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Inherit Money or Brains? The Role of Inheritance and Its Taxation in OLG Analysis*

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Abstract

Motivated by ongoing debates about the effectiveness of inheritance taxation in reducing wealth concentration—especially in contexts marked by strong family ties and rising wealth accumulation—this study employs an overlapping generations model calibrated to Polish data to investigate whether such policies truly mitigate inequality. By incorporating ex ante heterogeneity and human capital investment, I find that increasing inheritance taxes while lowering labor taxes fails to improve welfare or lessen inequality. Instead, these changes distort saving incentives, disproportionately lowering retirees' incomes and intensifying disparities across generations. Although redistributing tax revenues can benefit certain groups, it ultimately widens inequalities between retirees and working-age households. These results underscore the need for careful policy design and question the political feasibility of inheritance taxation, suggesting that inheritance taxation alone is insufficient as a policy tool for narrowing inequality.

JEL classification: D15, D64, H23, I38, D58

Keywords: Inheritance taxation, Welfare analysis, OLG modelling

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1 Introduction

There is a broad global political consensus that addressing inequality is one of the most pressing economic challenges of our time. To properly confront this issue, a vast body of empirical literature has emerged, documenting the historical evolution of inequality (e.g. Piketty and Saez 2003; Piketty and Saez 2014), alongside various theoretical approaches aimed at understanding its origins and components. Furthermore, since Piketty and Goldhammer (2014) introduced the idea of a universal wealth tax and highly progressive income taxation, numerous policy instruments targeting inequality reduction have become subjects of intense debate. While Piketty's proposal may have been deemed too radical by some, there exists a wide range of already implemented or planned to be implemented policy instruments that have yet to be fully examined within different economic models.

Inheritance taxation could play a particularly significant role in this context, especially given the growing attention that inherited wealth and parental background have received in recent academic literature (Boserup, Kopczuk, and Kreiner 2018; Abbott et al. 2019; Fogli and Guerrieri 2019). The development of finite-horizon models has made it possible to study intergenerational interactions within a macroeconomic framework, particularly in relation to taxation and other policies aimed at promoting equity. While intergenerational mobility has remained relatively stable in recent decades (Chetty et al. 2014), rising inequality has made the provision of equal opportunities increasingly important.

A standard approach in taxation literature evaluates policies based on two key criteria: equity and efficiency. While equity is difficult to quantify and requires explicitly defined measures, efficiency focuses on minimizing distortions to economic output. Historically, Chamley (1986) and Judd (1985) argued for the optimality of a zero capital income tax rate, which includes wealth, bequests, or any form of accumulated capital. This conclusion arises from the understanding that capital income taxation distorts intertemporal choices, discouraging savings and investment. However, Straub and Werning (2020) challenge this view, demonstrating that when the intertemporal elasticity of substitution is less than or equal to one, the long-run optimal tax rate on capital is positive. For elasticity values greater than one, the tax rate may converge to zero in the long run, though it could remain positive for extended periods prior to this. Notably, Straub and Werning (2020) are not the first to challenge the zero-tax rate result; Piketty and Saez (2013) also question it by relaxing standard model assumptions. Other studies, such as Krueger and Ludwig (2013), which account for idiosyncratic labor income shocks, and Blumkin and Sadka (2004), which include accidental bequests, similarly overturn the conclusion that capital should remain untaxed.

Intergenerational transfers—whether estates, inheritances, accidental bequests, or inter vivos gifts—can also be subjected to taxation, and recent literature indicates that taxing them can yield positive outcomes (Kopczuk 2013a; Piketty and Saez 2013; Saez and Stantcheva 2018; Stantcheva 2020). The rationale behind it is that taxes on transfers are highly progressive, and they level the wealth concentration. However, the literature presents conflicting results, often driven by model assumptions, such as the motivations behind bequeathal decisions (Slemrod, Hines, and Gale 2011; Cremer and Pestieau 2006) or the specification of utility functions (Heer 2001). For instance, Becker and Tomes (1979), followed by Davies (1986), demonstrate that in a dynastic framework with idiosyncratic labor productivity shocks, increasing estate taxes exacerbates long-term inequality and reduces welfare. Their model, based on pure dynastic altruism, incorporates the utility of future generations directly into the utility function of the current generation. In contrast, under the "joy-of-giving" motive, bequest taxation has an equalizing effect, as bequests are treated as a consumption expenditure in the final period of life. With Cobb-Douglas preferences, taxation becomes neutral to average wealth, and by reducing the variance of bequests, it leads to lower wealth inequality. Bossmann, Kleiber, and Wälde (2007), followed by Wan and Zhu (2019), show that the redistribution effect of taxation dominates the inheritance effect, reducing wealth variance while keeping average wealth constant, which in turn decreases inequality—measured not only by the coefficient of variation but also by the Gini coefficient. This finding holds across various forms of constant relative risk aversion (CRRA) utility functions (De Nardi and Yang 2016).

As noted by Cremer and Pestieau (2006), there are two primary approaches to taxing intergenerational wealth transfers: estate tax and inheritance tax. It is important to emphasize that both taxes can theoretically achieve optimal allocation. However, drawing from the literature on optimal taxation, it becomes clear that inheritance tax is generally preferable from a policy perspective. First, the optimal bequest tax formula depends not only on the magnitude of the inheritance but also on the behavioral responses of future generations (Piketty and Saez 2013). Consequently, taxing the donee (recipient) rather than the donor is more practical (Kopczuk 2013a). Second, when accounting for household heterogeneity in the number of children, it is impossible to derive an optimal estate tax formula that is independent of family size (Farhi and Werning 2010). In contrast, inheritance tax can implement the optimal allocation. Despite these theoretical insights, current wealth transfer taxation schemes remain far from the optimal formulas derived in the literature.

This article complements the existing literature on inheritance taxation. The objective is to study the mechanism behind inheritance tax and its role in reducing inequality within an OLG model featuring ex ante heterogeneity and human capital accumulation, calibrated to Polish data. The aim is to assess to what extent inheritance tax serves as a sufficient tool to address inequality within a family-based model and within the scope of available policy instruments.

The starting point is to position this work within the growing trend of modeling public policy instruments using the OLG approach. Addressing inequality as a priority in social policy requires quantitative analyses that not only distinguish between families across cohorts of different ages but also consider heterogeneity within the same cohort. While the proposed modeling approach aligns with the existing literature, this work seeks to contribute to the field in two key ways. First, building on prior work, this study employs the overlapping generations model not only for welfare analyses of various policy instruments (Auerbach and Kotlikoff 1987; Boersch-Supan and Ludwig 2010; Makarski, Tyrowicz, and Komada 2024), but also to examine the simultaneous effects of human capital accumulation and bequest taxation in the context of inequality. Although the model framework is similar to that of Lambrecht, Michel, and Vidal (2005) and Kunze (2014), these authors refrained from incorporating heterogeneity and did not attempt to analyze taxation in the context of inequality.

Second, this research contributes to the strand of literature that examines the drivers of inequality that can be mitigated through public policies. Significant contributions in this area include early works by Davies (1986), Castaneda, Díaz-Giménez, and Ríos-Rull (2003), and De Nardi (2004), and more recently by Pestieau and Sato (2008), De Nardi and Yang (2016), and De Nardi and Fella (2017). However, these issues have not yet been analyzed in the Polish context. Thus, both the approach and the research question are novel.

The results of the model, calibrated to Polish data, indicate that inheritance tax is not an effective tool for reducing inequality in a framework where individuals inherit from their parents and differ in their levels of human capital, while parents have a bequest motive. This outcome arises because, in the OLG model, such a tax distorts consumer choices, reducing the incentive to save, which leads to a decline in retirees' income and exacerbates inequality, particularly between age cohorts rather than within them.

The structure of the article is as follows. The introduction reviews the relevant literature to justify the construction of the model presented in Chapter 2. This chapter discusses the intuition behind the model and the potential pathways of its functioning. Chapter 3 provides a detailed overview of the model's calibration. The results are presented in Chapter 4, where a series of simulations is conducted to evaluate the welfare effects of changes in the inheritance tax rate. The conclusion highlights the implications for economic policy and suggests potential directions for future research.

2 Model set up

I modify the standard overlapping generation (OLG) model in the spirit of (Samuelson 1958) and Diamond (1965) to take account of the trade-off that parents face in choosing between their own consumption and family transfers – investments in children's education and leaving bequests. To this end, following Kunze (2014) and Delventhal, Fernández-Villaverde, and Guner (2021), I model altruistic parents by incorporating the disposable income level of their immediate descendants into their utility function. The disposable income might be enhanced by either transfers of human capital (education) or physical capital (inheritance). The model focuses on the case where successive generations are linked through income of descendants entering directly the utility function rather than considering a Barro's dynasty model (Barro 1974). This assumption makes altruistic model more tractable from a technical viewpoint than Barro's dynasty model and allows studying trade-off between investments in education and inheritance driven by relative returns. This trade-off might create wealth and income inequalities across households differing in altruistic behaviors and preferences. Moreover, De Nardi (2004) shows that these income linkages are important to explain the occurrence of large estates that characterize the upper tail of the wealth distribution in several countries.

2.1 Families

The economy in a period t consists of i types of families differentiated by the level of human capital H_i and the degree of altruism γ_i. The level of human capital and the degree of altruism is constant within the i-type family over time. The proportion of type i's families is defined by q_i and $\sum_i q_i = 1$. For simplicity, the unit in the model is a family, so we can treat a family as an individual. Since there is only one child born within the i-type family, the population is constant and the number of families every period is normalized to one, $N_t = 1$. People live for three periods: *childhood, adulthood* and *old-age*, but make economic decisions only during the latter two periods of life. During *childhood* young individuals are ensured with basic sustenance and education by their parents. Since they don't make any economic decisions, the first period of life is called *a dummy period*.

2.1.1 Families behavior

Adults born in $t-1$ and i-type work in period t and retire in $t+1$. While working, they allocate their budget for consumption ($c_{i,t}$) and savings ($s_{i,t}$) and provide the education expenditure ($e_{i,t}$) to their children. The costs of investing in education are only the opportunity costs of foregone wage income. Since adults supply work inelasticly, their labor income depends on their human capital level ($h_{i,t}$) and market wage (w_t). They also receive a non-negative inheritance from their parents ($b_{i,t}$) and a non-negative lump-sum benefit from the government ($a_{i,t}$), which can serve as a tax exemption or redistributive policy instrument. The government imposes taxes on the labor income (τ^L), bequests (τ^B) and collects pension contributions (τ^P). We assume that the tax rates are constant over time. Hence, the budget constraint of an *adult* born in $t-1$ is given by ($I^a_{i,t}$):

$$c_{i,t} + s_{i,t} + e_{i,t} = (1 - \tau^L - \tau^P)w_t h_{i,t} + (1 - \tau^B)b_{i,t} + a_{i,t} = I^a_{i,t} \tag{1}$$

When retired, families divide return on savings and pension benefits ($p_{i,t+1}$) between market good consumption ($d_{i,t+1}$) and bequests to their children ($b_{i,t+1}$):

$$d_{i,t+1} + b_{i,t+1} = R_{t+1}s_{i,t} + p_{i,t+1} = I^o_{i,t+1}, \tag{2}$$

where R_t is the gross interest rate.

2.1.2 Human capital accumulation

The level of education that children receive ($e_{i,t}$) and parental human capital ($h_{i,t}$) determine their level of human capital when they are adults. Following Croix and Doepke (2003) and

Delventhal, Fernández-Villaverde, and Guner (2021), human capital accumulates with:

$$h_{i,t+1} = (e_{i,t})^{\kappa}(h_{i,t})^{1-\kappa}, \tag{3}$$

where the parameter $\kappa \in (0, 1)$ is an elasticity of education effort to human capital and $1-\kappa$ represents intergenerational transmission of human capital within the family or dynamic complementarity of human capital.

2.1.3 Maximization problem

Households derive utility from their consumption and the income level of their children. That is, parents are altruistic and care about their children's disposable income (so-called *warm-glow motive*) and not about the use of this income.

The individual belonging to i-type family born at the beginning of period $t-1$ is endowed with $h_{i,t}$ units of human capital at the outset of the adulthood and chooses $\{c_{i,t}, d_{i,t+1}, s_{i,t}, e_{i,t}, b_{i,t+1}\}$ to maximize lifetime utility under constraints (1) and (2) and the human capital accumulation follows (3):

$$\begin{aligned}
\max_{(c_{i,t}, d_{i,t+1}, s_{i,t}, e_{i,}, b_{i,t+1})} \quad & u(c_{i,t}) + \beta u(d_{i,t+1}) + \gamma_i u(I^a_{i,t+1}) \\
s.to \quad & c_{i,t} + s_{i,t} + e_{i,t} = (1 - \tau^L - \tau^P)w_t h_{i,t} + (1 - \tau^B)b_{i,t} + a_{i,t} \\
& d_{i,t+1} + b_{i,t+1} = R_{t+1}s_{i,t} + p_{i,t+1} \\
& I^a_{i,t+1} = (1 - \tau^L - \tau^P)w_{t+1}h_{i,t+1} + (1 - \tau^B)b_{i,t+1} + a_{i,t+1} \\
& h_{i,t+1} = (e_{i,t})^{\kappa}(h_{i,t})^{1-\kappa} \\
& b_{i,t} \geq 0
\end{aligned} \tag{4}$$

The preferences are assumed to be logarithmic (hence, $u(\cdot)$ is increasing and concave, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and satisfies the Inada conditions) and depend on second and third period consumption and on the disposable income of the adult child. A detailed derivation of the consumer's solution to the problem can be found in the Appendix. The first-order conditions can be written as:

- Intratemporal choices:

$$c_{i,t} = \frac{I^a_{i,t+1}}{\gamma_i \kappa (1-\tau^L-\tau^P) w_{t+1} h^{1-\kappa}_{i,t} (e_{i,t})^{\kappa-1}} \tag{5}$$

$$c_{i,t} = \frac{I^a_{i,t+1}}{\gamma_i (1-\tau^B) R_{t+1}} \tag{6}$$

- Euler equation:

$$c_{i,t} = \frac{d_{i,t+1}}{\beta R_{t+1}} \tag{7}$$

As it follows from (5) and (6), more altruistic families (higher γ_i) devote less resources to their own consumption. Also, their consumption decreases in the elasticity of education effort to human capital (higher κ). Notice that with γ_i tending towards zero, families behave like life-cyclers as in the standard model with a tax policy but without intergenerational transfers.

Euler equation (7) is an intertemporal first-order condition, describing the consumption evolution along an optimal path. It implies that the marginal rate of substitution between consumption during *old-age* and *adulthood* is equal to the price ratio and the discount factor β. The higher the interest rate R_t, the higher income from voluntary savings and hence the consumption during the retirement becomes cheaper. Therefore, with increasing R_t, families choose to postpone their consumption until *old-age*. The same applies to the discount factor β - higher value of β implies more patient families.

Combining (5) with (6), if bequests are strictly positive, implies that the after-tax rate of return on education investments equals the net interest rate:

$$(1-\tau^B) R_{t+1} = \kappa (1-\tau^L-\tau^P) w_{t+1} h^{1-\kappa}_{i,t} (e_{i,t})^{\kappa-1} \tag{8}$$

If bequests are inoperative, the after-tax rate of return on education investments exceeds the net interest rate and the level of educational investment is suboptimal. From (8), the education expenditures can be written as:

$$e_{i,t} = \left[\frac{(1-\tau^B)R_{t+1}}{\kappa(1-\tau^L-\tau^P)w_{t+1}h_{i,t}^{1-\kappa}} \right]^{\frac{1}{\kappa-1}} \tag{9}$$

Since $\kappa \in (0,1), e_{i,t} = 0$ can never be optimal and $e_{i,t}$ is always interior on the interval $(0,1)$. Further, education always converges to its steady state value (see Ludwig and Vogel (2010) for proof). Therefore, the first-order condition for $e_{i,t}$ also holds with strict equality. Clearly, general equilibrium effects play a significant role in the education investments decisions, as $e_{i,t}$ depends on the price ratio $\frac{R_{t+1}}{w_{t+1}}$.

Combining (6) with (7), if bequests are strictly positive, implies that more altruistic families (higher γ_i) devote less resources to their own old-age consumption:

$$d_{i,t+1} = \frac{\beta I_{i,t+1}^a}{\gamma_i(1-\tau^B)} \tag{10}$$

2.2 Production

In every period t, firms produce a single output good according to a Cobb-Douglas production function combining physical capital K_t and labor capital $L_t = \bar{H}_t = \sum_i q_i h_{i,t}$:

$$Y_t = AK_t^\alpha \bar{H}_t^{1-\alpha} = Ak_t^\alpha \bar{H}_t = Ak_t^\alpha \sum_i q_i h_{i,t}, \tag{11}$$

where $\alpha \in (0,1)$ denotes output elasticity of capital, $A > 0$ is a productivity parameter and $k_t = \frac{K_t}{\bar{H}_t}$ denotes the physical to human capital ratio. Assuming total depreciation of the physical capital stock in one period, profit maximization gives the usual marginal productivity conditions, where marginal productivity equal to factor prices:

$$w_t = (1-\alpha)AK_t^\alpha \bar{H}_t^{-\alpha} = (1-\alpha)Ak_t^\alpha$$

$$R_t \quad = \quad \alpha A K_t^{\alpha-1} \bar{H}_t^{1-\alpha} = \alpha A k_t^{\alpha-1} \tag{12}$$

In OLG models with human capital accumulation, it is convenient to express variables per unit of effective human capital and not in per capita terms (see, for example, De La Croix and Michel (2002) and Croix and Doepke (2003)).

2.3 Government

The government levies taxes on bequests (τ^B) and labor income (τ^L) to finance a lump-sum benefit for working generation ($a_{i,t}$) and exogenously determined public spendings ($\overline{G_t}$). Public spendings are defined as a fixed share of the production. The government balances the budget every period t and its budget constraint is given:

$$\overline{G}_t + \sum_i q_i a_{i,t} = \tau^L \sum_i q_i h_{i,t} w_t + \tau^B \sum_i q_i b_{i,t} \tag{13}$$

Since the government spendings are defined as a fixed share of production in the economy, $\overline{G}_t = \bar{\lambda} Y_t$, it creates an externality. Higher production in the private sector increases public spendings and since the public spendings are financed through taxes, it reduces households resources for consumption, bequests and education investments.

The government also runs a pension system scheme. It collects pension contributions from the working generation (τ^P) to be distributed among currently retired ($p_{i,t}$), with a given replacement rate (φ). The pension system is balanced every period:

$$\sum_i q_i p_{i,t} = \tau^P \sum_i q_i h_{i,t} w_t \tag{14}$$

And the pensions are defined as:

$$p_{i,t} = \varphi w_{t-1} h_{i,t-1} \tag{15}$$

Hence, the pension contributions rate can be defined as:

$$\tau^P = \frac{\sum_i q_i p_{i,t}}{\sum_i q_i h_{i,t} w_t} = \frac{\sum_i p_{i,t}}{\sum_i h_{i,t} w_t} \tag{16}$$

For the government budget to be balanced in every period t, the labor income tax has to close the budget and hence:

$$\tau^L = \frac{\overline{G}_t + \sum_i q_i (a_{i,t} - \tau^B b_{i,t})}{\sum_i q_i h_{i,t} w_t} \tag{17}$$

2.4 Market clearing

The asset market clearing condition is given by:

$$\sum_i q_i s_{i,t} = K_{t+1} \tag{18}$$

The labour market clearing condition is given by:

$$L_t = \bar{H}_t = \sum_i q_i h_{i,t} \tag{19}$$

The goods market clearing condition is given by:

$$\sum_i q_i (c_{i,t} + e_{i,t} + d_{i,t} + s_{i,t}) = (1 - \bar{\lambda}) Y_t \tag{20}$$

Clearly, because of the externality created by the public spendings $(1-\bar{\lambda})$, the private marginal product of capital (K) and human capital ($\overline{H}$) is higher than the social marginal product of capital and human capital, respectively.

According to Walras' law, the equilibrium of the labour market implies that of the good and asset markets. In each period t we can substitute the adult's budget constraint (1) for the equilibrium condition of the good market (20):

$$\sum_i q_i(I^a_{i,t} + d_{i,t}) = (1-\bar{\lambda})Ak^\alpha_t \sum_i q_i h_{i,t} \tag{21}$$

2.5 Intratemporal choice

Substituting (12) into (8) gives the educational investment per unit of human capital within the family, $\bar{e}_{i,t} \equiv \frac{e_{i,t}}{h_{i,t}}$:

$$\bar{e}^{1-\kappa}_{i,t} = \frac{\kappa(1-\tau^L-\tau^P)(1-\alpha)}{(1-\tau^B)\alpha} k_{t+1} \tag{22}$$

The above expression shows that, for k_{t+1} given, an increase in inheritance taxation increases educational investments, which in turn increases the human capital growth rate. On the other hand, an increase in inheritance taxation has a negative impact on k_{t+1} via savings, since the higher the tax, the lower disposable income of the adults. This raises the question whether an increase in inheritance taxation speeds up or slows down growth in an economy.

Rearranging (3) gives:

$$\frac{h_{i,t+1}}{h_{i,t}} = \bar{e}^{\kappa}_{i,t}, \tag{23}$$

Hence, the expression (22) can be rewritten using (23) as follows:

$$\bar{e}_{i,t} h_{i,t} = \frac{\kappa(1-\tau^L-\tau^P)(1-\alpha)}{(1-\tau^B)\alpha} k_{t+1} h_{i,t+1} \tag{24}$$

Which further implies that:

$$k_{t+1} h_{i,t+1} = \frac{(1-\tau^B)\alpha}{\kappa(1-\tau^L-\tau^P)(1-\alpha)} \bar{e}_{i,t} h_{i,t} \tag{25}$$

The expression (24) suggests that regardless of the $i-$type of the family, the equation holds:

$$k_{t+1} \frac{\kappa(1-\tau^L-\tau^P)(1-\alpha)}{(1-\tau^B)\alpha} = \frac{e_{i,t}}{h_{i,t+1}} \tag{26}$$

2.6 The growth rate of the economy

Assume that $i = 2$ and the population is equally divided into two types families: A and B with human capital $H_1 = H_A$, $H_2 = H_B$ and $\gamma_1 = \gamma_A$, $\gamma_2 = \gamma_B$ where A is the group with less human capital and lower degree of altruism: $H_A < H_B$ and $\gamma_A < \gamma_B$. The share of the groups are given by $q_1 = q_A = 0.5 = q$ and $q_2 = q_B = 0.5 = q$. Recall that the shares are constant over time.

From (26) and (3), we get the following expression:

$$\frac{e_{1,t}}{(e_{1,t})^{\kappa}(h_{1,t})^{1-\kappa}} = \frac{e_{2,t}}{(e_{2,t})^{\kappa}(h_{2,t})^{1-\kappa}}$$

The above expressions suggest that regardless of the $i-$type of the family, the educational investment per unit of human capital in the family are the same. Therefore, the educational investment in the family will depend on the level of initial human capital.

Combining (10) and (21) gives the following expressions:

$$1) \quad qI^a_{1,t}\left[1 + \frac{\beta}{\gamma_1(1-\tau^B)}\right] + qI^a_{2,t}\left[1 + \frac{\beta}{\gamma_2(1-\tau^B)}\right] = (1-\bar{\lambda})Ak_t^{\alpha}q(h_{1,t} + h_{2,t}) \tag{27}$$

$$2) \quad qd_{1,t}\left[1 + \frac{\gamma_1(1-\tau^B)}{\beta}\right] + qd_{2,t}\left[1 + \frac{\gamma_2(1-\tau^B)}{\beta}\right] = (1-\bar{\lambda})Ak_t^{\alpha}q(h_{1,t} + h_{2,t}) \tag{28}$$

Assuming that the families with a given level of human capital and altruistic preferences have a share in production depending on the level of their human capital, (27) and (28) can be rewritten:

$$1) \quad I^a_{i,t+1} = (1-\bar{\lambda})\frac{\gamma_i(1-\tau^B)}{\gamma_i(1-\tau^B) + \beta}Ak^{\alpha}_{t+1}h_{i,t+1} \tag{29}$$

$$2) \quad d_{i,t+1} = (1-\bar{\lambda})\frac{\beta}{\beta + \gamma_i(1-\tau^B)}Ak^{\alpha}_{t+1}h_{i,t+1} \tag{30}$$

Assuming no redistributive policy, $a_{i,t} = 0$, the strictly positive bequest condition becomes:

$$\frac{I^a_{i,t+1} - (1 - \tau^L - \tau^P) w_{t+1} h_{i,t+1}}{(1 - \tau^B)} > 0 \tag{31}$$

and defines a lower bound on the tax policy:

$$\tau^L > 1 - \tau^P - \frac{\gamma_i (1 - \tau^B)(1 - \bar{\lambda})}{(1 - \alpha)\left(\gamma_i (1 - \tau^B) + \beta\right)} \tag{32}$$

The asset market clearing condition (18) divided by $\bar{H}_{t+1} = \sum_i q h_{i,t+1}$ becomes:

$$\frac{\sum_i q s_{i,t}}{\sum_i q h_{i,t+1}} = k_{t+1}$$

Using the fact that the shares q_1 and q_2 are equal and constant over time:

$$s_{1,t} + s_{2,t} = k_{t+1} h_{1,t+1} + k_{t+1} h_{2,t+1}$$

Inputing (24) gives:

$$s_{1,t} + s_{2,t} = (\bar{e}_{1,t} h_{1,t} + \bar{e}_{2,t} h_{2,t}) \frac{(1 - \tau^B)\alpha}{\kappa (1 - \tau^L - \tau^P)(1 - \alpha)} \tag{33}$$

Hence, using the assumption that the families with a given level of human capital and altruistic preferences have a share in aggregate savings depending on the level of their human capital, it can be derived:

$$s_{i,t} = \frac{(1 - \tau^B)\alpha}{\kappa (1 - \tau^L - \tau^P)(1 - \alpha)} \bar{e}_{i,t} h_{i,t} \tag{34}$$

Substituting (6) and (12) into (29) yields:

$$c_{i,t} = \frac{(1 - \bar{\lambda})(1 - \tau^B)}{\left(\gamma_i (1 - \tau^B) + \beta\right) \kappa (1 - \tau^L - \tau^P)(1 - \alpha)} \bar{e}_{i,t} h_{i,t} \tag{35}$$

Inserting (35) and (29) into (29) and dividing by $h_{i,t}$ yields:

$$\bar{e}_{i,t} = \frac{e_{i,t}}{h_{i,t}} = \frac{\frac{\gamma_i(1-\tau^B)(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}}{\left[\frac{(1-\tau^B)}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\left(\frac{(1-\bar{\lambda})}{\gamma_1(1-\tau^B)+\beta}+\alpha\right)+1\right]} A k_t^{\alpha} \tag{36}$$

The effect of inheritance taxation, for given stocks of physical and human capital, k_t and $h_{i,t}$, on savings is negative, because the tax slows down the accumulation of physical capital. At the same time, the investment in education increases with the tax, which accelerates the accumulation of the human capital.

The dynamics of the physical to human capital ratio k_t can be obtained by combining (22) and (36):

$$\bar{e}_{i,t}^{1-\kappa} = \left[\frac{\frac{\gamma_i(1-\tau^B)(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}}{\left[\frac{(1-\tau^B)}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\left(\frac{(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}+\alpha\right)+1\right]} A k_t^{\alpha}\right]^{1-\kappa} = \frac{\kappa(1-\tau^L-\tau^P)(1-\alpha)}{(1-\tau^B)\alpha} k_{t+1}$$

Rearranging the above equations gives:

$$k_{t+1} = \left[\frac{\frac{\gamma_i(1-\tau^B)(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}}{\left[\frac{(1-\tau^B)}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\left(\frac{(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}+\alpha\right)+1\right]} A\right]^{1-\kappa} \frac{(1-\tau^B)\alpha}{\kappa(1-\tau^L-\tau^P)(1-\alpha)} k_t^{\alpha(1-\kappa)} \tag{37}$$

The equations (37) describes the law of motion for capital per effective human capital unit.

2.7 Steady state

Assume that the economy is stable and the steady state can be obtained (see Lambrecht, Michel, and Thibault (2006) and Ludwig and Vogel (2010) for proofs and assumptions). As the focus is on the steady state, the time indices can be dropped. In the economy where investment in human capital is a source of endogenous growth, in order to obtain steady-state values, the model's variables must be expressed in the effective human capital unit as in the steady state all individual variables grow at the same rate. Hence, the variables needs to be expressed in a form that have constant steady-state values.

Rearranging (37) in steady state determines the long-run physical to human capital ratio:

$$k = \left[\frac{\frac{\gamma_i(1-\tau^B)(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}}{\left[\frac{(1-\tau^B)}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\left(\frac{(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}+\alpha\right)+1\right]} A \right]^{\frac{1-\kappa}{1-\alpha(1-\kappa)}} \left[\frac{(1-\tau^B)\alpha}{\kappa(1-\tau^L-\tau^P)(1-\alpha)} \right]^{\frac{1}{1-\alpha(1-\kappa)}} \tag{38}$$

In the steady state all individual variables grow at the same rate as human capital $\frac{h_{i,t+1}}{h_{i,t}}$, while the capital, the wage and the interest rate are constant. To obtain the growth rate of human capital and hence, the growth rate of the economy, use (3):

$$\frac{h_{i,t+1}}{h_{i,t}} = \frac{(e_{i,t})^{\kappa}(h_{i,t})^{1-\kappa}}{h_{i,t}} = g_i$$

Rearranging:

$$g_i = (e_{i,t})^{\kappa}(h_{i,t})^{-\kappa} = (\bar{e}_{i,t})^{\kappa}$$

Inserting into the above equation (36) and (38) gives:

$$g_i = \left[\frac{\frac{\gamma_i(1-\tau^B)(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}}{\left[\frac{(1-\tau^B)}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\left(\frac{(1-\bar{\lambda})}{\gamma_i(1-\tau^B)+\beta}+\alpha\right)+1\right]} A \right]^{\kappa+\frac{(1-\kappa)\kappa\alpha}{1-\alpha(1-\kappa)}} \left[\frac{(1-\tau^B)\alpha}{\kappa(1-\tau^L-\tau^P)(1-\alpha)} \right]^{\frac{\kappa\alpha}{1-\alpha(1-\kappa)}} \tag{39}$$

The above equations describe the growth rate of the individual variables in the steady state of the economy where the investment in human capital is the source of the endogenous growth.

Both taxes are clearly detrimental to growth. Government spending financed by taxes reduces households' disposable income, which could otherwise be spent on education, a key driver of human capital growth. In contrast, a higher preference for altruism positively impacts growth, as it leads to increased spending on education, thereby accelerating human capital accumulation. Additionally, it incentivizes households to save, enabling them to accumulate capital that can be transferred to the next generation in the form of inheritance.

The equations (38) and (36), describes the economy in the steady state, expressed in per unit of human capital. Moreover, using (36), the consumption per unit of human capital in the first period and savings per unit of human capital can be described as:

- Consumption:
$$\bar{c_i} = \frac{(1-\bar{\lambda})(1-\tau^B)}{(\gamma_1(1-\tau^B)+\beta)\,\kappa(1-\tau^L-\tau^P)(1-\alpha)}\bar{e}_1$$

- Savings:
$$\bar{s_i} = \frac{(1-\tau^B)\alpha}{\kappa(1-\tau^L-\tau^P)(1-\alpha)}\bar{e}_i$$

Additionally, the pensions per unit of human capital from (15) can be defined as:

$$\bar{p_i} = \varphi w$$

3 Calibration

To further elucidate the mechanisms of the model described in Section 2 and to analyze the impact of inheritance taxation on the model's behavior, a numerical exercise will be conducted. The economy will be studied under general equilibrium conditions. In this context, a perfectly competitive firm produces output using a Cobb-Douglas production function. Additionally, the government collects taxes on labor and inheritance and operates a social security system. These taxes fund a fixed amount of government spending. The analysis focuses on how changes in tax policy, specifically the composition of inheritance and labor income taxes, influence the economy and welfare. The impact of the tax policy change is evaluated by comparing the steady states to assess long-run differences.

In the numerical exercise, the model maintains the assumption of a lifespan of three periods for a single family. This assumption is primarily made to maintain computational simplicity, a common practice in the literature utilizing overlapping generations models (Diamond 1965; Auerbach and Kotlikoff 1987; Delventhal, Fernández-Villaverde, and Guner 2021). Extending the model to include more periods introduces complexities, particularly regarding the timing of inheritance transfers. Studies have shown that the timing of these transfers varies significantly across families and does not necessarily coincide with the end of life, as families adopt different strategies for transferring wealth between generations (Joulfaian 2005; Poterba 2001).

The model is calibrated and parameters are chosen to reflect Poland's economic situation, for several reasons. First, due to the numerous tax exemptions within various affinity groups, inheritance tax is practically negligible in Poland's current tax system, leaving room for theoretical exploration of its potential role and impact. Second, generational ties in Polish society are notably strong, with a pronounced bequest motive (Boguszewski 2013; Iacovou and Skew 2011). Third, as economic inequality continues to rise in Poland, the issue of reducing disparities is becoming increasingly prominent in public discourse, with one proposed measure being the enhancement of the inheritance tax's role in addressing this challenge (Sawulski, Brzeziński, and Bukowski 2024). This analysis has the potential to contribute to the theoretical framework underpinning the ongoing political discourse. The model parameters reflect the economic conditions of Poland in 2022 or is based on the most recent available data. Table 1 summarizes the parameters utilized in the model.

Production function Parameter values in the production function are selected based on established OLG literature (Heer and Irmen 2014). The parameter α, representing the elasticity of the production function with respect to capital, is set at 0.33, indicating a 33% contribution of capital to production. The intertemporal discount factor β, which reflects the rate at which future consumption is discounted, is assumed to be 0.99^{30}. Given the use of a three-period model, the discount factor is appropriately adjusted by raising it to a power roughly corresponding to the number of years that define each period in the model.

Pension system The assumptions regarding the pension system in the model approximate the key features of Poland's pay-as-you-go (PAYG) defined contribution system, as described by Góra (2013). However, the model simplifies by excluding minimum pensions, agricultural pensions, and widow's pensions. Consequently, the government's budget is balanced in each period, meaning that the sum of contributions collected is equal to the sum of pension payments at that time. The government's role is limited to the collection of contributions and the distribution of pensions. The contribution rate in the model was calibrated to reflect the replacement rate in the Polish pension system for a worker entering the pension system in 2022, which is approximately 30% (OECD 2023). Therefore, the contribution rate is determined endogenously within the model.

Government expenditure The parameter (λ) for government expenditure $\bar{G}$ is set as a fixed share of GDP, corresponding to an average of 18.2% of general government final consumption expenditure as a percentage of GDP over the period from 2013 to 2023, based on data from the World Bank's National Accounts Database.

Human capital accumulation function The parameter $1-\kappa$ determines the direct effect of parental human capital on the children's human capital, capturing the intergenerational transmission of ability, as well as human capital formation within the family. Empirical studies detect such effects, but they are relatively small—Rosenzweig and Wolpin (1994) find that an additional year of the mother's education at the high school level (roughly a 10% increase in education) raises a child's test scores by 2.4%. Following Croix and Doepke (2003) and Delventhal, Fernández-Villaverde, and Guner (2021), a moderate degree of intergenerational transmission of human capital is assumed, $(1-\kappa) = 0.3$. Hence, an elasticity of education effort to human capital (κ) is assumed to be 0.7.

Human capital initial distribution The distribution of initial human capital is calibrated to align with the Gini coefficient observed in Poland, which was reported by Statistics Poland to be 26.3 in 2022 (Statistics Poland 2023). Statistics Poland calculates this indicator based on household disposable income for the entire national population sample. Hence, the initial distribution of human capital is calibrated to be consistent with the Gini observed not only among the working population, but also among retirees. The initial human capital distribution is: $h_{01} = 0.825$ and $h_{02} = 2$.

Tax policy – inheritance tax In the Polish tax system, the so-called extended family is exempt from inheritance and gift tax. This group includes spouses, ascendants (parents, grandparents, great-grandparents), descendants (children, grandchildren, great-grandchildren), siblings, as well as stepchildren and stepparents. Consequently, property transfers, whether through inheritance or gifts, are not subject to taxation within this group, regardless of the value or the manner of the transferred assets. As a result, inheritance and gift taxes contributed only 0.04% of total tax revenue in Poland in 2019 (OECD). Accordingly, the inheritance tax rate in the model is set at 0.01 to reflect this minimal contribution of inheritance tax to the overall tax system.

Tax policy – labor income tax The labor tax rate is determined endogenously within the model to balance the government's budget and finance its consumption. In the initial steady state, the labor tax rate is approximately 27%. Labor tax revenues account for about 8% of GDP in Poland in 2021 (OECD).

Parental altruism The literature presents a range of debates surrounding the appropriate methodology for calculating the bequest-to-wealth ratio (Kopczuk 2013b). This ratio represents the proportion of total wealth of a household that is passed down through bequests across generations. The controversy in methodological approach to calculate the ratio stems from several factors. First, the availability and accuracy of data on wealth transfers can vary significantly across countries and over time. Many studies rely on inheritance tax records, which may not capture all forms of wealth transfer, particularly inter vivos gifts, such as real estate or college money. In Poland, for example, administrative wealth data are not open and available for researchers. Second, different studies use varying assumptions regarding savings behavior, life expectancy, and wealth accumulation. For instance, some studies finds that individuals accumulate wealth primarily for bequests, while others argue that wealth accumulation is driven by precautionary savings motives, and bequests are incidental (Lusardi 1998; Mastrogiacomo and Alessie 2014; Kopczuk and Lupton 2007). In the model used in this study, since all households dies with the certainty at the end of the third period, all bequest are intentional.

The macroeconomic literature often relies on the bequest-to-wealth ratio estimation proposed by Gale and Scholz (1994), which has been widely used in studies such as De Nardi and Yang (2016) or Nishiyama (2002). Gale and Scholz (1994) estimated that at least 20% of U.S. wealth is transferred to the next generation through bequests. When both bequests and inter vivos gifts are considered, this share rises to at least 51%. Furthermore, when expenditures on college education for children are included, the share of wealth transferred across generations becomes even larger, reflecting the significant role of both direct financial bequests and other forms of wealth transfers in shaping intergenerational wealth dynamics. The annual bequest-to-wealth ratio, which accounts for bequests, intended transfers, and educational expenditures such as college costs, is estimated to be approximately 1.7%. These numbers from U.S. are consistent with the findings based on the European data from Alvaredo, Garbinti, and Piketty (2017). Partially, this is due to the fact that real estate is the dominant asset type in many OECD countries. For example, in Poland, real estate accounts for approximately 50% of total wealth for households in the bottom wealth quintile, and up to 80% for those in the middle wealth quintile (OECD 2021). In the following model, a conservative approach is taken, with the bequest-to-wealth ratio set at 20%, aligned with Gale and Scholz (1994). To achieve the target value of the bequest-to-wealth ratio in the initial steady state, the parental altruism parameter γ has been calibrated to 0.65, which is also consistent with Delventhal, Fernández-Villaverde, and Guner (2021)

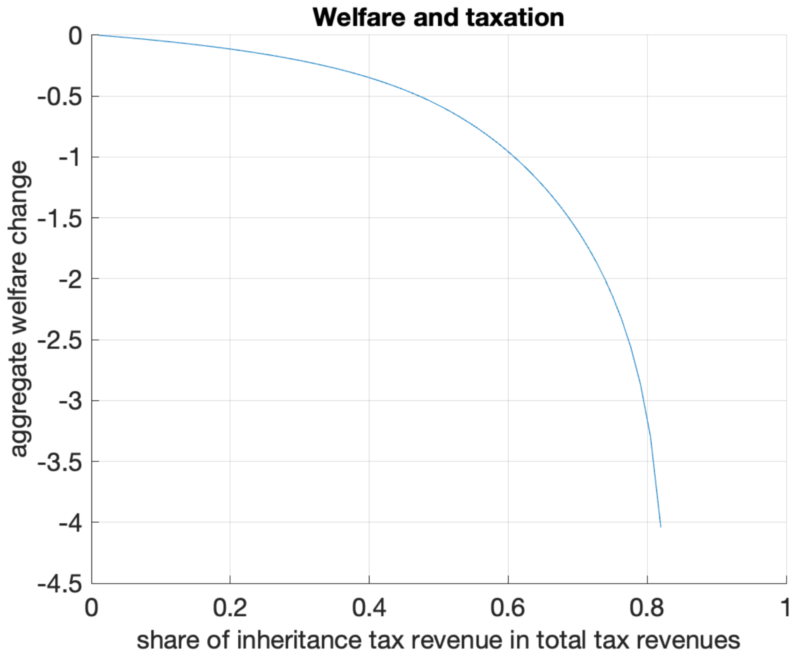
4 Quantitative analysis

The government controls two tax policy instruments: labor income tax and inheritance (bequest) tax. This analysis abstracts from the issue of tax optimality (see Section 1 for literature review on tax optimality) and instead focuses on identifying the tax mix that, given a fixed level of government consumption, maximizes the welfare of contemporary families. In doing so, it examines a second-best policy scenario where an exogenous constraint—such as the fixed level of government consumption—cannot be altered (Leach 2003). This approach allows for an exploration of how different combinations of taxes, such as labor income and inheritance taxes, influence overall welfare in a setting where the optimal tax structure is not attainable due to the constraint. The analysis is conducted in the steady-state framework.

4.1 Welfare analysis

To illustrate the role of inheritance taxation in welfare, the model is iteratively solved for varying inheritance tax rates, ranging from 0.01 (the initial steady-state value) to 0.99. All model parameters, except for the inheritance and labor tax rates, remain consistent with those in the initial steady state. The share of government consumption in GDP is held constant, requiring the labor tax rate to adjust in order to maintain a balanced government budget. As inheritance tax revenues increase, the labor tax decreases accordingly. Importantly, there is no redistribution mechanism for the collected taxes, meaning the reduction in labor taxes is the sole mechanism for rebalancing the budget in response to changes in inheritance tax revenue. The aggregate welfare is calculated as a sum of utilities of families differing in human capital at the beginning of the second period, according to Equation (4). In this sense, the government neither discriminates against nor favors any group, and assigns equal utility value to all individuals.

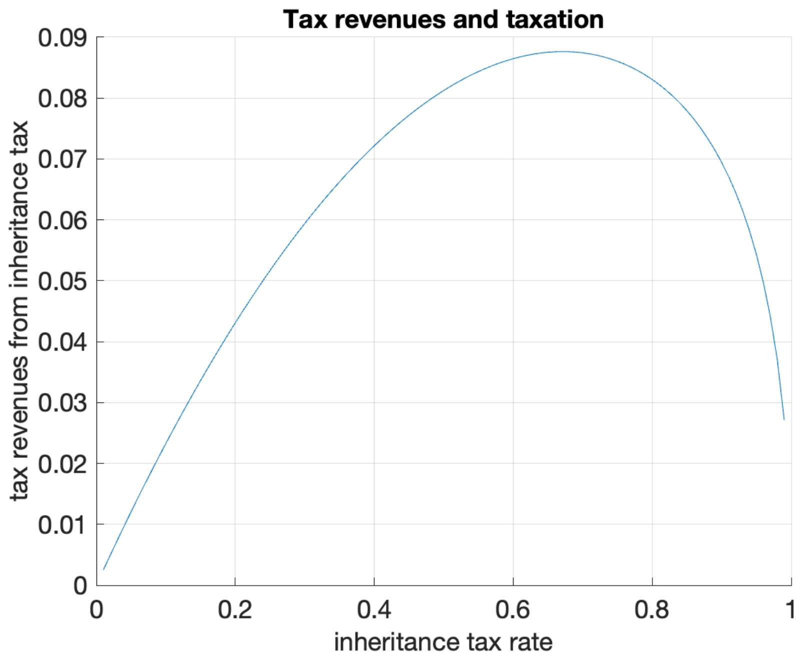
Figure 1: Welfare and inheritance taxation



Note: The aggregate change in welfare represents the difference between the welfare level in the simulation scenario and the welfare level in the initial steady state, expressed as a ratio of the welfare in the initial steady state.

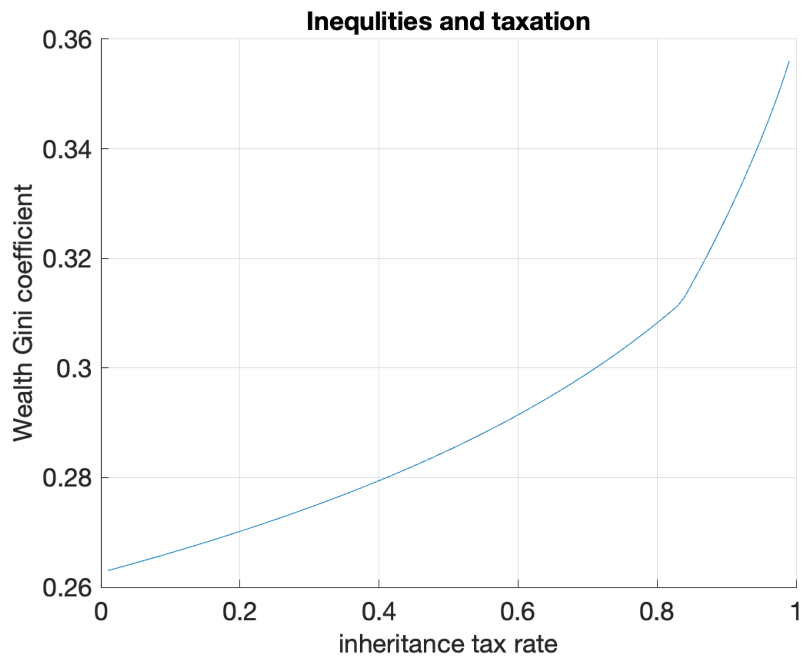
Figure 1 presents aggregate welfare as a function of the share of government revenue derived from inheritance taxes. Clearly, the larger the share of inheritance tax revenue in the government's budget, the lower the aggregate welfare. Consequently, no inheritance tax rate exists that would improve welfare compared to the status quo. This outcome arises for several reasons. First, the inheritance tax is highly distortionary to consumer choice. Since the labor supply is inelastic, reducing the labor tax rate does not significantly enhance household welfare compensating for the welfare loss following the increase of the inheritance taxation. Second, increasing the inheritance tax rate while reducing the labor tax rate negatively impacts the rate of human capital accumulation, which in turn slows GDP growth (as illustrated by Equation (39)). Third, as it follows from Equation (8), since the bequests in this model setting are operative and strictly positive, the higher the inheritance tax, the lower the after-tax rate of return on education investments. Fourth, inheritance taxation follows the Laffer curve. Figure 2 illustrates the relationship between inheritance tax revenue and the tax rate, showing that at very high rates, revenue decreases despite the higher tax rate. Furthermore, even at the maximum tax rate, revenues are insufficient to fully cover government consumption, meaning the labor tax is not eliminated, only reduced. In the simulation scenario with the maximum inheritance tax rate, the labor tax rate remains at 6.5%.

Figure 2: Tax revenues and inheritance taxation



One of the frequently raised arguments for increasing the share of inheritance tax in government revenue is to "level the playing field" for future generations. The idea is that by raising the inheritance tax rate, wealthier individuals may be discouraged from passing down large inheritances, thereby preventing their children from gaining an advantage over less affluent peers. Simultaneously, the reduction in labor tax would benefit both poorer and wealthier individuals. Moreover, increasing the inheritance tax could encourage greater investment in educational transfers rather than monetary ones, accelerating human capital accumulation and fostering economic growth. In summary, raising the inheritance tax could potentially reduce inequalities even without redistributing the revenues from it. However, in our framework, this does not occur. Figure 3 shows that as the inheritance tax rate increases, wealth inequality also rises. The mechanism behind this is not immediately obvious. As the inheritance tax rate rises, the incentive to save diminishes. This results in retirees' disposable income declining faster than that of working households, leading to increased inequality, as retirees are more adversely affected by reduced savings compared to working households. Moreover, the potential redistribution of revenues from inheritance tax would necessitate an increase in the overall tax burden, which has clear negative implications for human capital accumulation and economic growth. While an increase in the disposable income of working-age cohorts could reduce inequalities within this group, it would exacerbate disparities between working individuals and retirees. Consequently, overall inequality would deepen.

Figure 3: Inequalities and inheritance taxation



4.2 Inequality analysis

Building on the previously mentioned and frequently cited argument that inheritance tax could "level the playing field" for future generations, a simulation is performed within the model. Let's assume that bequests are inoperative and set to zero, meaning that transferring wealth to the next generation is not available. In line with a popular vote, in this scenario, inequality decreases, reaching a value of 0.214. The mechanism operates in such a way that the absence of monetary transfers equalizes disposable incomes both within and between age groups. In our framework, increasing the inheritance taxation excessively interferes with consumer choice, while equalizing the total bequests to zero has no such effect. This suggests that even though wealth transfers and the accumulation of multigenerational fortunes actually exacerbate inequality, the inheritance tax is not the appropriate tool to counteract this. The literature suggests that such an effect can be achieved, for instance, through a strongly progressive tax on savings or capital gains.

Given that the welfare analysis demonstrated that the rise in inequality is primarily driven by changes in the disposable income ratio between working-age cohorts and retirees, it becomes interesting to analyze the interaction of this model with pension system parameters. Lambrecht, Michel, and Vidal (2005) showed in a model with human capital accumulation and bequest motives that there is no case for unfunded public pensions in an economy where bequests are operative. According to the authors, such a case only exists when bequests are negative and families are sufficiently altruistic. However, the authors did not examine the inequality aspect, focusing solely on growth rates and human capital accumulation. Our model simulation shows that the absence of a pension system significantly exacerbates inequality, which in this scenario reaches 0.363—a value comparable to the simulation scenario with the maximum inheritance tax rate. Without a pension system, the disposable income of the working-age population increases due to the absence of a pension contribution ($\tau^P = 0$), but on the other hand, retired cohorts must rely solely on their savings. This deepens inequality, as families fail to adequately smooth their consumption over time.

5 Conclusions

The aim of this paper was to examine the welfare and inequality effects of changes in how government consumption is financed—specifically through a mix of labor tax and inheritance tax—and to assess the impact of the pay-as-you-go pension system on these outcomes. In doing so, the paper seeks to contribute to the ongoing debate on increasing the role of inheritance tax in fiscal policy as a tool for reducing inequality.

To achieve this, an overlapping generations model calibrated to Polish data was constructed, and counterfactual scenario simulations were conducted. These simulations evaluated the welfare effects of varying inheritance tax rates within the fiscal mix. The results indicate that increasing the inheritance tax rate, while simultaneously reducing the labor tax rate, leads to a deterioration in welfare. In the extreme case, where the inheritance tax rate is set at 0.99, the Gini coefficient rises from 0.263 in the initial steady state to 0.356, representing a 35% increase. As the inheritance tax rate increases, the incentive to save is reduced, causing a sharper decline in disposable income for retirees compared to working households. This leads to greater inequality, as retirees are disproportionately impacted by the drop in savings. Additionally, redistributing inheritance tax revenues would require raising the overall tax burden, which would negatively affect both human capital accumulation and economic growth. While higher disposable income for working-age cohorts could reduce inequalities within this group, it would simultaneously widen the gap between working individuals and retirees, further deepening overall inequality.

This study not only provides insights into the mechanism of inheritance taxation within the OLG model and its implications for Polish economic policy, but also offers a tool that can be enhanced and expanded to address other economic policy questions. First, for example, by incorporating additional tax structures, the interaction of inheritance tax within a broader tax context can be explored. Second, changing demographics and increasing life expectancy can be considered, which could be particularly important in addressing inequality, especially for older and retired cohorts. Third, various types of inequality can be analyzed—not only those driven by differences in human capital, but also disparities in life expectancy across different educational groups.

Moreover, in the context of potentially increasing the role of inheritance tax within the fiscal mix, in addition to welfare considerations, it is important to recognize that implementing such a solution may be practically unfeasible in Poland, even though it might intuitively seem fair. First, as previously mentioned, generational ties in Poland are strong, and the desire to influence the well-being of one's children is particularly pronounced. For individuals with strong preferences towards bequesting, inheritance tax would likely result in welfare deterioration, making it difficult to build political support for such a policy. Second, with an aging society, capital accumulation is increasing, meaning more people have assets to pass down. As a result, the issue of wealth transfer to children will affect an increasing number of voters, who are unlikely to support taxing themselves. Therefore, an interesting area for further development in the literature on inheritance taxation would be its expansion within the framework of political economy.

Appendix

Solving the consumer problem

Using the consumer maximization problem defined in (4) and denoting $(1-\tau^L-\tau^P)w_{t+1}(h_{i,t})^{1-\kappa} = \xi_i$, the first order conditions can be rewritten as:

$$
\begin{aligned}
c_{i,t}: \quad & \tfrac{1}{c_{i,t}} && = \lambda_t^1 \\
s_{i,t}: \quad & \lambda_{t+1}^2 R_{t+1} && = \lambda_t^1 \\
d_{i,t+1}: \quad & \tfrac{\beta}{d_{i,t+1}} && = \lambda_{t+1}^2 \\
e_{i,t}: \quad & \tfrac{\gamma_i \kappa \xi_i e_{i,t}^{\kappa-1}}{\xi_i e_{i,t}^{\kappa}+(1-\tau^B)b_{i,t+1}+a_{i,t+1}} && = \lambda_t^1 \\
b_{i,t+1}: \quad & \tfrac{\gamma_i(1-\tau^B)}{\xi_i e_{i,t}^{\kappa}+(1-\tau^B)b_{i,t+1}+a_{i,t+1}} && = \lambda_{t+1}^2
\end{aligned}
$$

Simplifying:

- Intratemporal choices:

$$
\begin{aligned}
c_{i,t} &= \frac{\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}}{\gamma_i \kappa \xi_i e_{i,t}^{\kappa-1}} \\
c_{i,t} &= \frac{\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}}{\gamma_i (1-\tau^B) R_{t+1}}
\end{aligned}
$$

- Euler equation:

$$c_{i,t} = \frac{d_{i,t+1}}{\beta R_{t+1}}$$

From intratemporal choices:

$$\gamma_i \kappa \xi_i e_{i,t}^{\kappa-1} = \gamma_i (1-\tau^B) R_{t+1}$$

Simplifying:

$$e_{i,t} = \left[\frac{(1-\tau^B)R_{t+1}}{\kappa\xi_i}\right]^{\frac{1}{\kappa-1}} \tag{40}$$

Using the second period budget constraint:

$$\frac{\beta\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right]}{\gamma_i(1-\tau^B)} + b_{i,t+1} = R_{t+1}s_{i,t} + p_{i,t+1}$$

Factoring out $s_{i,t}$:

$$s_{i,t} = \frac{\frac{\beta\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right]}{\gamma_i(1-\tau^B)} + b_{i,t+1} - p_{i,t+1}}{R_{t+1}}$$

Using the first period budget constraint:

$$\frac{\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}}{\gamma_i(1-\tau^B)R_{t+1}} + \frac{\frac{\beta\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right]}{\gamma_i(1-\tau^B)} + b_{i,t+1} - p_{i,t+1}}{R_{t+1}} + e_{i,t} = (1-\tau^L-\tau^P)w_t h_{i,t} + (1-\tau^B)b_{i,t} + a_{i,t}$$

Let's denote $(1-\tau^L-\tau^P)w_t h_{i,t} = \varsigma_i$. Hence:

$$\frac{\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}}{\gamma_i(1-\tau^B)R_{t+1}} + \frac{\frac{\beta\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right]}{\gamma_i(1-\tau^B)} + b_{i,t+1} - p_{i,t+1}}{R_{t+1}} + e_{i,t} = \varsigma_i + (1-\tau^B)b_{i,t} + a_{i,t}$$

Multiplying LHS by $\gamma_i(1-\tau^B)R_{t+1}$:

$$\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1} + \beta\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right] + \left[b_{i,t+1} - p_{i,t+1}\right]\gamma_i(1-\tau^B) + e_{i,t}\gamma_i(1-\tau^B)R_{t+1}$$

Rearranging:

- LHS:

$$(1+\beta)\left[\xi_i e_{i,t}^{\kappa} + (1-\tau^B)b_{i,t+1} + a_{i,t+1}\right] + \left[b_{i,t+1} - p_{i,t+1} + e_{i,t}R_{t+1}\right]\gamma_i(1-\tau^B)$$

- RHS:

$$\left[\varsigma_i + (1-\tau^B)b_{i,t} + a_{i,t}\right]\gamma_i(1-\tau^B)R_{t+1}$$

Factoring out $b_{i,t+1}$:

- LHS:

$$(1+\beta)(1-\tau^B)b_{i,t+1}+\gamma_i(1-\tau^B)b_{i,t+1}+(1+\beta)\left[\xi_i e_{i,t}^{\kappa} + a_{i,t+1}\right]+\left[e_{i,t}R_{t+1} - p_{i,t+1}\right]\gamma_i(1-\tau^B)$$

Hence:

$$b_{i,t+1} = \frac{\left[\varsigma_i + (1-\tau^B)b_{i,t} + a_{i,t}\right]\gamma_i(1-\tau^B)R_{t+1} - (1+\beta)\left[\xi_i e_{i,t}^{\kappa} + a_{i,t+1}\right] + \left[e_{i,t}R_{t+1} - p_{i,t+1}\right]\gamma_i(1-\tau^B)}{(1+\beta+\gamma_i)(1-\tau^B)}$$

Rearranging:

$$b_{i,t+1} = \frac{\left[R_{t+1}\left[\varsigma_i + (1-\tau^B)b_{i,t} + a_{i,t}\right] + e_{i,t}R_{t+1} - p_{i,t+1}\right]\gamma_i(1-\tau^B) - (1+\beta)\left[\xi_i e_{i,t}^{\kappa} + a_{i,t+1}\right]}{(1+\beta+\gamma_i)(1-\tau^B)}$$

In the steady state $b_{i,t+1} = b_{i,t} = b$ and all the time indices can be dropped:

$$b_i = \frac{\left[R\left[\varsigma_i + (1-\tau^B)b_i + a_i\right] + e_i R - p_i\right]\gamma_i(1-\tau^B) - (1+\beta)\left[\xi_i e_i^{\kappa} + a_i\right]}{(1+\beta+\gamma_i)(1-\tau^B)}$$

Factoring out b_i:

$$b_i = \frac{\left[R\left[\varsigma_i + a_i + e_i\right] - p_i\right]\gamma_i(1-\tau^B) - (1+\beta)\left[\xi_i e_i^{\kappa} + a_i\right]}{\left[(1+\beta+\gamma_i - R\gamma_i(1-\tau^B)\right](1-\tau^B)}$$

Inserting (40) to the above equation solves the consumer problem and ables to derive variables $b_i, c_i, s_i$and d_i using R, w and model's parameters only.

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