017). Thus, the Laubach-Williams NRI does not normally stabilise output in each and every period. At the same, it is not a long-run concept, and the NRI should not be expected to remain stable over longer periods of time.

The Laubach-Williams model has been amended in various ways – and proven not to be very robust to these changes. Kiley (2015), Juselius et al. (2017), and Krustev (2019) extend the framework with financial variables in different ways, while also making some other adjustments (Kiley resigns from an explicit link between NRI and potential growth, whereas

Krustev adds inflation expectations and unemployment), to obtain very different results – Kiley (2015) finds no decline in US NRI following the crisis, Juselius et al. (2017) find NRI to be consistently higher by approximately 0.5 percentage points than in the standard version of the model, while Krustev (2019) finds NRI to be more volatile, but otherwise largely consistent with the standard model. Pescatori and Turunen (2015) use shadow rates to account for unconventional policies and explicitly include drivers of NRI other than potential growth to find that NRI dipped into negative territory already before the crisis. Lewis and Vazquez-Grande (2019) allow the non-growth component of NRI to be stationary, obtaining much more volatile and pro-cyclical estimates of NRI than the standard model. Brand and Mazelis (2019) add Taylor rule to the framework and assume inflation is stationary around the target, obtaining wildly different estimates from the standard model that track the actual real interest rates much closer. According to their results, the US NRI is negative, fluctuating around -1% since the crisis.

An alternative to a small reduced-form model is a full-scale DSGE model. In these models there are two concepts that can be referred to as the natural rate: the short-term equilibrium rate, i.e. the real interest rate that would emerge absent of nominal rigidities, and the long-run, steady-state rate (Taylor and Wieland, 2016). The former reacts strongly to shocks as it stabilises output in every period. Consequently, it is very volatile – the estimates point to a 6-12 percentage point fall in the US NRI following the crisis (Barsky et al., 2014; Curdia, 2015; Del Negro et al., 2017; Gerali and Neri, 2018). The latter is assumed to be constant in the sample, but it can be estimated recursively. The estimates show that it has also fallen in the US, though the magnitude of this decline (from approximately 3% to 2%) is understandably much smaller than in the case of the short-term NRI (German Council of Economic Experts, 2015). Furthermore, a medium-term NRI can be obtained from a DSGE model by computing conditional forecasts of the short-term equilibrium rate over a given horizon. Del Negro et al. (2017) find that when computed over a 5-year horizon, such NRI turns out to be almost identical to the Laubach-Williams (2016) estimate.

An opposite alternative is a fully agnostic model that abstracts from theory. Lubik and Matthes (2015) estimate a time-varying parameter VAR model for the US, interpreting the 5-year conditional forecast for the real interest rate as NRI. Their results are also very similar to those obtained by Laubach and Williams (2016) – NRI falls from 2% to 0.5% following the crisis. Del

Negro et al. (2017) choose a different approach and estimate a VAR model with common trends to obtain the trend real interest rate. They find that it has fallen by approximately 1 percentage point since the crisis.

Moving away from macroeconomic models, one can estimate NRI using financial term structure models. Christensen and Rudebusch (2019) estimate affine term structure models with 3 or 4 factors on real yields using US TIPS data. Their estimate of NRI is an average expected real short rate over a five-year period starting five years ahead (5Y5Y). Their baseline estimate of US NRI is falling gradually from 1.5% to 0% following the crisis, but the results are highly sensitive to model specification and subject to significant ex-post revisions. Ajevskis (2018) estimates a shadow model (accounting for non-linearities close to zero lower bound) on nominal yields for the euro area, using expected inflation from swap data to construct the natural rate. NRI is found to have fallen from 1% to about -0.5% following the crisis, but the results are also sensitive to model specification and sample size.

The macroeconomic and financial approaches are mixed together by Johannsen and Mertens (2018) who estimate the shadow rate using both macroeconomic and yield curve data and get NRI as a 'by-product' in this process. They also find that the US NRI has fallen since the crisis, but by somewhat less than implied by Holston et al. (2017) – from 2% to approximately 1%.

All of the above studies are either conducted in a closed economy framework or control only for import or oil prices. This should not come as a surprise since these studies focus on the United States that can be successfully approximated as a closed economy. Only a handful of researchers take a different approach and attempt at estimating NRI in a framework more suitable for open economies. Wynne and Zhang (2018) estimate the Laubach-Williams model in a two-country setting in which home NRI is determined by both home and foreign potential growth. Berger and Kempa (2014) augment the Laubach-Williams framework with the exchange rate. Fries et al. (2018) estimate the model for 4 euro area countries, controlling for output gap in the other 3 countries. Hence, the 'openness' of these models may be described as very limited and insufficient. A somewhat more detailed structural model is built by Hledik and Vlcek (2018) who include i.a. actual and equilibrium exchange rate and foreign output gap. This model is not estimated, but calibrated, however.



For the economies we focus on – the CEE countries and the euro area – numerous studies have been conducted for the euro area, while studies for CEE are few and far between.

For the euro area, the results obtained by Holston et al. (2017) and Ajevskis (2018) are mentioned above. Brand and Mazelis (2019) from their modified version of the Laubach-Williams described above obtain a post-crisis decline in NRI from about 2% to -1%. Hristov estimates standard and modified versions of Smets-Wouters DSGE model and obtains a decline in NRI from 3-4% to -2% or -3% following the crisis, while Gerali and Neri (2018) for their version of a DSGE model find that NRI fell from about 3% to -1%. Fries et al. (2018) finds that while NRI for euro area as a whole fell from 2% to 0% after the crisis, it has been quite volatile, but the differences across countries are small. Constancio (2016) applies the VAR, Laubach-Williams and medium-term DSGE frameworks to find that NRI has fallen by 1-3 percentage points since the crisis and is most likely negative. Brand et al. (2018) give a nice overview of NRI studies for the euro area, including some of the unpublished papers.

For CEE, most of the studies were conducted before the crisis (Brzoza-Brzezina, 2006; Horvath, 2009). In recent years, the only study that we are aware of is Hledik and Vlcek (2018) who find NRI in Czechia to be relatively stable, fluctuating around 1%.

## **2.2 The starting point**

Having all of the above in mind, we decide to use to the Laubach-Williams framework, and in particular its latest Holston et al. (2017) version, as a starting point for our model. There are several reasons for this choice.

Firstly, the medium-term approach applied in this framework is most relevant from the policy perspective. Under flexible inflation targeting, inflation is usually expected to return to target in the medium term. Hence, monetary policy decisions are made with this time horizon in mind and policymakers need a relevant benchmark to judge their monetary policy stance against. Immediate stabilisation of inflation and output is not the aim, and hence the usefulness of short-term equilibrium rates obtained from DSGE models is limited. Their usefulness is further limited by high volatility and model-dependence. Similarly, long-term

equilibrium rates are not very relevant for monetary policy that is usually assumed to have no effect on the real economy in the long run.

Secondly, the Laubach-Williams framework gives most flexibility when it comes to making amendments to it. The possibility of adding variables to a VAR model is limited due to a fast-growing number of parameters to be estimated. In case of CEE countries, sample sizes are additionally quite small, which further limits degrees of freedom. At the same time, augmenting DSGE models, because of their complexity and tight theoretical frames, is not a straightforward task. While term structure models might seem to be an interesting alternative, estimates of expected short-term rates – and thus NRI – are highly model-dependent. In particular, estimates of term premia – and thus expected short-term rates - tend to differ substantially in levels, while being more consistent in trends and dynamics, prompting some of the authors to argue that their levels should not be interpreted (e.g. Cohen et al., 2018).

Having said that, the Laubach-Williams framework also has its flaws. It is sensitive to model specification, NRI estimates are often imprecise, especially when Phillips and IS curves are close to being flat (Fiorentini et al., 2018), and subject to significant ex-post revisions. Output gap estimates tend to differ quite substantially from alternative estimates, using i.a. the production function approach, which via potential growth estimates affects NRI estimates. Our model extensions, presented in the next sections, are aimed at combatting these issues.

At this stage, we choose not to use any of the amended versions of the Laubach-Williams model discussed in the previous subsection, even though eventually some of our extensions are similar (e.g. lending spread similar to corporate spread in Kiley, 2015; inflation expectations like in Krustev, 2019). However, we also discard some of the adjustments made in these papers - these modelling choices are discussed in detail in sections 3 and 4.

The starting point Holston et al. (2017) model comprises of the following equations:

- Phillips curve:

$$\pi_t = \sum_{i=1}^4 b_{\pi,i} \pi_{t-i} + b_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \quad (1)$$

- IS curve:

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \epsilon_{\tilde{y},t} \quad (2)$$

- NRI equation:

$$r_t^* = g_t + z_t \quad (3)$$

- Remaining equations:

$$y_t = y_t^* + \tilde{y}_t/100 + \epsilon_{y,t}$$

$$y_t^* = y_{t-1}^* + g_{t-1}/400$$

$$g_t = g_{t-1} + \epsilon_{g,t}$$

$$z_t = z_{t-1} + \epsilon_{z,t}$$

Where  $\pi_t$  - core inflation,  $y_t$  - ln GDP,  $y_t^*$  - ln potential GDP,  $\tilde{y}_t$  - output gap (as percentage – that is why it is divided by 100),  $r_t$  - real interest rate,  $r_t^*$  - natural rate of interest,  $g_t$  - potential GDP growth (annual in per cent – that is why it is divided by 400),  $z_t$  - other determinants of NRI, and  $\epsilon_t$  are shocks. The sum of  $b_\pi$  is constrained to 1, reflecting the assumption that the Phillips curve is vertical in the long run.

The model comprises of basic New Keynesian Phillips and IS curves, where expectations of inflation and output gap are replaced with their past levels. Natural rate of interest is a sum of potential growth and an agnostic other factor, which are modelled as random walks. Based on previous results obtained by Laubach and Williams (2003, 2016), who show that the elasticity of NRI to potential growth is not significantly different from 1 and imprecisely estimated, Holston et al. (2017) assume that this elasticity is equal to 1.

### 3. Theoretical framework

The Holston et al. (2017) specification is based on the New Keynesian model. Thus, as a first step we derive the theoretical specification of our model from the Gali-Monacelli (2005) open economy NK model. Along the way, we make additions and adjustments to that model in order to control for potentially important drivers of inflation and output gap.

#### 3.1 The Phillips curve

The open economy Phillips curve is virtually the same as in the basic model, the only difference being that it explains domestic inflation (the rise in prices of goods produced domestically) rather than overall inflation (Gali and Monacelli, 2005):

$$\pi_t^h = \beta E_t(\pi_{t+1}^h) + \kappa \tilde{y}_t$$

Where  $\beta$  is a model parameter (the discount factor) and  $\kappa$  is a function of model parameters (the slope of the Phillips curve); both take positive values. However, in practice there is no good indicator of domestic inflation. Some authors argue that domestic price growth can be approximated with PPI (Gali, 2008), but the measures of PPI often do not cover services. Moreover, monetary policy is primarily interested in CPI inflation, and thus we would like the model to be specified in terms of this indicator. The relationship between domestic inflation and overall (CPI) inflation is as follows (Gali and Monacelli, 2005):

$$\pi_t^h = \pi_t - \alpha \Delta s_t = \pi_t - \alpha(\Delta e_t + \pi_t^f - \pi_t^h)$$

$$\pi_t^h = \frac{\pi_t}{1 - \alpha} - \frac{\alpha}{1 - \alpha}(\Delta e_t + \pi_t^f)$$

Where  $s_t$  – ln terms of trade,  $e_t$  – ln nominal exchange rate (increase denotes depreciation),  $\pi_t^f$  – foreign core inflation, and  $\alpha$  is a model parameter (degree of openness,  $>0$ ). Plugging the latter equation into the Phillips curve, we get the following:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(1 - \alpha)\tilde{y}_t + \alpha(\Delta e_t + \pi_t^f) - \beta\alpha E_t(\Delta e_{t+1} + \pi_{t+1}^f)$$

Hence, in an open economy we have two additional determinants of inflation – the exchange rate and foreign inflation - that influence price growth through their impact on import prices.

In the more general version of the NK model, inflation is driven by marginal costs, and only under certain assumptions movements in marginal costs are equivalent to movements in output gap. In the open economy version of the model, marginal costs can be described as follows (Gali and Monacelli, 2005):

$$mc_t = -v + (\sigma_\alpha + \varphi)y_t + (\sigma_\alpha - \sigma)y_t^f - (1 + \varphi)a_t$$

Where  $mc_t$  is ln marginal cost,  $y_t^f$  is ln GDP abroad,  $a_t$  is ln productivity,  $\varphi$  and  $\sigma$  are model parameters, and  $v$  and  $\sigma_\alpha$  are functions of model parameters (for details about model parameters please refer to Gali and Monacelli, 2005). By equating the marginal cost to  $-\mu$ , where  $\mu$  is the optimal flex-price mark-up, potential output can be obtained (Gali and Monacelli, 2005):

$$y_t^* = \frac{v - \mu}{(\sigma_\alpha + \varphi)} + \frac{(\sigma - \sigma_\alpha)}{(\sigma_\alpha + \varphi)}y_t^f + \frac{(1 + \varphi)}{(\sigma_\alpha + \varphi)}a_t$$

However, the above expression for potential output is derived under the assumption that prices are flexible in the home economy, but remain sticky abroad. If one instead defines domestic potential output in a more coherent manner as the level of output that would emerge if prices were flexible in the whole world economy, the following expression can be obtained:

$$y_t^* = \frac{v - \mu}{(\sigma_\alpha + \varphi)} + \frac{(\sigma - \sigma_\alpha)}{(\sigma_\alpha + \varphi)}y_t^{*f} + \frac{(1 + \varphi)}{(\sigma_\alpha + \varphi)}a_t$$

Where  $y_t^{*f}$  is ln potential GDP abroad. In such a case, fluctuations in marginal costs are no longer equivalent to fluctuations in output gap:

$$\widehat{mc}_t = (\sigma_\alpha + \varphi)\tilde{y}_t + (\sigma_\alpha - \sigma)\tilde{y}_t^f$$

Where  $\widehat{mc}_t$  is the ln deviation of marginal costs from the flex-price level and  $\tilde{y}_t^f$  is output gap abroad. Hence, output gap abroad enters the Phillips curve:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(1 - \alpha)\tilde{y}_t + \alpha(\Delta e_t + \pi_t^f) - \beta\alpha E_t(\Delta e_{t+1} + \pi_{t+1}^f) + \frac{(\sigma_\alpha - \sigma)}{(\sigma_\alpha + \varphi)}\kappa(1 - \alpha)\tilde{y}_t^f$$

This specification accommodates the hypothesis that - as a result of globalisation and growing importance of global value chains - global output gap directly affects inflation even after controlling for domestic economic conditions (Borio and Filardo, 2007). While Borio and

Filardo use ad-hoc Phillips curve specifications in their study, we show that the same relationship can be derived from the NK model.

The textbook open economy NK model remains very simple and does not incorporate any shocks other than productivity shocks. In particular, cost-push shocks are omitted. Cost-push shocks are defined as the drivers of marginal costs that do not affect output gap (Clarida et al., 1999). Hence, they drive a wedge between efficient (potential) output and natural (flexible price) output:

$$\begin{aligned}\varepsilon_t^s &= \kappa(1 - \alpha)(y_t^* - y_t^n) \\ y_t - y_t^n &= \tilde{y}_t + \frac{1}{\kappa(1 - \alpha)} \varepsilon_t^s\end{aligned}$$

Where  $\varepsilon_t^s$  is the cost-push shock and  $y_t^n$  is ln natural output. In practice, fluctuations in oil prices, or, in a wider sense, in energy prices, are the most important source of cost-push shocks. They might be of relevance especially in highly fossil-fuel dependent CEE economies. Hence, we add them to the Phillips curve:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(1 - \alpha)\tilde{y}_t + \alpha(\Delta e_t + \pi_t^f) - \beta\alpha E_t(\Delta e_{t+1} + \pi_{t+1}^f) + \frac{(\sigma_\alpha - \sigma)}{(\sigma_\alpha + \varphi)}\kappa(1 - \alpha)\tilde{y}_t^f + \varepsilon_t^s$$

The textbook NK model is also purely forward-looking: inflation depends only on expectations about its future level, not on its past level. This is not necessarily the case in reality - empirical work has demonstrated that inflation is persistent, i.e. adding past inflation to the Phillips curve specification that already includes inflation expectations improves the model fit (see Mavroeidis et al., 2014 for a review). Gali and Gertler (1999) accommodate this empirical finding by assuming that a fraction of firms is backward-looking. As a result, past inflation enters the Phillips curve:

$$\begin{aligned}\pi_t &= \gamma_f E_t(\pi_{t+1}) + \gamma_b \pi_{t-1} + \kappa(1 - \alpha)\tilde{y}_t + \alpha(\Delta e_t + \pi_t^f) - \beta\alpha E_t(\Delta e_{t+1} + \pi_{t+1}^f) \\ &\quad + \frac{(\sigma_\alpha - \sigma)}{(\sigma_\alpha + \varphi)}\kappa(1 + \alpha)\tilde{y}_t^f + \varepsilon_t^s\end{aligned}\tag{4}$$

Where  $\gamma_f$  and  $\gamma_b$  are functions of model parameters ( $>0$ ).

### 3.2 The IS curve

Similarly as in the case of the Phillips curve, the IS curves obtained from the open economy NK model and the basic model do not differ substantially. The only difference is that the real interest rate is derived from domestic inflation rather than overall inflation (Gali and Monacelli, 2005):

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} (i_t - E_t(\pi_{t+1}^h) - r_t^*)$$

Where  $i_t$  is the nominal interest rate. Domestic inflation can be expressed as (Gali and Monacelli, 2005):

$$\pi_t^h = \pi_t - \alpha \Delta s_t = \pi_t - \frac{\alpha}{1 - \alpha} \Delta q_t$$

Where  $q_t$  is ln real exchange rate (increase denotes depreciation). Plugging this into the IS curve we get:

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} (i_t - E_t(\pi_{t+1}) - r_t^*) - \frac{\alpha}{\sigma_\alpha(1 - \alpha)} E_t(\Delta q_{t+1})$$

Expected depreciation in the exchange rate has a negative impact on output gap. In the model depreciation decreases consumption at a given output as imports become more expensive. When expected consumption shrinks, for the Euler equation to hold, current consumption – and hence output - has to fall as well.

In the real world, however, currency depreciation tends to improve economic conditions as it increases the competitiveness of domestic production. The magnitude of this effect can vary as it depends on the way export and import prices are formed (in what currency they are quoted). At the same time, if FX debt is present, depreciation has a negative impact on the economy via increasing debt servicing costs and negative wealth effects.

In CEE, domestic currency debt dominates, but export prices are usually quoted in foreign currency, and hence the exchange rate fluctuations do not have a direct and automatic effect on export competitiveness. Therefore, currency depreciation is expected to have a positive, but rather small and potentially insignificant effect on output gap.

The natural interest rate  $r_t^*$  can be expressed in the following way:

$$r_t^* = \rho + \sigma_\alpha E_t(\Delta y_{t+1}^*) + \sigma_\alpha \alpha \theta E_t(\Delta y_{t+1}^f)$$

Where  $\theta$  is a function of model parameters. Similarly as in the basic NK model, NRI is determined by consumer time preferences and potential growth. Additionally, output growth abroad plays a role.

Similarly as in the case of the expression for potential growth discussed in the previous subsection, the above expression is derived assuming that domestic prices are flexible, but they remain sticky abroad. If the natural rate is instead defined as the real interest rate under elastic prices in the whole world economy, potential growth abroad enters the equation for the natural rate, while foreign output gap moves to the IS curve:

$$r_t^* = \rho + \sigma_\alpha E_t(g_t) + \sigma_\alpha \alpha \theta E_t(g_t^f)$$

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} (i_t - E_t(\pi_{t+1}) - r_t^*) + \frac{\alpha}{\sigma_\alpha(1-\alpha)} E_t(\Delta q_{t+1}) + \alpha \theta E_t(\Delta \tilde{y}_{t+1}^f)$$

Where  $g_t^f$  – potential GDP growth abroad. Output gap is now additionally determined by the movements in the real exchange rate and foreign output gap, reflecting the importance of exports for small open economies and the synchronisation of their business cycles with major economies. In CEE, over 30% of gross value added is exported, hence accounting for the exchange rate movements and fluctuations in economic activity abroad is very important.

The above specification implicitly assumes that the central bank has complete control over nominal interest rates. In practice, however, the transmission mechanism can be highly imperfect and interest rates economic agents face might fluctuate significantly even when monetary policy is not adjusted, especially at a time of financial market disruption. This channel might have played a very significant role especially during and after the global financial crisis. Kiley (2015) shows that accounting for varying spread between central bank and market interest rates indeed matters for the estimation of NRI.

The easiest way to control for financial market shocks in an NK model is by adding demand shock to the IS curve:

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} (i_t - E_t(\pi_{t+1}) - r_t^*) + \frac{\alpha}{\sigma_\alpha(1-\alpha)} E_t(\Delta q_{t+1}) + \alpha \theta E_t(\Delta \tilde{y}_{t+1}^f) + \varepsilon_t^d \quad (5)$$

Where  $\varepsilon_t^d$  is the demand shock.



### 3.3 The exchange rate equation

Most of the new variables in Phillips and IS curves are either clearly exogenous for a small open economy (foreign inflation, energy prices and foreign output gap) or are unlikely to be affected by the model's state variables (inflation expectations). However, this is not necessarily the case for the exchange rate. Thus, the signal equation for the exchange rate needs to be specified.

In the open economy NK model, the exchange rate can be linked either to the interest rate differential or the difference in output (Gali and Monacelli, 2005):

$$E_t(\Delta e_{t+1}) = i_t - i_t^f$$

$$q_t = (1 - \alpha)s_t = (1 - \alpha)\sigma_\alpha(y_t - y_t^f)$$

Where  $i_t^f$  is the nominal interest rate abroad. The uncovered interest rate parity can be expressed in real terms:

$$E_t(\Delta e_{t+1}) = (1 - \alpha)E_t(\Delta s_{t+1}) - E_t(\Delta \pi_{t+1}^f) + E_t(\Delta \pi_{t+1})$$

$$(1 - \alpha)E_t(\Delta s_{t+1}) = r_t - r_t^f$$

$$E_t(\Delta q_{t+1}) = r_t - r_t^f$$

Where  $r_t^f$  is the real interest rate abroad. The two conditions can be linked together:

$$q_t = r_t^f - r_t + E_t(q_{t+1}) = r_t^f - r_t + (1 - \alpha)\sigma_\alpha E_t(y_{t+1} - y_{t+1}^f)$$

Let us now decompose output into potential and gap and take the first difference of the whole expression:

$$q_t = r_t^f - r_t + (1 - \alpha)\sigma_\alpha E_t(y_{t+1}^* - y_{t+1}^{*f}) + (1 - \alpha)\sigma_\alpha E_t(\tilde{y}_{t+1} - \tilde{y}_{t+1}^f)$$

$$\Delta q_t = \Delta(r_t^f - r_t) + (1 - \alpha)\sigma_\alpha E_t(g_t - g_t^f) + (1 - \alpha)\sigma_\alpha E_t(\Delta \tilde{y}_{t+1} - \Delta \tilde{y}_{t+1}^f) \quad (6)$$

The equation implies that the rise in interest rates abroad (or a fall in domestic rates) leads to currency depreciation in line with the uncovered interest rate parity. First differences are taken so that we can abstract from risk premia that lead to the UIP puzzle – implicitly, it is assumed that risk premia are constant over time.

From the model perspective, the exchange rate reacts to changes in outputs so that terms of trade stabilises the economy and assures market clearing. In practice, there are also other channels not present in the model that work in the opposite direction. Therefore, the impact of output gap and potential growth on the exchange rate is somewhat ambiguous.

Firstly, higher potential growth than abroad, i.e. real convergence, is likely to lead to price convergence via the Balassa-Samuelson effect - as productivity and wages in the tradable sector are increasing, so do the wages in the non-tradable sector, even though productivity is rising there at a slower pace. This generates pressure to increase prices. However, price convergence does not necessarily have to take place through higher inflation than abroad – it might also happen via currency appreciation. Empirical studies show that this is indeed the case for CEE counties (Egert et al., 2003; Rubaszek and Rawdanowicz, 2009).

Secondly, increasing output gap tends to lead to currency appreciation via decreasing exchange rate risk premium (Greszta et al., 2011). As economic conditions are improving, the perceived risk and risk aversion in financial markets are decreasing. As a result, capital starts flowing from the ‘core’ to ‘peripheries’, i.e. from major advanced economies to emerging markets and minor advanced economies, and ‘peripheral’ currencies are appreciating.

### 3.4 Remaining equations

The remaining two equations are the ones for potential growth and, most importantly, the NRI. As already shown above, in the open economy version of the NK model both variables depend on foreign output growth – and after our amendments, on foreign potential growth:

$$g_t = \frac{(1 + \varphi)}{(\sigma_\alpha + \varphi)} \Delta a_t + \frac{(\sigma - \sigma_\alpha)}{(\sigma_\alpha + \varphi)} g_t^f$$

$$r_t^* = \rho + \sigma_\alpha E_t(g_t) + \sigma_\alpha \alpha \Theta E_t(g_t^f) \quad (7)$$

It should be noted, however, that the coefficients in front of foreign potential growth are likely to be low and differ across countries (and time) depending on i.a. the degree of openness. Theoretically, they can even be negative. Assuming standard parameters and the degree of

openness  $\alpha = 0.3$ ,<sup>3</sup> the coefficients are equal to 0.11 in the potential growth equation and 0.2 in the NRI equation.

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<sup>3</sup>  $\sigma$  (intertemporal elasticity of substitution) = 1,  $\varphi$  (labour supply elasticity) = 1,  $\eta$  (elasticity of substitution between domestic and foreign goods) = 1.5,  $\gamma$  (elasticity of substitution between foreign goods) = 1.5 (Adolfson et al., 2007). In such a case  $\sigma_\alpha = 0.8$  and  $\theta = 0.85$ .

## 4. Empirical specification and estimation

In this section, theoretical specifications derived in the section 3 are transformed into empirical specifications, bearing in mind the Holston-Laubach-Williams model described in subsection 2.2.

### 4.1 The Phillips curve

Equation ( 4 ) constitutes the theoretical base for our empirical specification of the Phillips curve (in red additions to equation ( 1 ) of the Holston et al., 2017 specification):

$$\pi_t = \sum_{i=1}^4 b_{\pi,i} \pi_{t-i} + \left( 1 - \sum_{i=1}^4 b_{\pi,i} \right) \pi_t^e + b_{\pi,o} (\pi_{t-4}^o - \pi_{t-4}) + b_e \Delta e_{t-2} + b_y \tilde{y}_t + b_{y,f} \tilde{y}_{t-1}^f + \epsilon_{\pi,t}$$

Where  $\pi_t^e$ - inflation expectations, and  $\pi_t^o$ - energy price inflation.

Several adjustments are made when compared to the theoretical model. Firstly, core inflation is the dependent variable, similarly as in Holston et al. (2017), as we try to focus on these elements of the CPI basket that are sensitive to domestic economic conditions.

Secondly, four lags of inflation are used as independent variables instead of one. This is also in line with Holston et al. (2017) who use these lags of inflation as a measure of inflation expectations, effectively assuming that expectations are adaptive. We have a separate measure of expectations, but keep these lags as controls for inflation persistence. Having more than one lag significantly improves the model fit, hence we stick to the Holston et al. (2017) specification in this respect. Similarly, also the lags to the exchange rate movements, output gaps and energy prices are chosen in order to optimise the model fit. In particular, contemporaneous output gap is used – like in the theoretical specification – instead of its 1st lag as in the Holston et al. (2017) model since such a specification fits the data better.

Thirdly, the Phillips curve is assumed to be vertical in the long run. Therefore, the sum of coefficients in front of lagged and expected inflation is constrained to be one and energy price inflation is measured against core inflation. The same approach has been chosen by Laubach and Williams (2003) in their seminal paper.

Fourthly, expected exchange rate movements and foreign inflation are disregarded as we focus on their contemporaneous and lagged fluctuations. Expected foreign inflation turns out to be highly colinear with current/lagged foreign inflation. As for the exchange rate, expectations about its movements could be proxied with the interest rate differential. However, this leads to a substantial reverse causality bias as central banks change interest rates in reaction to inflation movements.

Finally, foreign inflation is omitted from the specification as it is either highly insignificant or enters the equation with the wrong sign for all analysed economies.

Several additional comments need to be made. Firstly, energy price inflation is our measure of cost-push shocks. We choose it over more often used oil price changes because it is broader and better reflects cost pressures faced by enterprises. Moreover, prices of energy other than gasoline do not necessarily follow oil prices as they can be driven by individual supply and demand factors and are often heavily regulated.

Secondly, energy prices are included in the specification even though core inflation is used as the dependent variable. Even when energy prices do not have a direct impact on inflation, their indirect impact on prices of virtually all goods is hard to question and should be particularly significant in a highly fossil-fuel dependent CEE economies.

Thirdly, we pick the exchange rate and foreign inflation over import price deflator. In the open economy NK model, foreign inflation and the exchange rate together measure the impact of changes in import prices on inflation. A seemingly easier way to control for import prices would be to include the import price deflator in the specification. However, such a deflator has a couple of disadvantages. It covers not only 'core' goods, but also food and energy. Since all the investigated economies are oil importers, import price deflator is highly correlated with the price of oil, whose effect on inflation is already included in the model via energy price inflation. Moreover, import price deflators tend to be very volatile and contain a lot of noise. As a result, their correlation with core inflation is rather poor.

## 4.2 The IS curve

Equation ( 5 ) serves as a theoretical base for our empirical specification (in red additions to equation ( 2 ) of the Holston et al., 2017 specification):

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + a_e \Delta q_{t-3} + a_f \Delta \tilde{y}_t^f + a_l ls_t + \epsilon_{\tilde{y},t}$$

$$cu_t = a_c \tilde{y}_t + \epsilon_{cu,t}$$

Where  $cu_t$  is a survey measure of capacity utilisation and  $ls_t$  is a deviation of lending spread from its mean.

Similarly as for the Phillips curve, several amendments are made in the empirical specification when compared to the theoretical model. Firstly, expectations about domestic output gap, movements in the exchange rate and foreign output gap are replaced with lags (or contemporaneous values) of these variables. This is a standard approach in literature. For the real interest rate gap, we keep two lags from the Holston et al. (2017) specification. For the exchange rate and foreign output gap we choose lags to optimise the model fit. For domestic output gap we include only one lag, unlike Holston et al. (2017).

More importantly, however, the deviation from the mean spread between average interest rate on bank loans and central bank interest rates (lending spread) is our measure of the financial demand shock. In the literature, corporate bond spread (a spread between the yield on medium-rated corporate bonds and sovereign bonds) is usually used in this character (e.g. Kiley, 2015). However, since European economies are more bank-dominated, corporate bond markets are relatively shallow in CEE countries and the data on bank loans is more readily available, we use average interest rates on new bank loans as a measure of market interest rates and compare them directly with central bank interest rates (interest rates on bank loans are often variable and directly linked to interbank interest rates, which are closely aligned with central bank rates).

Finally, we use survey data to improve the accuracy of output gap estimates. It is a well-established fact in the literature that output gap estimates are imprecise and subject to large ex-post revisions (i.a. Orphanides and van Norden, 2002). At the same time, several studies have shown that using survey data on capacity utilisation and economic conditions improves

the accuracy of estimates both in terms of ex-ante errors and ex-post revisions (Marcellino and Musso, 2011; ECB, 2015; Hulej and Grabek, 2015). We incorporate the survey data into our specification in the simplest possible way - by assuming that the survey measure of capacity utilisation reflects contemporaneous output gap with an idiosyncratic error. Hence, output gap can be regarded as a partially observable variable in our model.

High precision of output gap (and hence, potential growth) estimates is crucial for the estimation of NRI not only because potential growth is a key determinant of NRI, but also because it gives the model more power to estimate  $z_t$  (the other determinants of NRI). In practice, without the aid of capacity utilisation survey data, the model often estimates the IS curve to be flat (interest rate gap to have no impact on output gap), and thus lacks power to estimate  $z_t$  and sets it to zero. As a result, NRI is equal to potential growth and output gap becomes a 'by-product' of the differential between the real interest rate and potential growth (the exchange rate and foreign output gap staying constant). When output gap becomes partially observable, the IS curve is rarely estimated to be completely flat, giving the model more power to extract the interest rate gap that leads to a given output gap. As a consequence, NRI can be more precisely estimated.

Several additional comments are valid here. Fiorentini et al. (2018) show that for the natural interest rate to be precisely estimated (identifiable) in the Laubach-Williams model, Phillips and IS curves have to be steep enough. After adding capacity utilisation, steep Phillips curve is no longer necessary, as output gap is precisely estimated (and thus NRI is more precisely estimated) even when Phillips curve is flat. Steep IS curve continues to be necessary, however. Fiorentini et al. (2018) show that this condition can also be dropped if one assumes the interest rate gap to be stationary. One cannot, however, make this assumption and use capacity utilisation data at the same time, since the conditions of state space model formulation are not fulfilled<sup>4</sup>. Having to choose between the two, we opt for capacity utilisation – with stationary interest rate gap, output gap estimates tend to be as unrealistic as in the standard Laubach-Williams model, calling into question the quality of NRI estimates. With capacity utilisation

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<sup>4</sup> Either signal variable (real interest rate or interest rate gap) has to enter the state equation (for output gap), or (if the IS curve is moved to a signal equation for GDP, as in Fiorentini et al., 2018) contemporaneous signal variable (GDP) has to enter another signal equation (for capacity utilisation) – both formulations are not possible in a state space model.

this problem is (usually) solved, while IS curve is more than not steep enough to obtain relatively precise NRI estimates.

An alternative to using capacity utilisation would be augmenting the model with unemployment in the form of Okun's law. Brand and Mazelis (2019) and Krustev (2019) use this approach either in their baseline (Krustev) or alternative (Brand and Mazelis) specifications. The downsides of this approach are two-fold: firstly, it requires the estimation of more parameters and additional state variables (NAIRU and unemployment gap), which could require augmenting the model further (e.g. with wage or ULC equation); secondly, output gap estimates might still continue to be unrealistic, as demonstrated by the euro area estimates in Brand and Mazelis (2019). Capacity utilisation is also viewed as a better proximate measure of demand pressure and thus output gap, while unemployment gap is a better measure of wage pressure. For these reasons, we prefer the use of capacity utilisation.

### 4.3 The exchange rate equation

Equation ( 6 ) serves as a theoretical base for our empirical specification:

$$q_t = q_{t-1} + c_g(g_t - g_t^f) + c_y\Delta(\tilde{y}_t - \tilde{y}_t^f) + c_r\Delta(r_t - r_t^f) + \epsilon_{e,t}$$

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^f$$

Only minor adjustments are made in the empirical specification when compared with the theoretical model: expected output gaps are replaced with contemporaneous output gaps and lags to the interest rate differential are adjusted to optimise the model fit.

### 4.4 Remaining equations

Theoretical NRI equation – equation ( 7 ) – includes parameters to be estimated in front of domestic and foreign potential growth. However, Holston et al. (2017) notice the difficulty of estimation of coefficients inside the NRI equation, given the imprecision and fragility of NRI estimates. In practice, the coefficients in front of additional variables (such as foreign potential growth here) tend to be overestimated and as a result, NRI follows these variables too closely



(Pescatori and Turunen, 2015 encounter this problem). This is why we decide to retain the agnostic Holston et al. (2017) specification of the NRI equation – equation ( 3 ) - in our benchmark model. The impact of the foreign sector on the NRI should still be captured by the  $z_t$  factor – in our analysis of drivers of NRI other than potential growth we investigate whether this is the case. Additionally, as a part of our robustness analysis, we check whether including foreign potential growth explicitly in the NRI equation has a significant impact on the results. Since productivity is not modelled, we also stick to the random walk specification of potential growth. As domestic productivity growth and foreign potential growth are likely to co-move very closely, this should not be a problem.

Hence, our specifications of the remaining equations are as follows:

$$\begin{aligned}
 r_t^* &= g_t + z_t \\
 g_t &= g_{t-1} + \epsilon_{g,t} \\
 z_t &= z_{t-1} + \epsilon_{z,t} \\
 y_t &= y_t^* + \tilde{y}_t/100 \\
 y_t^* &= y_{t-1}^* + g_{t-1}/400
 \end{aligned}$$

We make one adjustment compared to the Holston et al. (2017) model – there is only a shock to potential growth rate, but no shock to potential GDP level. In practice the model has a hard time distinguishing between the two, and the variance of the shock to the potential level converges towards zero. Therefore, we choose to omit it in the specification.

## 4.5 Estimation

Similarly as Holston et al. (2017), we use Kalman filter for estimation and link together the variances of shocks to potential growth and output gap (as well as other determinants of NRI  $z_t$ ). There are two reasons for doing so. Firstly, this makes it easier for the filter to distinguish between output gap and potential output. Secondly, maximum likelihood estimates of shocks to potential growth and  $z_t$  can be biased towards zero due to a so-called ‘pile-up problem’ (Stock, 1994).

Unlike Holston et al. (2017), however, we do not use the Stock-Watson (1998) median unbiased estimator to estimate the ratio of shocks. In our sample this estimator tends to be very sensitive to the model specification and gives a wide range of often highly unrealistic results. Therefore, we choose an alternative approach and set the ratio of variances of shocks to potential growth and output gap to be equal to the ratio of variances of potential growth and output gap that are obtained from the HP filter:

$$\frac{\text{var}(\epsilon_{g,t})}{\text{var}(\epsilon_{\tilde{y},t})} = \frac{\text{var}(g_t^{HP})}{\text{var}(\tilde{y}_t^{HP})}$$

The same value is chosen for the ratio of variances of shocks to  $z_t$  and output gap:

$$\frac{\text{var}(\epsilon_{z,t})}{\text{var}(\epsilon_{\tilde{y},t})} = \frac{\text{var}(g_t^{HP})}{\text{var}(\tilde{y}_t^{HP})}$$

The estimates obtained from the HP filter are likely to be biased, however. On the one hand, all variation in output gap is treated as if it had source in shocks, while in the model there are other important drivers of output gap (interest rate gap, foreign output gap, lending spread, the exchange rate). Therefore, the shock to output gap is likely to be overestimated and the shock to potential growth underestimated. On the other hand, potential growth is modelled as a random walk, while output gap is a stationary process. Hence, the ratio of variances of potential growth and the shock to potential growth is likely to be higher than the same ratio for output gap. This leads to an overestimation of the shock to potential growth and underestimation of the shock to output gap.

Despite these problems, we prefer an estimator that is biased, but this bias is relatively well understood, to an estimator that is theoretically unbiased, but in practice can give highly imprecise results.

Additionally, for the Kalman filter to be able to better distinguish between potential output and output gap, we set initial values and variances for state variables. Similarly as in Holston et al. (2017), initial values for output gap, potential output and potential growth are taken from the HP-filtered data, while initial values for  $z_t$  are set to zero. Initial variances for output gap, potential output and potential growth are also taken from HP-filtered data (as variances from the whole sample) and the initial variance of  $z_t$  is set to equal the initial variance of potential growth.

The model is estimated separately for each country. As mentioned in section 3, lags to most of the variables are chosen to optimise the model fit. Variables that enter the equations with a wrong sign or are highly insignificant are dropped. These changes are described in Table 1.

Interestingly, energy prices enter the specification only for the euro area and Hungary, while foreign output gap enters the Phillips curve only for Poland, suggesting that these variables are not crucial for the estimation of NRI. As a part of our robustness checks we investigate the sensitivity of the results to removing these and other variables.

Country	Variables dropped	Lags changed
euro area	Phillips curve: $\pi_{t-2}$ to $\pi_{t-4}$ , $\tilde{y}_{t-1}^f$	Phillips curve: $\pi_{t-5}^o$ , $\Delta e_{t-1}$
Czechia	Phillips curve: $\pi_{t-4}^o$ , $\tilde{y}_{t-1}^f$ IS curve: $ls_t$ Exchange rate equation: $\Delta(r_t - r_t^f)$	Phillips curve: $\tilde{y}_{t-1}$
Hungary	Phillips curve: $\tilde{y}_{t-1}^f$	Phillips curve: $\tilde{y}_{t-3}$ IS curve: $\Delta q_{t-1}$ , $ls_{t-2}$ Exchange rate equation: $\Delta(r_{t-2} - r_{t-2}^f)$
Poland	Phillips curve: $\pi_{t-4}^o$ IS curve: $(r_{t-2} - r_{t-2}^*)$ Exchange rate equation: $(g_t - g_t^f)$	IS curve: $\Delta q_{t-2}$ , $\Delta \tilde{y}_{t-1}^f$ Exchange rate equation: $\Delta(r_{t-1} - r_{t-1}^f)$

**Table 1** Changes in specifications for each country in comparison with the baseline

## 5. Data

Our sample covers the euro area and Poland, Czechia and Hungary: three CEE economies with independent monetary policy and inflation targeting in place for about 20 years, which guarantees sufficiently long sample. We use quarterly data which spans from 1996Q2 to 2017Q4.

Euro area is used as a proxy for the foreign sector of CEE economies as about 60% of CEE foreign trade is conducted with this economic zone. The model for the euro area is estimated primarily to obtain model consistent estimates of output gap and potential growth that are subsequently used in the estimation of NRI in CEE countries. For the euro area, the US is used as a proxy for the foreign sector with output gap and potential growth obtained from the HP filter.

Variable	Construction	Source
$\pi_t$	Core inflation (excluding food and energy) QoQ SAAR	OECD, own calculations
$y_t$	ln GDP in constant prices	OECD
$r_t$	Central bank benchmark interest rate* minus annual core inflation	Bloomberg, Wu and Xia (2017), OECD
$\pi_t^e$	Consumer inflation expectations over the next 12 months (balance statistics) transformed such that mean expectations = mean headline inflation and standard deviation of expectations = 0.6 standard deviation of headline inflation	Eurostat, own calculations
$\pi_t^o$	Energy price inflation QoQ SAAR	OECD, own calculations
$q_t$	ln real effective exchange rate (increase denotes appreciation)	BIS
$ls_t$	Deviation of lending spread (mean interest rate on new bank loans minus central bank benchmark interest rate) from sample average (linear trend for Hungary)	NBP, ECB, CNB, MNB, IMF, own calculations**
$cu_t$	Deviation from linear trend of percentage of firms reporting insufficient demand as a factor limiting activity, with an opposite sign	Eurostat, GUS, own calculations

**Table 2** Data construction and sources

\* For Hungary 3M interbank offer rate (BUBOR 3M) and for the euro area the Wu-Xia (2017) shadow rate are used instead.

\*\* Comprehensive and comparable data on bank interest rates is available for the EU since 2004; for the earlier period, either incomprehensive data from the national sources (Poland, Hungary) or the IMF data on lending rates (euro area, Czechia) is used.

Table 2 describes the data used in the estimation. The whole dataset – together with estimation codes - is available online (Stefański, 2019).

Several comments have to be made. Firstly, real interest rates are computed ex-post (deflated with current annual inflation), similarly as in the Holston et al. (2017) paper, even though inflation expectations are computed and used in the estimation. We stick with the ex-post rates as they are easiest to compute, relatively uncontroversial and most often used in policy analysis. Using ex-ante rates is more controversial in a sense that there are many possible ways to compute inflation expectations. Having said that, we use ex-ante rates in one of our robustness checks.

Secondly, in two cases we do not use central bank benchmark interest rates in the computation of real interest rates. For Hungary the 3M interbank offer rate (BUBOR 3M) is used as in 2016 the central bank effectively (though not officially) changed the policy rate from 3M deposit (earlier 2-week deposit) to overnight deposit. More importantly, for the euro area the Wu-Xia (2017) shadow rate is used in order to account for the effects of unconventional policies.

## 5.1 Inflation expectations

The computation of inflation expectations requires a more detailed deliberation. There are basically six ways in which this can be done:

- perfect foresight,
- adaptive expectations,
- surveys of forecasters,
- surveys of enterprises,
- surveys of households,
- financial market data (inflation-linked bonds).

In practice, financial market data is not available for the CEE economies: inflation-linked bonds have been introduced relatively recently and the market for them is shallow. Similarly, since past inflation enters our Phillips curve explicitly, adaptive expectations are not an option.

Surveys of forecasters are most readily available and hence most often used in econometric studies. However, they are unlikely to capture expectations of economic agents. Moreover, their tendency to revert to the mean might even lead to spurious correlation with inflation,

which could bias the estimates of other drivers of inflation. Similar comments can be made about the assumption of perfect foresight of inflation.

Expectations of price setters, i.e. of enterprises, seem to be most relevant from the policy perspective. Unfortunately, survey results are not available for some of the economies for a sufficiently long sample period (Poland, euro area).

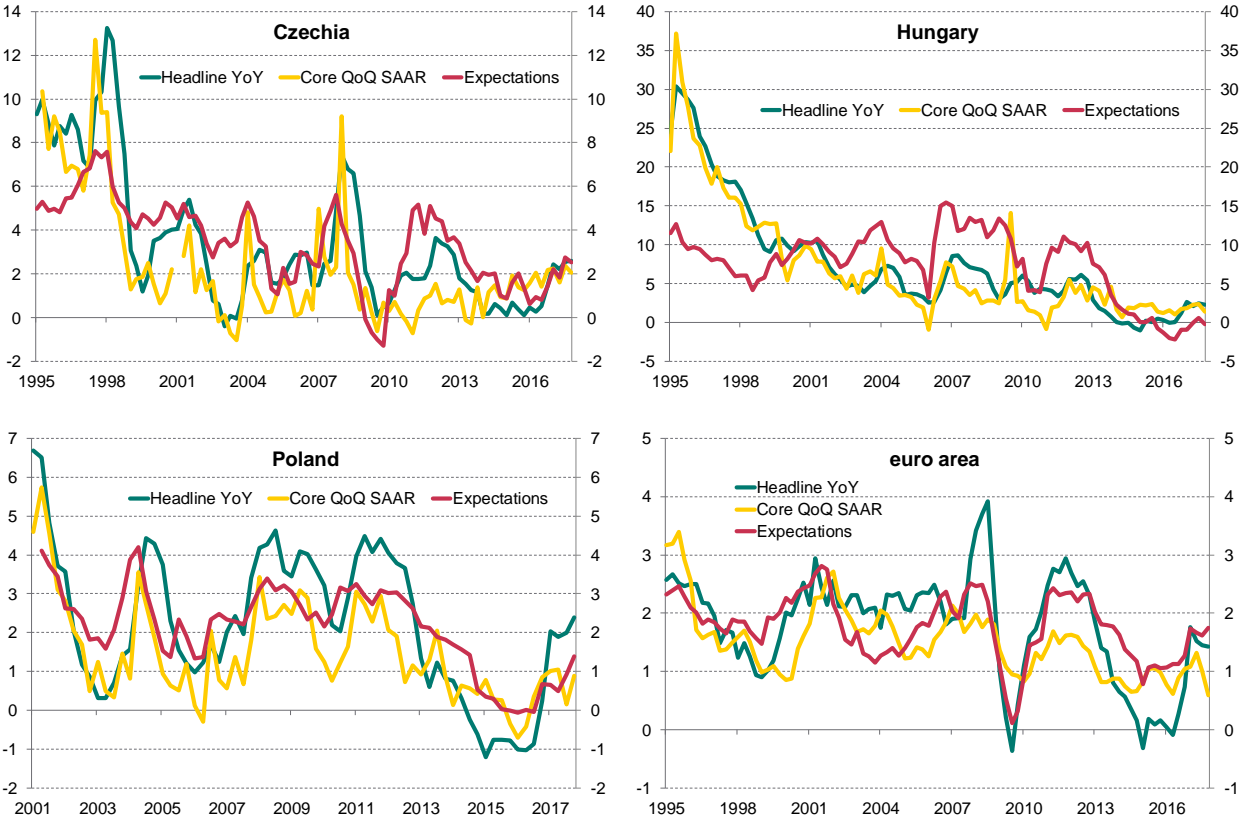
Consumer inflation expectations seem to come second in terms of relevance as enterprises are likely to take them into account while setting prices. The data is available for most of the analysed period. Moreover, using consumer expectations enables us to control for the Coibion-Gorodnichenko hypothesis (2015) which states that consumer inflation expectations, closely aligned with the level of oil prices, have been responsible for the post-crisis break-up in the Phillips curve relationship. For these reasons, we decide to use consumer surveys as our measure of inflation expectations.

However, the data on consumer inflation expectations is available in the form of balance statistics and needs to be transformed into expected inflation before it is incorporated into our specification of the Phillips curve. Consumers are asked whether inflation will be higher, the same, lower or negative over the next 12 months. Because of the way the question is asked, the standard Carlson-Parkin (1975) method of quantification assumes that consumer expectations are closely aligned with current inflation, to such extent that they are no longer informative. At the same time, the balance statistic tends to co-move quite closely with inflation, suggesting that it might be a good approximation of inflation expectations on its own, despite the way the question is asked. Several central banks, including the NBP and CNB, have begun to use the balance statistic in such fashion in recent years.

Therefore, we do not use standard methods of quantification, but decide to create our own method which simply rescales balance statistic into expected inflation. We assume no constant error in inflation expectations: mean expected inflation is equal to mean headline inflation. This assumption is not completely uncontroversial as it has been proven in literature that inflation perceived by consumers is higher than actual inflation (e.g. Hałka and Łyziak, 2015) and so are expectations. However, in such a case inflation would tend to go up and ultimately explode, at least according to a standard NK model. Thus, it seems safe to assume that this constant upward bias in inflation expectations has no influence on the price setting behaviour.

As for the standard deviation of inflation expectations, we investigate the surveys in which consumers are asked a quantitative question about expected inflation.<sup>5</sup> The standard deviation of expected inflation in these surveys turns out to be about 0.6 of the standard deviation of annual headline inflation (between 0.54 and 0.67, to be precise). Therefore, the standard deviation of inflation expectations is assumed to equal 0.6 of the standard deviation of headline inflation.

The resulting measures of inflation expectations are presented in Figure 3.



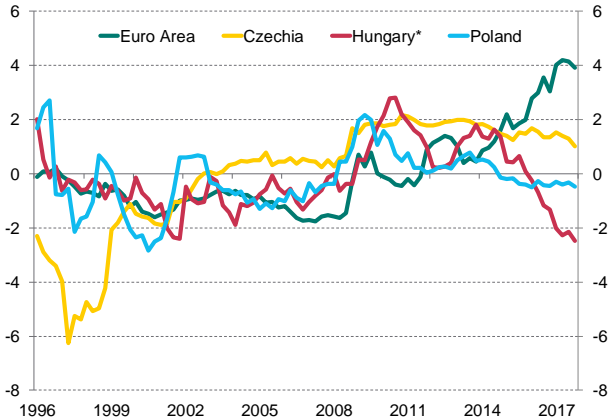
**Figure 3** Consumer inflation expectations and inflation (%)  
 For Czechia, core inflation in 2001Q1 (-14%) is eliminated from the sample as an outlier. For Poland, the data on consumer inflation expectations starts in 2001Q2.  
 Source: OECD data and own calculations based on Eurostat data.

### 5.2 Lending spread

The data on lending rates is comprised primarily from national sources (central bank websites) as the weighted mean interest rate on new (new business including renegotiations) bank loans

<sup>5</sup> Such surveys are/used to be conducted e.g. in the US, UK and Czechia.

(total loans including credit cards and overdrafts). Comprehensive and comparable data for the EU countries is in general available since 2004 (one or two years earlier for some countries). For Poland and Hungary, partial data for the earlier period is available from national sources – it is used after correcting for structural breaks. For Czechia and the euro area, the IMF data on lending rates from the International Financial Statistics Database is used, also after correcting for structural breaks.



**Figure 4** Deviation of lending spread from sample mean (pp)

\* For Hungary, deviation from linear trend

Source: Own calculations based on central bank and IMF data.

To obtain lending spread, nominal central bank interest rates (the same as the ones used in the computation of real interest rates) are subtracted from the interest rate on bank loans. In the estimation, the deviation of this spread from its sample mean is used. The exception is Hungary, where the lending spread increased substantially after the crisis, partially reflecting changes in the composition of lending (an increase in the share of overdrafts). Therefore, the deviation from linear trend is used for this country, which turns out to fit the model much better.

The data on the deviation of lending spread from the sample mean is shown in Figure 4. What stands out is exceptionally low spread in Czechia in 1997-98. This is related to the Czech currency crisis, which broke out in May 1997 (see Horvath, 1999 for more details). To combat the crisis, the central bank was forced to raise interest rates sharply (from 12.4% to 39% in early June 1997). Even though the situation stabilised pretty quickly and interest rates were cut, they remained at somewhat elevated levels throughout 1997 and 1998 due to the risks associated with Asian and Russian crises. According to the IMF data, lending rates did not increase to the same extent as central bank interest rates, resulting in lower spreads.



Such a low spread during a period of high market stress and low economic activity generates a positive correlation between the spread and output gap. As a result, lending spread is omitted from the IS curve in the specification for Czechia (see Table 1).

### 5.3 Capacity utilisation

Capacity utilisation data is often available only for the manufacturing sector. In the EU, the capacity utilisation survey is conducted also in the construction sector.<sup>6</sup> Still, these two sectors cover only 20-40% of the economy. Moreover, production in manufacturing and construction tends to be more volatile and more susceptible to external shocks than production in the rest of the economy, thus capacity utilisation in these two sectors is not necessarily representative for the economy as a whole.

Therefore, other survey data has been proposed and used to measure output gap. In particular, ECB (2015) proposes to take the data on the share of firms reporting insufficient demand as a factor limiting production since this time series is available for all the major sectors of the economy and comes from the European Commission survey, which implies uniform methodology across the EU countries. Using this data, they obtain sensible output gap estimates that are subject to much smaller ex-post revisions than alternative estimates, e.g. from the production function.

We follow the same approach. The data on the share of firms reporting insufficient demand as a factor limiting production from the European Commission survey is aggregated for the whole economy. For Poland we use national (GUS) data instead as in the EC data there is a large structural break in 2013. Before 2003, when the survey for services was not conducted, we use the data for industry and construction, after correcting for the structural break.

The (indirect) measure of output gap is computed as the deviation of survey data from linear trend, with an opposite sign (since low share of firms reporting insufficient demand is equivalent to high output gap). The deviation from trend is preferred over the deviation from mean as survey measures of output gap can be subject to trends. The most well-known

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<sup>6</sup> In Poland the alternative survey conducted by NBP covers also services, but this is an exception.

example is that of capacity utilisation in the US manufacturing that has been subject to a clear downward trend over the last 50 years. Indeed, in our data a similar trend can be noticed particularly for Poland.

## 6. Results

The estimates of model parameters are presented in Tables 3-5.

From the point of view of NRI estimation, the most important parameter is the slope of the IS curve ( $a_r$ ) that reflects the impact of the real rate gap on output gap. Only when this parameter is significant and relatively large, the model is able to estimate the real rate gap (and hence NRI) that drives given movements in output gap. Otherwise the filter sets the  $z_t$  term to zero and NRI is by assumption equal to potential growth which is estimated with the help of the business survey data.

Parameter	Euro Area	Czechia	Hungary	Poland
$\sum b_{\pi,i}$	0.939*** (0.000)	0.824*** (0.000)	0.946*** (0.000)	0.709*** (0.000)
$b_{\pi,e}$	0.061 (0.190)	0.176 (0.141)	0.054 (0.308)	0.291*** (0.000)
$b_{\pi,o}$	0.0016 (0.553)	-	0.049 (0.204)	-
$b_e$	-0.0129 (0.187)	-0.118** (0.018)	-0.130*** (0.000)	-0.054 (0.113)
$b_y$	0.027 (0.154)	0.134 (0.102)	0.103 (0.326)	0.048 (0.337)
$b_{y,f}$	-	-	-	0.062 (0.440)

**Table 3** The Phillips curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Overall, the model does quite a good job estimating  $a_r$ . For Poland and the euro area, the parameter is large and highly significant. For Czechia and Hungary, the model performs a bit worse, but the real rate gap still has an economically relevant effect on output gap. In the Holston et al. (2017) paper  $a_r$  is estimated to be smaller on average (0.01 to 0.07). In particular,

for the euro area the coefficient is about 6 times as small (0.036). This suggests that our model has more power to estimate NRI than the original Laubach-Williams model.

Parameter	Euro Area	Czechia	Hungary	Poland
$a_y$	0.847*** (0.000)	0.940*** (0.000)	0.910*** (0.000)	0.648*** (0.000)
$a_r$	-0.205** (0.022)	-0.069* (0.061)	-0.068 (0.161)	-0.266*** (0.000)
$a_e$	-0.040 (0.181)	-0.034 (0.268)	-0.063* (0.080)	-0.029 (0.375)
$a_f$	0.412*** (0.000)	0.687*** (0.000)	0.681** (0.014)	0.213 (0.483)
$a_l$	-0.220* (0.098)	-	-0.165 (0.264)	-0.425** (0.016)

**Table 4** The IS curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$c_g$	0.072 (0.713)	0.173 (0.381)	0.406 (0.150)	-
$c_y$	-0.048 (0.927)	0.766 (0.199)	-0.246 (0.658)	-1.308*** (0.007)
$c_r$	0.642 (0.237)	-	0.649 (0.177)	0.903 (0.130)

**Table 5** The exchange rate equation estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

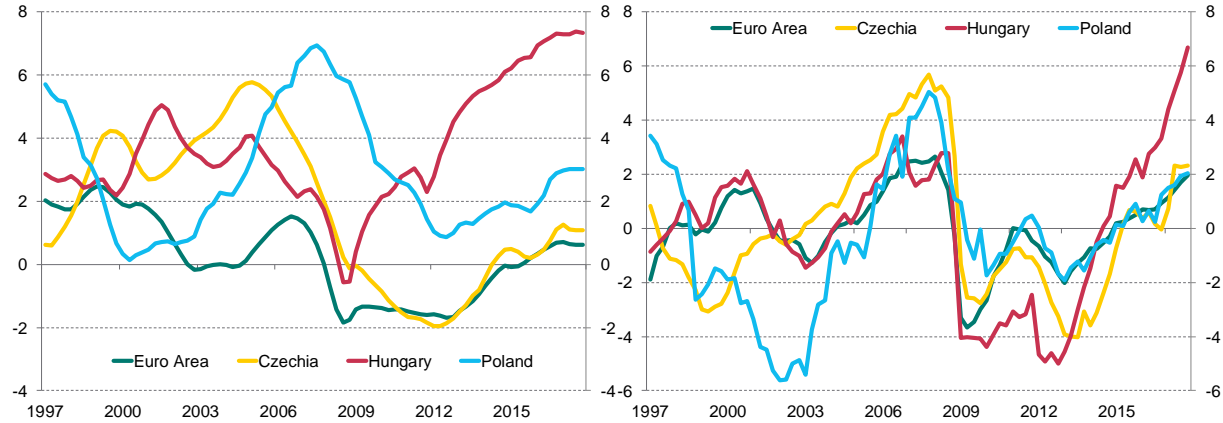
Looking at other coefficients, in the Phillips curve energy prices and foreign output gap are not significant, if they enter the equation at all. As mentioned above, in the robustness checks we investigate the model sensitivity to removing these variables. At the same time, the exchange rate is significant most of the time, suggesting it is an important addition to the model. The slope of the Phillips curve is positive and economically relevant, but not

significant, supporting the hypothesis that the Phillips curve relationship could have broken up in

years, as several studies have found.

In the IS curve, foreign output gap turns out to be very important for most of the analysed economies. Lending spread also seems to be an important addition, while the exchange rate is not statistically significant most of the time, but economically relevant. The exchange rate equation does not do a good job explaining exchange rate movements, which is not very surprising given the difficulties of modelling and forecasting exchange rates. The Balassa-Samuelson effect is to some extent present in Hungary and Czechia (though it is not statistically significant), while output gap is not such a good proxy for the exchange rate risk premium. Interest rate parity seems to work in all countries bar Czechia (though again, it is not statistically significant).

Turning to NRI, Figure 5 presents estimates for all countries, while Figure 7 presents decomposed estimates with 95% confidence bands for each economy separately. Here and in all the following graphs, two sided and smoothed estimates are reported.



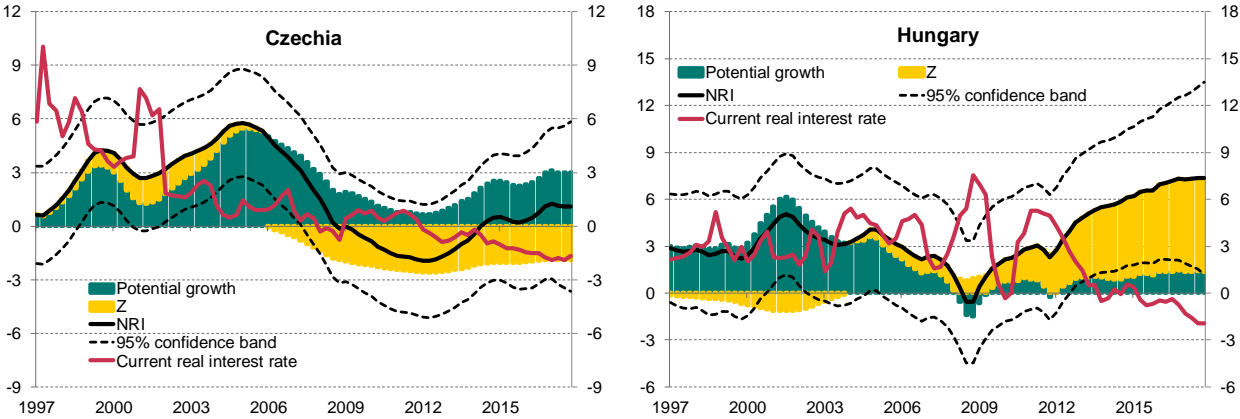
**Figure 5** Natural rate of interest – baseline estimates (%) **Figure 6** Output gaps (%)

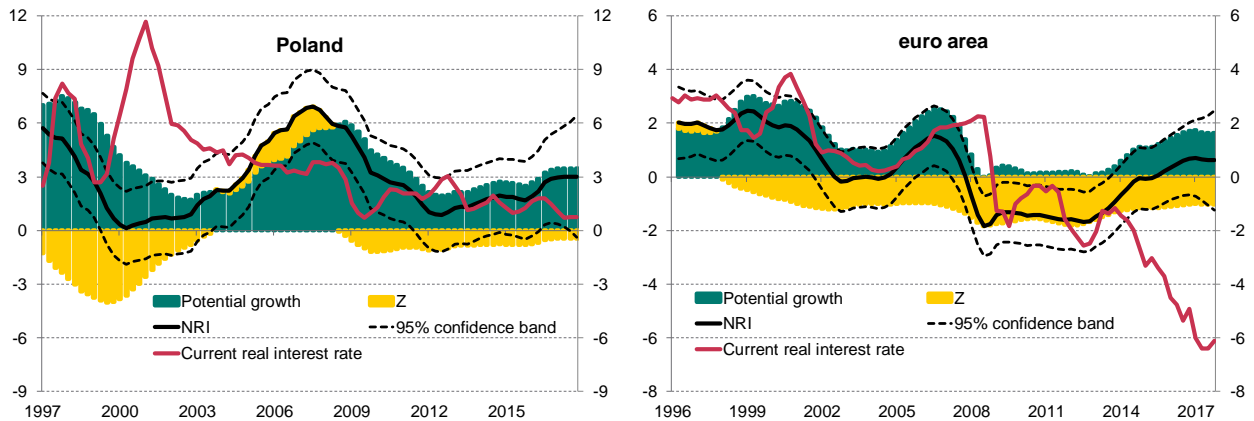
Looking at the point estimates at the end of 2017, the natural interest rate is positive in all analysed economies. In the euro area it stays at 0.6% and in Czechia it is only slightly higher at 1.1%. In Poland NRI is significantly higher (and has been so for the last decade), at 3%. Even higher – perhaps unrealistically – is the estimate for Hungary, amounting to 7.3%.

NRI has been far from stable during the analysed period in all the economies. In Poland and Czechia, it had been rising during the pre-crisis boom, only to fall dramatically during the crisis (6-8 pp from peak to trough). Over 2012-17, these losses have been partially made up for. In the euro area the natural rate behaved in a similar fashion as it somewhat rose before the crisis and dropped considerably afterwards, but recovered most of these losses after 2013. At the same time, Hungary is a clear outlier, as its NRI did not rise much during the pre-crisis boom and entered an upward trend already in 2009, staying at significantly higher levels than in other analysed economies after 2012.

Over the whole sample, the cross-country correlation in NRI is weak at best, except for the euro area – Czechia pair (correlation of 0.64; Table A.1 in the Appendix). However, over time some co-movement seems to be emerging, which is particularly visible since 2012 when the natural rate has been increasing in all the analysed countries, resulting in high cross-country correlations (Table A.2). This increasing co-movement in NRI is consistent with the growing integration of CEE economies into the European single market.

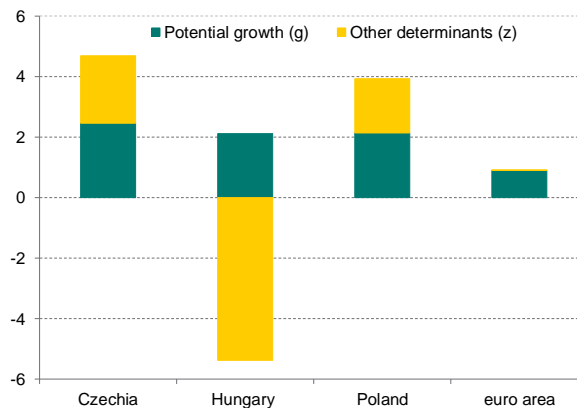
At the same time, NRI seems to be pro-cyclical. Correlation with output gap (Figure 6) is especially high for Poland (0.79), but it is also significantly positive for other economies, ranging from 0.32 (Hungary) to 0.45 (euro area).





**Figure 7** Decomposed NRI vs current real interest rate (%)

Looking at the decompositions, both potential growth and other factors ( $z_t$ ) play a significant role in NRI developments. In Poland and Czechia, both factors have a similar contribution to the variance in NRI and the natural rate being lower at the end of 2017 than in the pre-crisis peak (Figure 8). For the euro area, the importance of  $z_t$  is somewhat lower as it is responsible for 37% of fluctuations in NRI and at the end of the sample in 2017, it stays at a very similar level as in the pre-crisis peak. However, these other factors are a constant drag on NRI, bringing it significantly below the potential growth rate. To the contrary, for Hungary  $z_t$  is a dominant force, especially since the crisis when its increase more than offset a fall in potential growth. Over the whole sample, it is responsible for 58% of fluctuations in the natural rate.



**Figure 8** The drivers of a fall in the NRI from the pre-crisis peak (pp)\*

\* 2005Q1 for Hungary and Czechia, 2007Q3 for Poland and 2006Q3 for the euro area.

The precision of estimates remains unsatisfactory, but it is significantly better than for the original Laubach-Williams model as estimated by Holston et al. (2017). The standard errors range from 0.6 pp on average in the sample for the euro area to 2 pp for Hungary. The precision

of estimates for the euro area seems to be sufficiently high to inspire confidence and facilitate their use for policy purposes. It is also much higher than in the original Laubach-Williams framework – Holston et al. (2017) obtain standard errors for the euro area that are 7 times as large (4 pp).

The precision of estimates for CEE countries is lower, probably as a result of higher macroeconomic volatility and economic transition that had been taking place over a large part of the sample. Having said that, for Poland it is still relatively high with standard errors of 1 pp on average. However, the increase in the standard errors at the end of the sample makes it more difficult to draw any policy conclusions. For Czechia and Hungary, the estimates are so imprecise that relying on any point estimates becomes very risky. It should be noted, however, that the standard errors obtained for these countries are still significantly smaller than would be obtained using the original Laubach-Williams framework.

What does our estimates of NRI say about the monetary policy stance as of 2017 and earlier? Before looking into this, it should be noted that it is not optimal for real interest rate to follow the natural rate all the time. Since NRI is a medium-run concept, the actual real interest rate should drop below the natural rate during recessions and exceed it during booms.

Bearing this in mind, in all analysed economies monetary policy seems to be expansionary – especially so in the euro area and Hungary where the size of monetary expansion is staggering. In Czechia, monetary policy tends to be highly pro-cyclical: the authorities failed to sufficiently tighten policy before the crisis and loosen it after the crisis. A similar error seems to have been made after 2014 when the economic recovery has been supported by monetary policy despite the economy being close to or above potential. Similar comments could be made about Poland where monetary policy was too loose before the global financial crisis, too tight during the euro area crisis and again too loose in 2017. In general, in both countries monetary policy seems to be too passive, not reacting enough to cyclical fluctuations. In Hungary and the euro area, the real interest rate seems to have tracked the natural rate pretty well before the crisis (which was not sufficient as monetary policy should have been tightened more during the boom), but monetary policy failed to be sufficiently loosened during the crisis and was excessively loose after 2014.



As a check for the sensibility of obtained estimates, one might look at the output gap estimates that are crucial especially for the estimation of the  $z_t$  factor. For the euro area, Poland and Czechia they all seem very reasonable and do not deviate very substantially from the HP filter estimates (see Appendix, Figure A.1). They only tend to be slightly more volatile, pointing to deeper recessions and larger booms. They also suggest that as of 2017, output gaps were somewhat higher than implied by HP filter (by 0.8-1 pp). For the euro area and Czechia, the estimates additionally imply that before the crisis output gaps turned positive earlier.

For Hungary, HP and our estimates align well before the crisis, but diverge significantly afterwards. Our estimate suggests that output gap in Hungary was very high in 2017 and exceeded 6%, while the gap obtained from HP filter is just above 1%. While the HP filter estimate is likely to be biased downwards<sup>7</sup>, output gap above 6% seems highly unlikely as inflationary pressures remain muted with inflation relatively stable and below the 3% target. The overestimation of output gap would explain unrealistically large estimates of NRI – other things being equal, quickly rising output gap must be explained by a highly negative real rate gap, i.e. high natural interest rate.

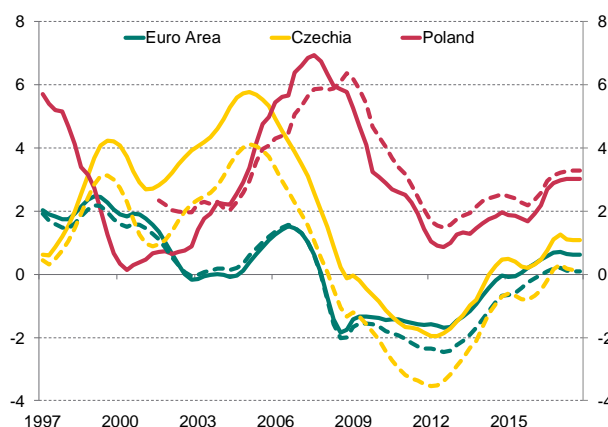
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<sup>7</sup> Among other factors, the record low unemployment of 3.8% would suggest so; OECD estimates output gap in Hungary to be 2.4% on average in 2017 compared to 0.8% obtained from HP filter and 5.5% from our estimates.

## 7. Robustness checks

### 7.1 Ex-ante real interest rates

As a first robustness check, we investigate what is the sensitivity of the results to the use of ex-ante real interest rates. Ex-ante real interest rates are computed as a difference between nominal interest rate and our measure of consumer inflation expectations. The results are shown in Figure 9.



**Figure 9** NRI estimates: baseline vs ex-ante real interest rate specification (%)

Solid line - baseline, dashed line – ex-ante real interest rate.

Hungary not included as with ex-ante real interest rate, the IS curve relationship between real rate gap and output gap breaks up.

Overall, the use of ex-ante rather than ex-post real interest does not materially influence the results. For the euro area, both estimates are very closely aligned before the crisis, while after the crisis, NRI from the ex-ante specification is about 0.6 pp lower on average. This results, however, solely from the fact that ex-ante real interest rate is about 0.6 pp lower than the ex-post rate during this period. Thus, policy recommendations obtained from the analysis of ex-ante rates are virtually the same as the ones obtained from the analysis of ex-post rates.

For Czechia, ex-ante NRI is consistently lower (on average by 1.3 pp) than the ex-post NRI. Again, this is related to the fact that ex-ante real rate is on average 1.3 pp lower than the ex-post rate (as headline inflation, and hence consumer inflation expectations, has been higher than core inflation). After accounting for this, the absolute difference between estimates is similarly low as for the euro area (0.34 pp on average). For Poland, the absolute difference between estimates is somewhat larger (0.72 pp on average). However, this largely stems from

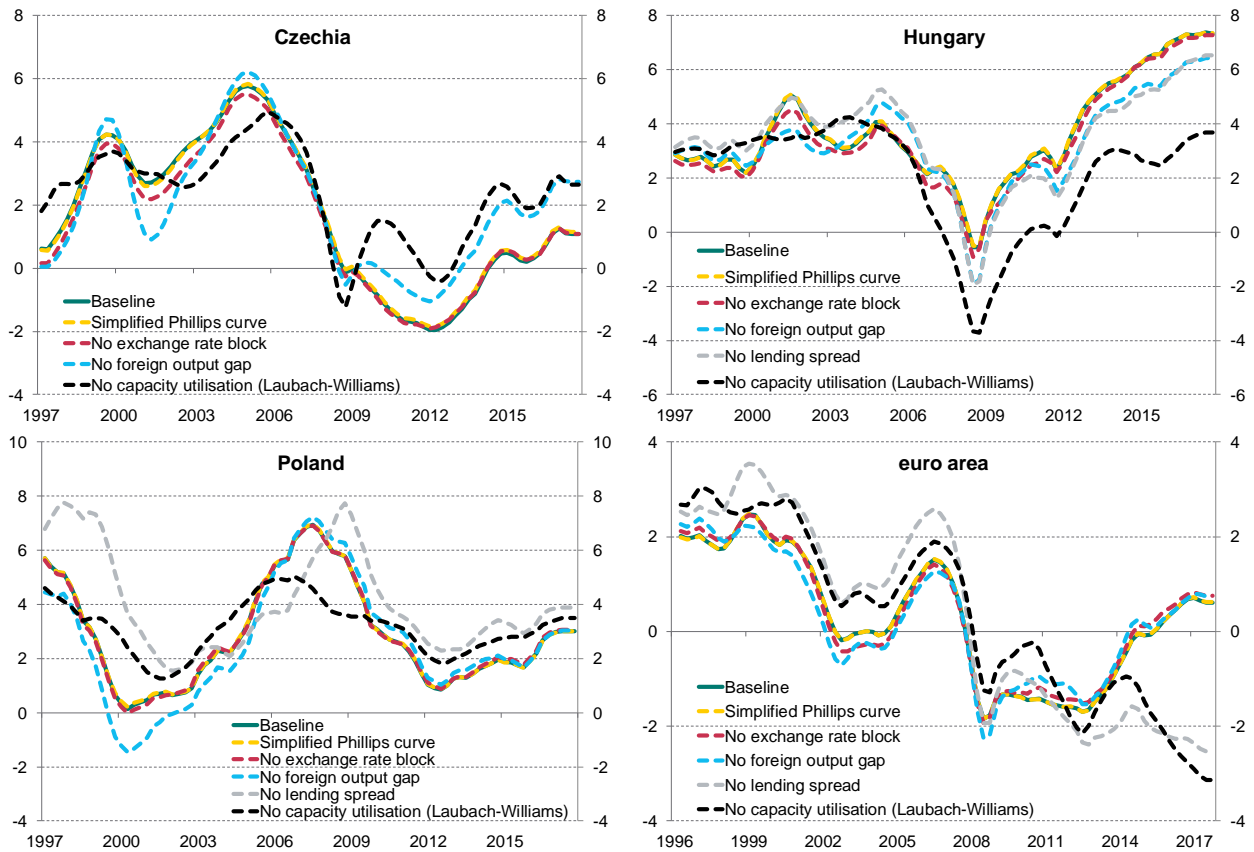
the difference in samples (data on inflation expectations starts only in 2001). When ex-ante NRI is compared with ex-post NRI estimated from the same sample, the mean absolute difference between estimates shrinks to only 0.25 pp. For Hungary, estimates are not reported since with ex-ante real interest rates, the relationship between real rate gap and output gap is found to be positive, discrediting the model.

Looking at the parameter estimates and overall performance of the models (See Appendix, Tables A.3-A.5), for the euro area and Czechia these are very similar across baseline and alternative specifications. Key parameters (the slopes of Phillips and IS curves) are only slightly less significant in the ex-ante specification, while point estimates and the overall model fit (as proxied by log likelihood) are virtually the same. For Poland, the slope of the IS curve turns insignificant, several other parameters also become less significant and the overall model fit deteriorates. However, this mostly stems from the difference in the sample size. When the alternative specification is compared with the baseline model estimated for the same, shorter, sample, the conclusions are the same as for Czechia and the euro area.

## **7.2 Exclusion of variables**

As a second robustness check, variables that have been added to the original Holston et al. (2017) specification are subsequently excluded, starting with the ones that are least significant. Variables are excluded cumulatively - every next specification excludes additional variables on the top of the ones from the previous specifications. The results are shown in Figure 10.

In the first specification ('simplified Phillips curve') three variables are excluded from the Phillips curve: foreign output gap, energy prices and inflation expectations. These variables do not seem to have much influence on inflation, despite theoretical rationale. The former two enter only some of the country specifications and even then, they are insignificant. The latter seems to matter more, though it is significant only for Poland. Moreover, the Phillips curve specification matters less for the estimation of NRI – the slope of IS curve and hence the whole specification of the IS curve is crucial in this regard. That is why Phillips curve is simplified at first.



**Figure 10** NRI estimates: baseline vs simplified specifications (%)

In the simplified specifications, variables are excluded cumulatively e.g. the “no exchange rate block” specification excludes the same variables as the “simplified Phillips curve” specification, plus the exchange rate variables.

Simplified Phillips curve: no energy prices, foreign output gap and inflation expectations in the Phillips curve; No exchange rate block: additionally no exchange rate in Phillips and IS curves and no exchange rate equation; No foreign output gap: additionally no foreign output gap in the IS curve; No lending spread: additionally no lending spread in the IS curve; No capacity utilisation: additionally no capacity utilisation equation (specification equivalent to Holston et al., 2017).

The exclusion of these variables has virtually no impact on NRI estimates – the absolute deviation at no point exceeds 0.1 pp. Looking at parameter estimates (Tables A.6-A.8 in the Appendix), the slope of the Phillips curve turns significant for the euro area and Poland. Other parameters are largely unaffected. The overall model fit deteriorates, but only slightly. Overall – the exclusion of these variables is recommendable.

In the second specification (‘no exchange rate block’) the exchange rate is excluded from Phillips and IS curves and the exchange rate equation is eliminated, on the top of previous exclusions. This potentially could affect the results somewhat – the exchange rate is mostly significant in the Phillips curve and while it is not significant in the IS curve, its effect on the output gap is economically relevant. Moreover, where output gap has some effect on the exchange rate (Czechia, Poland) the elimination of the exchange rate equation could potentially play a role.

The exclusion of the exchange rate turns out to have very little impact, however. The mean absolute deviations range from 0.07 (Poland) to 0.24 (Hungary). Looking at the parameter estimates (Table A.9), the IS curve is largely unaffected, while there are substantial differences in the Phillips curve – the parameter for the first lag of inflation increases by about 0.2 and the slope changes. The overall model fit deteriorates substantially. Despite this, having in mind that excluding the exchange rate significantly decreases the number of parameters to be estimated (by up to 6), the omission of the exchange rate seems recommendable.

The third specification ('no foreign output gap') excludes foreign output gap from the IS curve. This variable is highly significant for all countries bar Poland; therefore, it is expected to have an impact on NRI estimates. And this is indeed the case. Unsurprisingly, it has least impact on the euro area, with mean absolute deviation of 0.25 pp. For CEE countries, the deviation ranges between 0.63 and 0.8 pp. The largest role it plays in Czechia post-crisis, where the simplified model estimates NRI to be 1.3 pp higher on average than baseline.

Looking at the parameter estimates (Table A.10), the IS curve is affected, but perhaps surprisingly the omission of foreign output gap makes other parameters larger and more significant (except for Czechia). Thus, the model has even more power to estimate  $z_t$  and NRI. What might also be surprising, the overall model fit does not worsen much compared to the previous specification (except for Poland). Despite this goodness of fit, the omission of a significant and theoretically legitimate variable leads to omitted variable bias in the estimates of NRI. Hence, foreign output gap in the IS curve is a desirable addition to the model.

The fourth and penultimate specification ('no lending spread') excludes lending spread from the IS curve. This variable, which captures the difference between central bank and market interest rates, might explain the weakness of transmission of central bank's actions to the economy, and as such should have a significant impact on the slope of IS curve and hence the NRI estimates.

Our results confirm that this is indeed the case. Mean absolute deviation from baseline grows considerably for the euro area (from 0.25 to 1.04 pp) and Poland (from 0.63 to 1.65 pp), but less so for Hungary, where lending spread is least significant in the first place (from 0.67 to 0.82 pp). For Czechia, lending spread is not included in the baseline model. The most significant deviation (up to 3 pp) is visible for the euro area by the end of the sample. This stems from the

fact that in the baseline specification the effects of a decreasing shadow rate are offset by an increasing lending spread (as lending rates do not track the shadow rate). When this effect is not present, NRI is estimated to be significantly lower.

Looking at the parameter estimates (Table A.11), the coefficient on lagged output gap increases, while the slope of the IS curve decreases compared to the previous specification. The latter effect is particularly strong for Poland, where real rate gap from being very highly significant turns entirely insignificant (p-value 0.9). As a result, the  $z_t$  shows very little variation and the NRI estimate tracks potential growth very closely. Therefore, lending spread turns out to be a very important addition to the model.

Finally, the last specification ('no capacity utilisation') excludes the equation that links output gap to the business survey measure of capacity utilisation. This specification is therefore largely equivalent to the Holston et al. (2017) model. The resignation from the use of survey data makes output gap entirely unobservable and as such, it limits the model's power to estimate the  $z_t$  factor and NRI. Thus, NRI estimates are likely to be less accurate and more closely aligned with potential growth.

At the first glance, the results obtained from the Laubach-Williams model are comforting in a sense that they show similar trends as the baseline estimates. For the euro area, they are even quite closely aligned with each other, bar the last 3 years. However, mean absolute deviations are substantial, ranging from 0.93 pp (the euro area) to 1.85 pp (Hungary). At certain points in time the differences between estimates are really large – e.g. for Hungary since 2008 the L-W estimate is consistently below the baseline, by 3.1 pp on average. A significant deviation in this period can also be observed in Czechia, while in Poland the largest difference can be seen in 2007-08.

After looking at the parameter estimates (Table A.12), the picture becomes even more gloomy. Parameters tend to differ very substantially from baseline, even 10-fold. In Czechia and the euro area, the IS curve steepens somewhat, generating some movement in the estimates of the  $z_t$  factor. The Phillips curve, which now plays a much larger part in the NRI estimation,<sup>8</sup> is virtually flat, however. As a result, output gap estimates are highly imprecise. To the contrary,

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<sup>8</sup> Without survey data, the Phillips curve pins down output gap, which by influencing potential growth estimates has an impact on NRI.

for Hungary and Poland the IS curve is virtually flat, while the Phillips curve becomes very steep. As a result, the  $z_t$  factor does not fluctuate and is close to zero. Thus, NRI is very closely aligned with potential growth.

It seems that the filter estimates output gap either from the real rate gap (euro area, Czechia) or the Phillips curve (Hungary, Poland), being unable to reconcile information from these two sources. As an additional check, one might look at the output gap estimates (Appendix, Figure A.1)<sup>9</sup>. For the euro area and Czechia, estimates as of 2017 are unrealistically high (6-7%). Such high output gap seems to be nigh impossible particularly in the euro area, given heightened levels of unemployment in several of its member countries. For Poland and Hungary, output gap estimates are closely aligned with the HP filter estimates. This is unsurprising given that the IS curves for these countries are virtually flat.

Thus, while certainly something can be made from the Laubach-Williams results, their reliability seems to be very low given suspicious parameter and output gap estimates.

Specification	Euro Area	Czechia	Hungary	Poland
Baseline	0.58	1.59	2.04	1.05
Simplified Phillips curve	0.58	1.64	2.05	1.04
No exchange rate block	0.56	1.70	2.05	1.06
No foreign output gap	0.49	2.57	1.73	1.09
No lending spread	0.71	-	1.86	4.85
Laubach-Williams	1.91	3.02	3.88	4.15

**Table 6** Mean NRI standard errors (pp)

The conclusions from the above-conducted analysis do not change when one looks at the accuracy of NRI estimates (Table 6). Standard errors do not change significantly after the exclusion of energy prices, inflation expectations, foreign output gap (from the Phillips curve) and the exchange rate. The exclusion of foreign output gap and lending spread from the IS curve has a differing impact – while in Poland standard errors explode, in Hungary they even decrease. Following the resignation from the capacity utilisation data, standard errors are significantly larger (two to four times) than in the baseline specification for all analysed

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<sup>9</sup> In the previous robustness check specifications output gaps are virtually the same as in the baseline specification, hence they are not reported.

economies, however. Therefore, the augmented model performs considerably better in terms of ex-ante accuracy of NRI estimates than the original Laubach-Williams model.

All in all, it seems that our model can be simplified by excluding energy prices, inflation expectations, the exchange rate and foreign output gap (the latter only from the Phillips curve) without influencing NRI estimates much. This decreases the number of parameters to be estimated by up to 9, which can be beneficial especially in samples that are even smaller than ours. At the same time, foreign output gap in the IS curve, lending spread and especially the use of business survey data to pin down output gap seem to be very useful – and desirable – additions to the model that significantly improve the performance of the Laubach-Williams framework.

### 7.3 Alternative NRI specification

As a third robustness check, we make amendments to the specification of the NRI equation. The open economy New Keynesian model analysed in section 3 suggests that potential growth abroad should be explicitly included in the NRI equation. On the other hand, several studies that investigate the behaviour of real interest rates in long samples cast doubt at whether potential growth should be regarded as the main determinant of the natural rate (Hamilton et al., 2016; Borio et al., 2017). This calls for an even more agnostic specification of the NRI equation.

Hence, we estimate three alternative versions of the model with the following NRI equations:

$$r_t^* = d_d g_t + d_f g_t^f + z_t$$

$$r_t^* = d_c g_t + (1 - d_c) g_t^f + z_t$$

$$r_t^* = z_t$$

Where  $d_d$ ,  $d_f$  and  $d_c$  are model parameters, while  $g_t$  and  $z_t$  continue to be defined as random walk processes.

The first equation is an unconstrained version of the equation implied by our version of the open economy NK model (see section 3.5). With standard parameterisation, the sum of coefficients in front of potential growth rates is close to 1. That is why in the second equation



the sum of parameters is constrained to be 1. Finally, the third equation is a fully agnostic one in which NRI is simply a random walk with no explicit drivers included.

Before analysing NRI estimates, let us look at the estimates of the NRI equation parameters. These are presented in Table 7.

Parameter	Euro Area	Czechia	Hungary	Poland
$d_d$	1.006 (0.156)	-0.245 (0.656)	-1.395 (0.122)	0.253 (0.524)
$d_f$	0.212 (0.611)	2.557*** (0.001)	3.225 (0.116)	1.002 (0.371)
$d_c$	0.752* (0.065)	-0.837 (0.177)	-1.335 (0.124)	0.283 (0.456)

**Table 7** The estimation of alternative specifications of the NRI equation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Several observations can be made. Firstly, the estimates are not in line with the theory (except for the euro area). For Czechia and Hungary, domestic potential growth is even negatively correlated with NRI and in the unconstrained specification, the sum of parameters is considerably larger than 1 (though for Hungary this difference is not statistically significant due to very high standard errors). Secondly, for CEE countries potential growth abroad is a more important driver of NRI than domestic growth. This corroborates the findings of Hamilton et al. (2016) and Borio et al. (2017) that domestic potential growth does not seem to be such an important driver of NRI as suggested by the theory. Thirdly, estimates are imprecise, which is in line with what Laubach and Williams (2003) find for their version of the model.

NRI estimates are shown in Figure 11.

In specifications where foreign potential growth is explicitly included and domestic growth is estimated to have a negative impact on NRI (Czechia and Hungary), NRI estimates are more volatile (standard deviation higher by 20-79%) and at times diverge substantially from baseline (1.3-2.4 pp on average, but 6.3 pp at the peak in 2001 for Hungary). Where parameter estimates seem to be more sensible (euro area and Poland), NRI estimates are either very close to the

baseline (euro area) or less volatile (Poland). In the agnostic specification, the model falls apart for Czechia as the IS curve becomes flat. For other economies, NRI estimates are less volatile (standard deviation lower by 13-39%), but overall show similar trends and do not differ terribly from the baseline results (mean absolute deviation less than 1.1 pp). For Hungary and Poland, the model fit also improves, as evidenced by log likelihood and Akaike criterion.

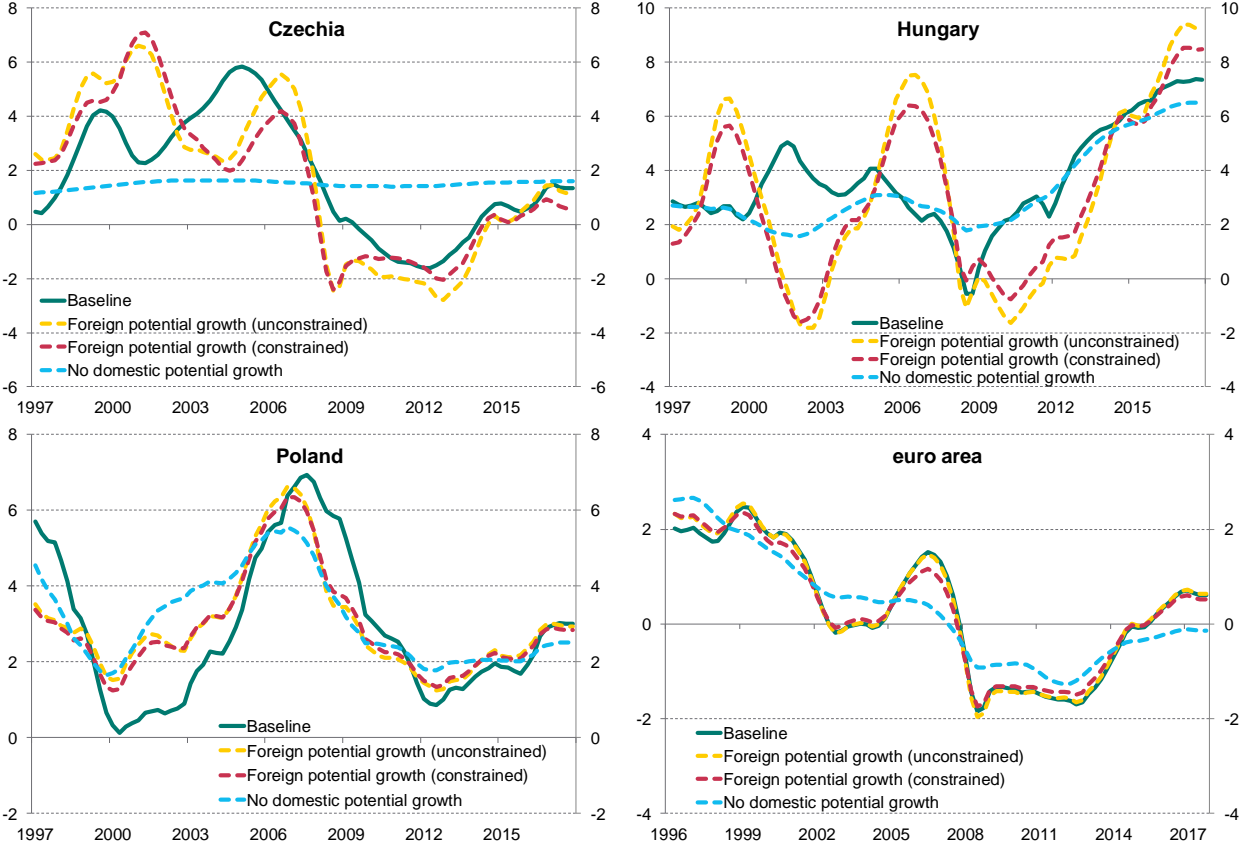


Figure 11 NRI estimates: baseline vs alternative NRI specifications (%)

Overall, the experiments with the NRI specification seem to confirm that domestic potential growth should not be regarded as the key determinant of the natural rate. When foreign potential growth is added, it is estimated to be a more important driver of NRI than domestic growth. However, parameter estimates are imprecise and seem unrealistic, and NRI estimates become even more volatile. On the other hand, when domestic growth is excluded from the specification, NRI estimates are not very different from the baseline estimates, but become less volatile; at the same time, the model fit improves in some cases. Therefore, the agnostic specification might be preferred over baseline when the model has enough power to estimate  $z_t$  (countries other than Czechia in our case).

## 7.4 Ex-post revisions

As a final robustness check, the sensitivity of the baseline estimates to ex-post revisions is investigated. The estimates of the seminal Laubach-Williams model are known to be subject to significant revisions – in particular, just before the global financial crisis NRI was estimated to be about 1 pp higher than the current estimate suggests.<sup>10</sup> In order to investigate whether our model encounters similar problems, we estimate it for an extending sample, the first sample ending in 2006Q1. Importantly, current data is used in these estimations.<sup>11</sup> While the results obtained in this way are not entirely “real-time”, they are not contaminated by the (possibly differing across time and countries) ability of statistical offices to estimate GDP in real-time. Hence, the effect of changing samples on NRI estimates is singled out – ultimately, this is what we are interested in.

Let us first analyse the sensitivity of parameter estimates to ex-post revisions. Table 8 presents the standard deviation of parameter estimates as a percentage of the baseline estimate for chosen parameters, while Figure 12 shows the estimates of the most important parameter – the slope of the IS curve. Other parameter estimates are presented in the Appendix (Figures A.2-A.10).

Parameter	Euro Area	Czechia	Hungary	Poland
$b_y$	80.2	69.4	71.4	105.6
$a_r$	24.3	49.5	33.4	8.0
$a_f$	29.2	27.6	24.3	130.3
$a_l$	89.6	-	37.9	15.6
$a_c$	11.2	33.3	41.7	12.4

**Table 8** Standard deviations of parameter estimates as a percentage of the baseline estimate

$b_y$  – slope of the Phillips curve,  $a_r$  – slope of the IS curve,  $a_f$  – elasticity of domestic output gap to foreign output gap,  $a_l$  – elasticity of domestic output gap to lending spread,  $a_c$  – elasticity of the capacity utilisation measure to domestic output gap

Parameter estimates turn out to be highly sensitive to the chosen sample, and hence they are subject to substantial ex-post revisions. The standard deviation of estimated parameters is on average equal to about 50% of the final (baseline) estimate. While this statistic differs

<sup>10</sup> The sensitivity of Laubach-Williams estimates to ex-post revisions can be easily investigated since their estimates are regularly updated and published on the San Francisco Fed website: [https://www.frbsf.org/economic-research/economists/john-williams/Laubach\\_Williams\\_real\\_time\\_estimates.xlsx](https://www.frbsf.org/economic-research/economists/john-williams/Laubach_Williams_real_time_estimates.xlsx)

<sup>11</sup> However, deviations from linear trend or mean for business survey and lending spread data are recalculated for given samples.

significantly across countries and parameters, on average it is not any lower for the parameters that have the highest impact on NRI estimates (these are listed in Table 8 based on the results obtained in the previous subsection). Importantly, parameter estimates not only vary substantially over time, but also tend to flip signs quite often. This happens to 22% (16 out of 73) of parameters – most often for Hungary and the euro area (6 parameters each). In Hungary, even the slope of the IS curve changes sign – it is virtually flat for the samples ending up to 2014 and becomes negative only afterwards. A very important coefficient in front of the lending spread also changes sign for both Hungary and the euro area. Unsurprisingly, estimates vary especially pre-crisis vs post-crisis – this is not the only source of variability, however.

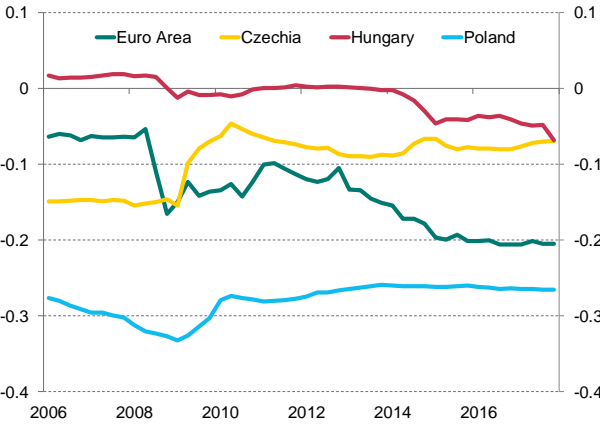
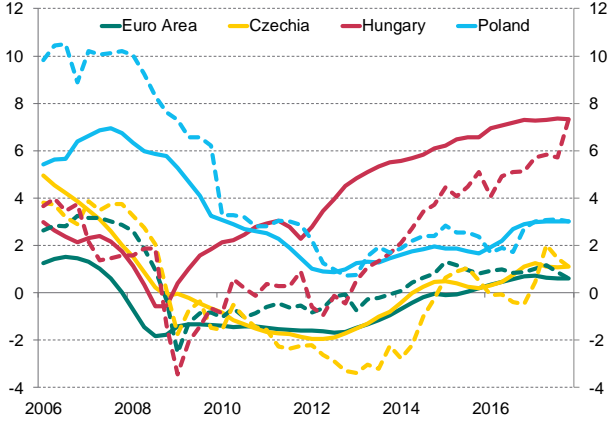


Figure 12 Real-time estimates of the slope of the IS curve

Varying parameters lead to significant ex-post revisions of NRI estimates (Figure 13). On average these revisions range from 0.9 pp in Czechia to 2.3 pp in Hungary, but around the global financial crisis and the euro area crisis they tend to be larger, reaching up to 5 pp in Hungary and Poland. Having said that, for Czechia, Poland and the euro area revisions most (about 60%) of the time do not exceed 1 pp. Estimates are quite precise especially after the crisis (post-2009) in Poland with mean revisions of less than 0.5 pp, which is probably a reflection of Poland having the most stable slope of the IS curve (Table 8). In Hungary, real-time estimates are consistently below the baseline after the crisis (2.7 pp on average), which is probably a reflection of the IS curve being virtually flat in most of the real-time samples.

Potentially, ex-post revisions could also stem from differences in input data – deviations from linear trend/mean for the capacity utilisation and lending spread data are recalculated for each sample. Especially the former could have an impact on NRI by influencing potential growth

estimates via output gap estimates. In order to investigate whether this effect plays a role we estimate real-time NRI and compute ex-post revisions with input data held constant, i.e. the survey data is detrended for the full sample. Such revisions should result only from changing parameter estimates.



**Figure 13** NRI estimates: baseline vs real-time estimates (%)

Solid line - baseline, dashed line – real-time estimates.

For Czechia and Hungary, these revisions turn out to be even (substantially) larger than in the benchmark case (1.8 pp on average vs 0.9 pp for Czechia and 3.7 pp vs 2.3 pp for Hungary; Figure A.12). Therefore, recomputing survey data in real time tends to mitigate the effects of shifts in parameter estimates. For Poland and the euro area, the revisions decrease somewhat (0.9 pp vs 1.4 pp for Poland and 0.9 pp vs 1.1 pp for the euro area; Figure A.11). Differences in input data are thus responsible for 20-33% of total ex-post revisions.

In terms of ex-post revisions, our model fares worse than the simpler Laubach-Williams model does for the US. This probably stems from their sample being significantly larger (it starts in 1960) and hence their parameter estimates being much more stable.<sup>12</sup> Their potential growth estimates are also significantly less volatile (standard deviation/mean ratio of 11% compared to 44-81% for our estimates) and thus less prone to revisions.

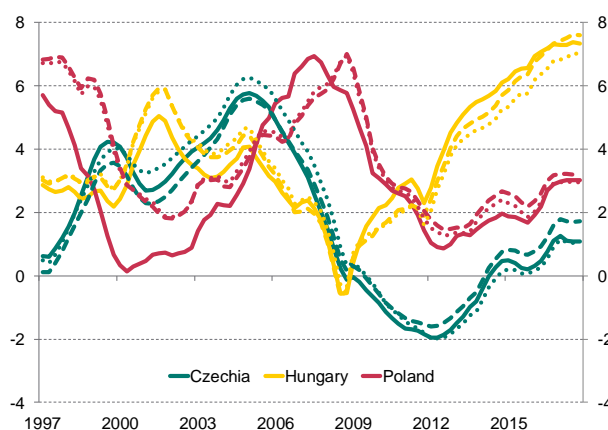
<sup>12</sup> Laubach and Williams do not publish their real-time parameter estimates, hence this hypothesis cannot be tested.

## 7.5 Panel estimation

Instead of estimating the model separately for each country, one can assume that coefficients are the same across countries and estimate the model simultaneously, taking advantage of a larger, pooled sample. This can potentially lead to more precise and stable estimates – if cross-country differences in parameters are relatively small.

We estimate the model for a panel of CEE countries, using the previously obtained estimates for the euro area. Simultaneous estimation for all 4 economies is not feasible since contemporaneous output gap in the euro area enters the output gap equation for CEE countries, which is not allowed in a simultaneously estimated state space model.<sup>13</sup> Coefficients are fixed across countries, but variances of shocks are allowed to differ. Initial values and variances as well as relations between variances of shocks are set in the same way as in the baseline specification.

We estimate two panel specifications – one as presented in section 4 and one adjusted to optimise the model fit.<sup>14</sup> The estimation results are shown in Tables A.13-A.15 of the Appendix. NRI estimates are shown in Figure 14.



**Figure 14** NRI estimates: baseline vs panel estimates (%)

Solid line - baseline, dashed line – baseline panel estimates,  
dotted line – optimal fit panel estimate.

Most of the time, parameter estimates understandably fall somewhere in between the individual country estimates, while their statistical significance is somewhat higher than

<sup>13</sup> Contemporaneous state variables cannot enter the state equations.

<sup>14</sup> Energy prices and foreign output gap are excluded from the Phillips curve and the 2nd lag of real interest rate is excluded from the IS curve; 1st lag of domestic output gap is used in the Phillips curve and 1st lag of real exchange rate is used in the IS curve; lending spread adjusted for linear trend is used for Czechia.

country average – the latter can be best seen when looking at the slope of the Phillips curve, which becomes significant while all the country estimates are not. There are important exceptions to this ‘rule’, however - the slope of the IS curve and the impact of lending spread are smaller than in individual estimations. This results from the inclusion of Czech lending spread, which is positively correlated with output gap (see subsection 5.2) and thus excluded from the baseline specification for Czechia. Its inclusion not only drives the lending spread parameter towards zero, but also flattens the Czech IS curve, driving the panel estimate of the IS curve slope towards zero.

Despite these intricacies, NRI estimates do not diverge substantially from baseline for Czechia and Hungary - the absolute deviation is 0.3-0.6 pp on average, never exceeding 1 pp (Figure 14). The deviations are relatively small because panel estimates of the IS curve slope – which is a crucial parameter in the estimation of NRI – come quite close to the individual estimates for these countries (minus 0.55-0.67 for panel against minus 0.68-0.69 individually).

For Poland, however, the deviations from baseline are quite large – about 1 pp on average, exceeding 3 pp in 1999. Radically different estimate of the IS curve slope is guilty here – the panel estimate of the slope is about 4 times as low as the individual estimate. This plays a role especially during the disinflation period of late 1990s/early 2000s, when real interest rates were high, putting a large drag on economic activity. Knowing the size of this drag, the slope of the IS curve decides how far the equilibrium real interest rate was from the actual rate. Thus, large deviations in the IS curve slope during this period cause large deviations in the estimate of NRI.

Overall, differences across analysed economies seem to be too large to successfully use panel estimation. The model estimates for Poland clearly differ from the ones for Czechia and Hungary, especially when it comes to the crucial estimate of the IS curve slope, which weighs on the NRI estimate. At first glance, one might be more optimistic about pooling Czechia and Hungary together – however, NRI estimates for these countries are similar to baseline because Poland is also in the sample. Were it not for Poland, the above-mentioned problems with Czech lending spread would drive the IS curve slope further towards zero, resulting in larger discrepancies against baseline estimates.





## 8. Implications and limitations

The results presented in this paper have important implications for the conduct of monetary policy. The natural rate not only varies quite substantially over time, which requires central banks to closely monitor its developments, but it is also pro-cyclical. In practice this means that central banks should more strongly react to economic fluctuations, which influence optimal policy not only via movements in output gap, but also the natural rate.

Given high ex-ante and ex-post errors in NRI estimation, optimal policy may be proxied by a Taylor rule with constant NRI and higher than usual sensitivity to output gap. This is illustrated in Figure 16 for Poland, which shows real interest rates according to different specifications of the Taylor rule. Baseline (yellow line) is the Taylor rule with classic parameters:

$$r_t = r_t^* + 0.5\tilde{y}_t + 0.5(\pi_t - \bar{\pi}_t)$$

Where  $\bar{\pi}_t$  is the inflation target. Red line is the Taylor rule with constant NRI, equal to the sample mean ( $\bar{r}^*$ ):

$$r_t = \bar{r}^* + 0.5\tilde{y}_t + 0.5(\pi_t - \bar{\pi}_t)$$

And the blue line shows the Taylor rule with constant NRI and sensitivity to output gap ( $\beta$ ) fitted to match the baseline rule:

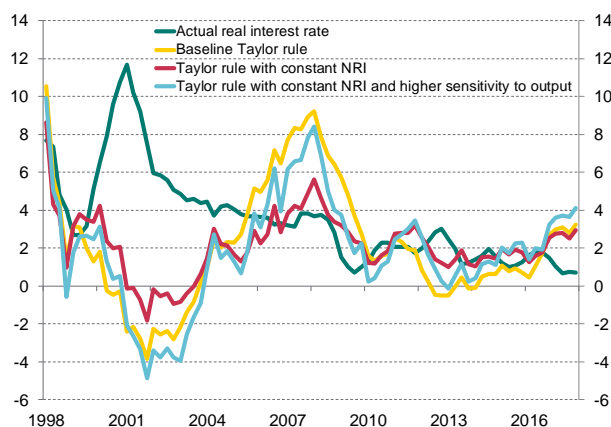
$$r_t = \bar{r}^* + \beta\tilde{y}_t + 0.5(\pi_t - \bar{\pi}_t)$$

The estimated  $\beta$  is 1.08.

Assuming that the baseline Taylor rule describes optimal monetary policy, not accounting for movements in the natural rate might result in significant policy errors. Constant NRI Taylor rule deviates (in absolute terms) by on average 1.5 pp from baseline, but the deviation exceeds 4 pp during the pre-crisis boom and 2 pp during the disinflation-dotcom crisis. Taylor rule with higher sensitivity to output does a significantly better job – the average error drops to 1 pp and extreme cases are much more limited as the error never exceeds 2.5 pp.

Indeed, it seems that Polish monetary policy has quite closely followed the constant NRI Taylor rule since 2006 (average absolute deviation of 0.9 pp, maximum 2.2 pp), which –

assuming classic Taylor rule describes optimal policy – resulted in significant policy errors (2 pp on average), especially before the crisis (up to a staggering 5.5 pp!).



**Figure 16** Real interest rates in Poland: actual vs Taylor rules (%)

Baseline Taylor rule specification:  $r_t = r_t^* + 0.5\tilde{y}_t + 0.5(\pi_t - \bar{\pi}_t)$ , where  $\bar{\pi}_t$  is inflation target. Constant NRI = sample mean of NRI. Higher sensitivity to output: 1.08 (estimated as the best fit of the constant NRI Taylor rule to the variable NRI Taylor rule).

The results have important implications also for the monetary policy stance as of 2017. While the natural interest rate is lower than in the past, its decline is not large enough to justify the continuation of loose or even ultra-loose monetary policy when economic conditions are improving and inflation is rising, as it was the case in 2017. As a result, assuming standard Taylor rules (the baseline specification above) monetary policy is found to have been overly expansionary at the end of 2017 in all analysed economies – the ‘error’ is lowest in Poland (interest rates too low by 2.5 pp), followed by Czechia (3.9 pp), the euro area (7 pp due to aggressive unconventional policies resulting in highly negative shadow rates) and Hungary (11.8 pp, though this result should be taken with more than a pinch of salt, as discussed below).

There are many caveats to the results obtained in this paper, however. Firstly, the NRI estimates are heavily dependent on survey data used in the estimation process – the capacity utilisation data in particular, but also the lending spread data. In CEE countries the survey data time series are usually not that long and even if they are available, these economies were undergoing substantial structural changes at the beginning of the sample, which puts the reliability of the data into question. This is especially the case for Hungary, which probably partially explains the dubious character of NRI and output gap estimates for this country.

Another reason why NRI in Hungary is probably overestimated is not accounting for the majority of unconventional monetary policies conducted by the Hungarian Central Bank (MNB) since 2013. Over this period, MNB has launched several – often very original - unconventional monetary policy programmes, such as the Funding for Growth Scheme providing cheap credit to small and medium enterprises (Bokor et al., 2016) or the Self-financing Programme incentivising banks to purchase government bonds (Hoffman and Kolozsi, 2016). As there are no estimates of the shadow rate for Hungary, interbank market rate is used to compute the real interest rate (see section 5) – as a result, these unconventional policies are not taken into account. The actual monetary policy stance is most likely (much) more expansionary (the shadow rate is lower), which – assuming that our model estimates the difference between the current and the natural rate well – implies that NRI is lower.

Secondly, while our NRI estimates are more precise and more robust than those obtained from the standard Laubach-Williams framework, they remain vulnerable to sample choice and prone to significant ex-post revisions due to shifts in parameter estimates and changes in detrended survey data.

Further research into methods of NRI estimation and its developments should focus on improving robustness to sample changes and ex-post revisions. In particular, models with time-varying parameters can be used to account for apparent structural shifts in the relationships between interest rates, economic activity and inflation. Applicability to countries with fewer observations and less reliable data should also be studied and further improved.

There is still a lot to do in the study of determinants of NRI, including the cross-country interrelations. Previous studies have utilised long time series and trend real interest rates, which creates problems due to structural changes and significant and persistent deviations of real rates from the natural level (e.g. during disinflation periods). Ideally, shorter (but not too short) samples and estimates of NRI for a wide range of countries should be used. This, however, requires the development of NRI estimation framework which could be relatively easily applied to a large number of countries. This paper is a first step in this direction, but only a first step.

## **Acknowledgements**

The views expressed in this paper belong to the author only, and have not been endorsed by SGH. I am grateful to Andras Balatoni and an anonymous NBP Working Papers referee for very useful comments and suggestions.

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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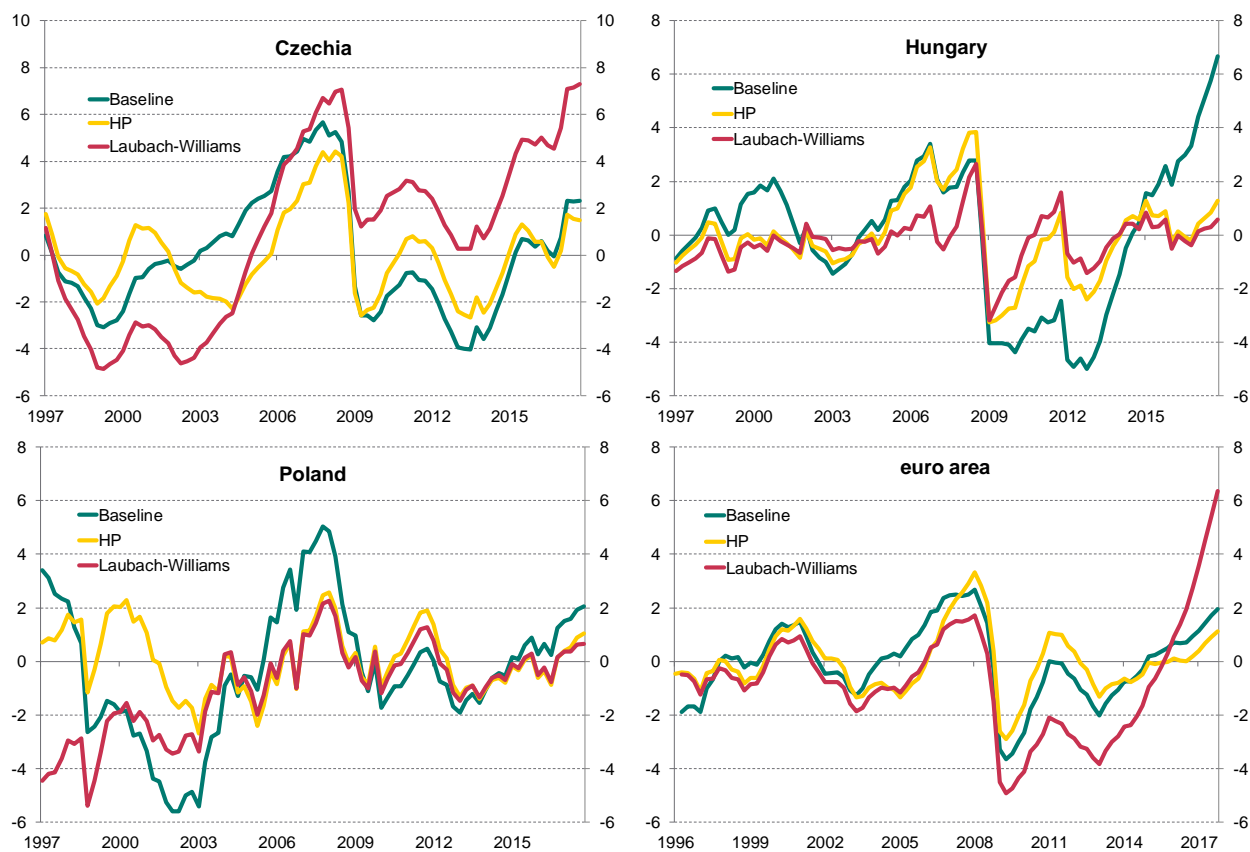
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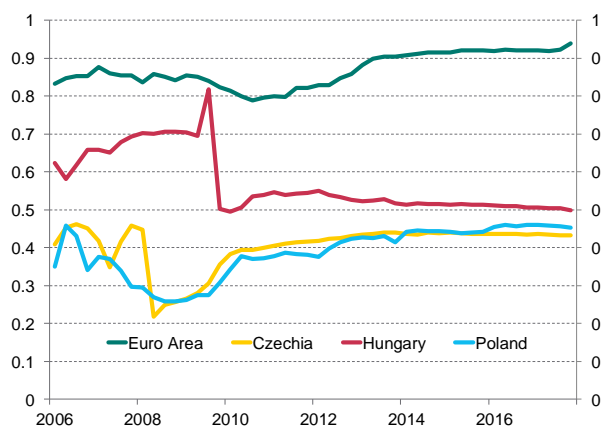
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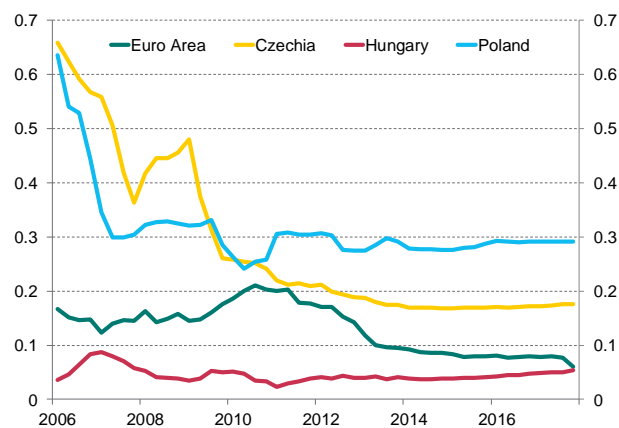
# Appendix



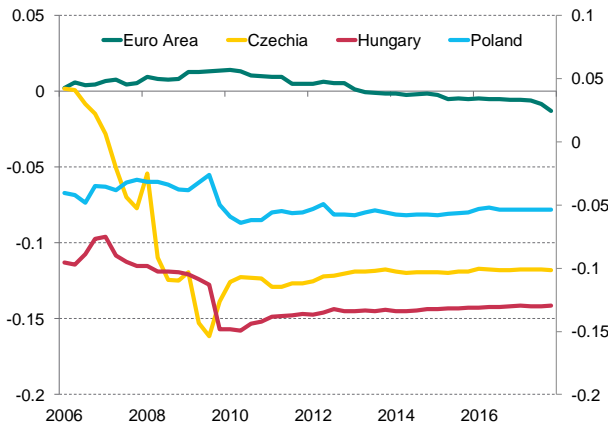
**Figure A.1** Output gap estimates (%)



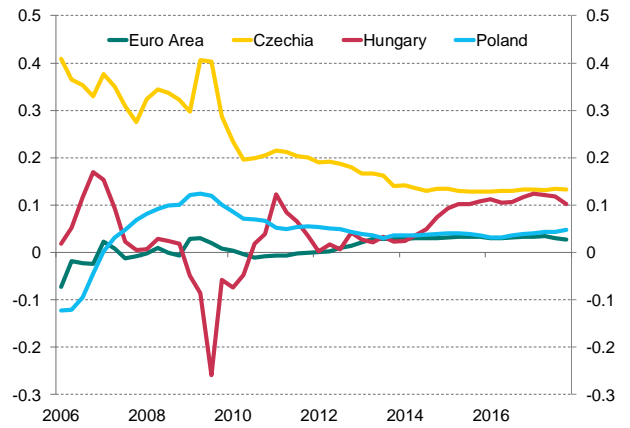
**Figure A.2** Real-time estimates of the coefficient in front of the 1<sup>st</sup> lag of inflation in the Phillips curve



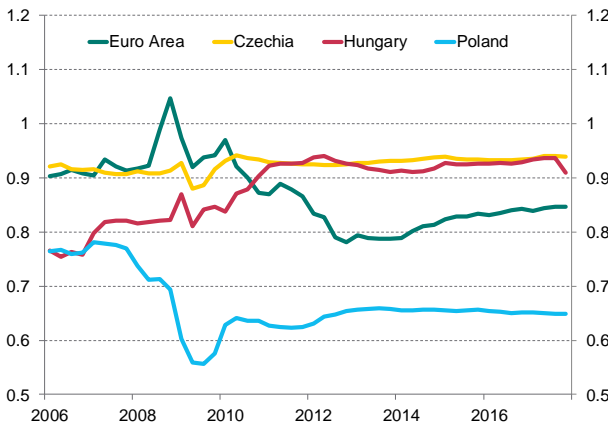
**Figure A.3** Real-time estimates of the coefficient in front of inflation expectations in the Phillips curve



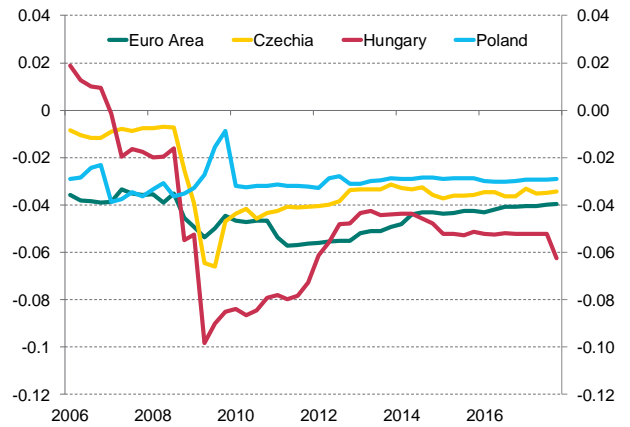
**Figure A.4** Real-time estimates of the coefficient in front of the exchange rate in the Phillips curve



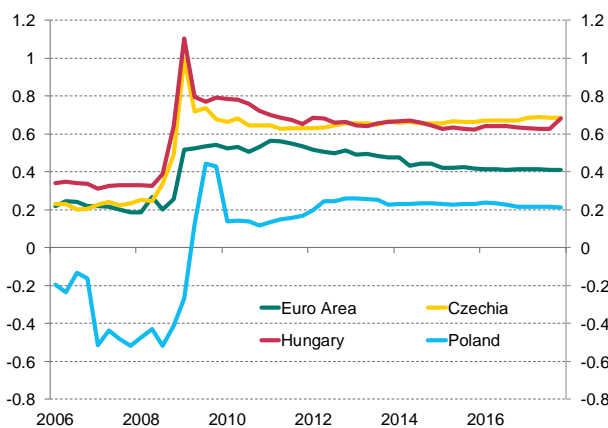
**Figure A.5** Real-time estimates of the slope of the Phillips curve



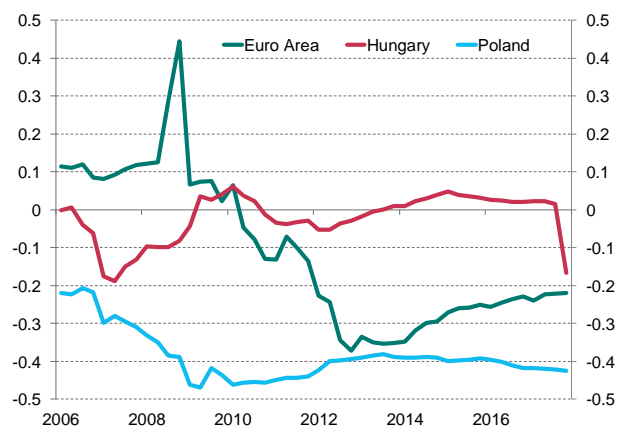
**Figure A.6** Real-time estimates of the coefficient in front of the 1<sup>st</sup> lag of output gap in the IS curve



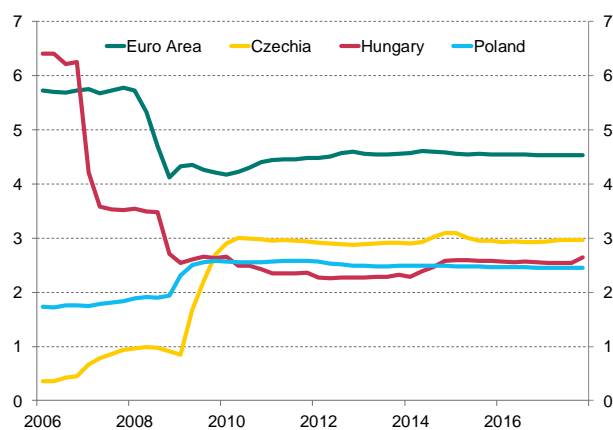
**Figure A.7** Real-time estimates of the coefficient in front of the exchange rate in the IS curve



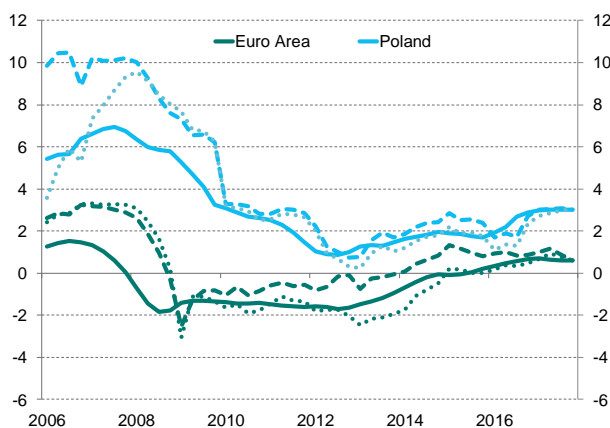
**Figure A.8** Real-time estimates of the coefficient in front of foreign output gap in the IS curve



**Figure A.9** Real-time estimates of the coefficient in front of the lending spread in the IS curve

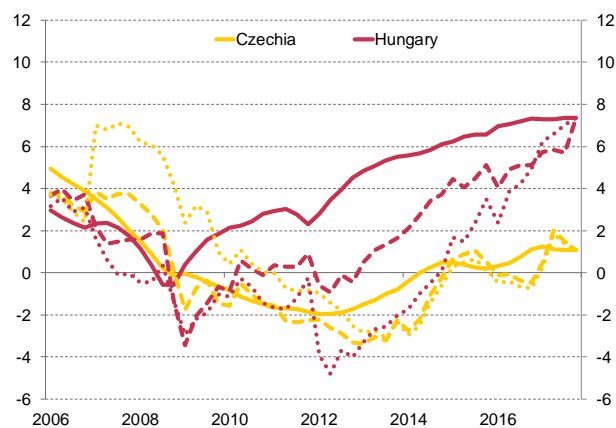


**Figure A.10** Real-time estimates of the slope of the capacity utilisation equation



**Figure A.11** NRI estimates: baseline vs real-time estimates in the euro area and Poland (%)

Solid line - baseline, dashed line – real-time estimates, dotted line – real-time estimates with input data held constant.



**Figure A.12** NRI estimates: baseline vs real-time estimates in Czechia and Hungary (%)

Solid line - baseline, dashed line – real-time estimates, dotted line – real-time estimates with input data held constant.

	euro area	Czechia	Hungary	Poland
euro area	1.00	0.64	0.09	0.10
Czechia	0.64	1.00	-0.10	0.09
Hungary	0.09	-0.10	1.00	-0.48
Poland	0.10	0.09	-0.48	1.00

**Table A.1** Cross-country correlations of NRI estimates: full sample

	euro area	Czechia	Hungary	Poland
euro area	1.00	0.98	0.95	0.92
Czechia	0.98	1.00	0.95	0.93
Hungary	0.95	0.95	1.00	0.89
Poland	0.92	0.93	0.89	1.00

Table A.2 Cross-country correlations of NRI estimates: 2012Q1-2017Q4

Parameter	Euro Area	Czechia	Poland
$\sum b_{\pi,i}$	0.939*** (0.000)	0.823*** (0.000)	0.699*** (0.000)
$b_{\pi,e}$	0.061 (0.196)	0.177 (0.144)	0.301*** (0.000)
$b_{\pi,o}$	0.0016 (0.558)	-	-
$b_e$	-0.0129 (0.197)	-0.118** (0.020)	-0.055 (0.145)
$b_y$	0.027 (0.155)	0.130* (0.099)	0.071 (0.342)
$b_{y,f}$	-	-	0.025 (0.821)

Table A.3 Ex-ante real interest specification: the Phillips curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Poland
$a_y$	0.814*** (0.000)	0.945*** (0.000)	0.803*** (0.000)
$a_r$	-0.188 (0.117)	-0.066* (0.070)	-0.179 (0.323)
$a_e$	-0.034 (0.265)	-0.031 (0.323)	-0.035 (0.305)
$a_f$	0.438*** (0.000)	0.707*** (0.000)	0.186 (0.582)
$a_l$	-0.185 (0.180)	-	-0.485 (0.172)

**Table A.4** Ex-ante real interest specification: the IS curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Poland
$c_g$	0.088 (0.713)	0.170 (0.400)	-
$c_y$	-0.095 (0.927)	0.834 (0.142)	-2.207*** (0.002)
$c_r$	0.981* (0.054)	-	0.156 (0.774)

**Table A.5** Ex-ante real interest specification: the exchange rate equation estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$b_e$	-0.0137 (0.170)	-0.146*** (0.000)	-0.142*** (0.000)	-0.115*** (0.000)
$b_y$	0.047*** (0.006)	0.131 (0.113)	0.114 (0.326)	0.166*** (0.006)

**Table A.6** Simplified Phillips curve specification: the Phillips curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$a_y$	0.846*** (0.000)	0.942*** (0.000)	0.910*** (0.000)	0.643*** (0.000)
$a_r$	-0.203** (0.019)	-0.064* (0.068)	-0.067 (0.158)	-0.268*** (0.000)
$a_e$	-0.040 (0.161)	-0.034 (0.263)	-0.063* (0.079)	-0.029 (0.419)
$a_f$	0.413*** (0.000)	0.687*** (0.000)	0.685** (0.013)	0.216 (0.491)
$a_l$	-0.218* (0.095)	-	-0.167 (0.252)	-0.424** (0.021)

**Table A.7** Simplified Phillips curve specification: the IS curve estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$c_g$	0.075	0.167	0.405	-
	(0.703)	(0.415)	(0.148)	
$c_y$	-0.048	0.790	-0.240	-1.312***
	(0.929)	(0.189)	(0.663)	(0.009)
$c_r$	0.642	-	0.650	0.903
	(0.221)		(0.163)	(0.123)

**Table A.8** Simplified Phillips curve specification: the exchange rate equation estimation

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$b_y$	0.048***	0.069	0.154	0.113*
	(0.002)	(0.288)	(0.256)	(0.081)
$a_y$	0.844***	0.938***	0.908***	0.646***
	(0.000)	(0.000)	(0.000)	(0.000)
$a_r$	-0.224***	-0.061*	-0.070	-0.274***
	(0.004)	(0.064)	(0.122)	(0.000)
$a_f$	0.373***	0.745***	0.595***	0.171
	(0.000)	(0.000)	(0.000)	(0.412)
$a_l$	-0.254*	-	-0.126	-0.432***
	(0.056)		(0.329)	(0.005)

**Table A.9** No exchange rate block specification: estimation results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Euro Area	Czechia	Hungary	Poland
$b_y$	0.049***	0.068	0.157	0.064
	(0.002)	(0.371)	(0.233)	(0.309)
$a_y$	0.789***	0.938***	0.905***	0.566***
	(0.000)	(0.000)	(0.000)	(0.000)
$a_r$	-0.346***	-0.036	-0.111***	-0.307***
	(0.004)	(0.389)	(0.009)	(0.000)
$a_l$	-0.425***	-	-0.107	-0.722***
	(0.000)		(0.416)	(0.000)

**Table A.10** No foreign output gap specification: estimation results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%

p-values in parentheses.

Parameter	Euro Area	Hungary	Poland
$b_y$	0.050***	0.154	0.058
	(0.002)	(0.239)	(0.305)
$a_y$	0.891***	0.936***	0.920***
	(0.000)	(0.000)	(0.000)
$a_r$	-0.193***	-0.095***	-0.006
	(0.002)	(0.009)	(0.902)

**Table A.11** No lending spread specification: estimation results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%

p-values in parentheses.



Parameter	Euro Area	Czechia	Hungary	Poland
$b_y$	0.001	0.019	0.729**	0.339***
	(0.931)	(0.686)	(0.010)	(0.003)
$a_y$	0.983***	0.926***	0.802***	0.886***
	(0.000)	(0.000)	(0.000)	(0.000)
$a_r$	-0.400***	-0.195*	-0.021	-0.012
	(0.000)	(0.092)	(0.612)	(0.752)

**Table A.12** No capacity utilisation specification: estimation results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Baseline panel specification	Optimal model fit
$\sum b_{\pi,i}$	0.859***	0.863***
	(0.000)	(0.000)
$b_{\pi,e}$	0.141***	0.137***
	(0.006)	(0.003)
$b_{\pi,o}$	-0.0073	-
	(0.702)	
$b_e$	-0.077***	-0.080***
	(0.001)	(0.001)
$b_y$	0.102**	0.099***
	(0.035)	(0.001)
$b_{y,f}$	-0.0048	-
	(0.962)	

**Table A.13** Panel estimation: the Phillips curve results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Baseline panel specification	Optimal model fit
$a_y$	0.933*** (0.000)	0.934*** (0.000)
$a_r$	-0.055 (0.154)	-0.067* (0.064)
$a_e$	-0.0001 (0.974)	-0.044** (0.049)
$a_f$	0.662*** (0.000)	0.699*** (0.000)
$a_l$	-0.012 (0.833)	-0.046 (0.528)

**Table A.14** Panel estimation: the IS curve results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.

Parameter	Baseline panel specification	Optimal model fit
$c_g$	0.627 (0.107)	0.697* (0.074)
$c_y$	-0.465 (0.252)	-0.467 (0.253)
$c_r$	-2.884*** (0.001)	-2.902*** (0.000)

**Table A.15** Panel estimation: the exchange rate equation results

\*\*\* - significant at 1% level, \*\* - at 5%, \* - at 10%  
p-values in parentheses.