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Abstract

A model of cyclical signals, trend and noise is developed, allowing more than one cycle per each variable and the composition of cycles to differ for each macroeconomic aggregate. Estimation and best-model selection result in very good fit to the data, revealing that many cycles drive macroeconomic aggregates, which undergo fundamentally different cyclical fluctuations. The most pronounced cycles are those below the traditional business-cycle band. The constructed model allows to test whether the growth of an economy is balanced. Results demonstrate that economies may undergo unbalanced growth. It has been shown that small open economies' cycles are not necessarily driven by the rest-of-the-world conditions. A constructed method of evaluating whether government's consumption is procyclical and whether its noncyclical part is reactive to or actively shaping output growth is presented and applied. The devised framework constitutes a research and policy tool that enables to discover the rich cyclical structure of macroeconomies.

JEL codes: C51, C52, E32
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1. Introduction

The existing econometric models representing economic fluctuations uncover only a single cycle, or use just a single frequency for many stochastic cycles. Nonetheless, the actual process of the evolution of economy in time may be a composite of many cycles whose frequencies differ. Moreover, the hypothesis that frequencies with high, but not the highest values of the periodogram matter as well, i.e., that the actual economic processes consist of a few or many cycles, remains untested. If it is true, then the business-cycle fluctuations, which in fact are just another expression for cycles of economic growth, are still not fully understood.

Which economic cycles have the most influence on the growth rates of an economy? Surprisingly, the most commonly given answer has changed little since the seminal work of Burns and Mitchell (1946); the majority of econometric studies have focused on finding a single cycle with a period in the range of $6-40$ quarters on the basis of a filtered data's periodogram, or on using only one such cycle in estimation. Such an approach, however, raises a few questions. The first one is how much information on the full cyclical part of the studied process is lost by bandpass filtering and filtering in general. If the strength of higher- and lower-, than the usually studied band, frequency components is not assessed – due to the use of filters and made assumptions – and compared to the traditional business-cycle frequencies, then such a question will remain unanswered. The second issue is how much of the real-world process' path is actually explained by the discovered cycle? If a single cycle of frequency from the band traditionally accepted as 'business cycle frequencies' in fact fails to explain most of the non-filtered series' volatility, why would we treat it as economically relevant?

Furthermore, focusing on a single frequency, even if a stochastic cycle model is used, presumes that economic growth is balanced along the business cycle, in the sense that all macroeconomic sectors undergo the same, perfectly coordinated fluctuations. That precludes by assumption the possibility that various macroeconomic aggregates undergo fundamentally different fluctuations in terms of their amplitudes, frequencies and phases. Nonetheless, if the true data-generating process is composed of a few or more cycles with different periods, an assumption of a single stochastic cycle can lead to biased estimates and imprecise forecasts. Additionally, a model with a single stochastic cycle may blur the information of actual cyclical fluctuations, hindering our comprehension of economic activity.

The possibility of many cycles shaping economic fluctuations and potential differences of these variations between macroeconomic sectors matter for prediction of the evolution of those economy-wide aggregates. Identifying such various cyclical patterns would increase the understanding of economic processes and could provide invaluable policy guidance. A comparison of the strength of the cycles of which each of the macroeconomic variables, such as gross domestic product (GDP), capital formation, household or government consumption, are composed of would provide insight into what forces are driving these aggregates and whether the cycles of which they are composed are the same in terms of their periods and phases. In other words, an analysis and comparison of differing cyclical patterns of various macroeconomic aggregates would inform us about which variables lead others, which are synchronised, and at which frequencies the most pronounced cycles occur, providing potentially crucial insights for economic policy. Among else, this is what is presented in this paper.

A model of economic cyclical signals is created. After an evaluation of periodograms, explicit representations of various potentially relevant cycles of the gross growth rates of various macroeconomic aggregates are constructed and estimated, using data on a small open economy. This method allows to uncover that more than just one cycle is relevant in considerations of economic fluctuations and growth. Moreover, the most pronounced in terms of their contribution to the growth rates, and in some cases the only ones that are relevant, are cycles of periods below and at the bottom of the traditional 6-32 quarter band. For discovering this, however, not seasonally adjusted data is needed. This choice does not affect the estimates because these fluctuations are accounted for by the explicit high-frequency cycles (discussion regarding the trading days and holidays, and in general the methods of seasonal adjustment is contained in the next section).

Results show that in all considered cases, including the most studied one, the GDP itself, having more than one higher-frequency cycle in a model of economic growth is more justified than the single cycle of a period of $6-40$ quarters. That is, using a few cycles yields a better fit than including more or just one cycle in the model. This does not mean that cycles from the traditional business-cycle frequency band do not exist in real-world economies. Conversely, they appear among the top 12 values of periodograms of almost all series that were studied in this article, and were included in some of the best-fit models of gross growth rates, e.g., gross capital formation. Most of the studied aggregates are characterised by rich, but differing cyclical patterns, which in most cases account for a very large part of the data's volatility. The performed estimation and analysis show that not only the composition of the cyclical signal, but also the extent to which variables are driven by cycles differs. Such an investigation allows better understanding of the mechanisms of economic cycles and their role alongside trends and sector-specific idiosyncratic disturbances in the growth rates of macroeconomic sectors.

This highlights the second issue of interest, namely assessing whether economic growth is balanced or various macroeconomic sectors undergo different trends and patterns of fluctuations. The two issues are interrelated, given that in vast majority of econometric studies regarding the business cycle, both economic cycles and a trend are estimated. In this paper it is demonstrated, using a small open economy (Poland) as an example, that macroeconomic aggregates are characterised both by different cyclical patterns and different exponential trends. In light of these results, balanced growth assumptions made in theoretical macroeconomics may seem more controversial. These findings also raise questions about the often assumed common, or single, cycles assumptions. In light of this, the focus only on GDP's cyclical patterns in economic or econometric inquiries of the dynamics of business cycles turns out to be less justified.

By allowing a rich decomposition of the signal of a macroeconomic aggregate into various cycles, the devised framework provides a method to assess the procyclicality of government's expenditure, and activeness or reactiveness of its non-cyclical part. Due to the decomposition of the studies series into exponential trend, cycle and an error, the phases of the cycles of the same period/frequency in GDP and the government's consumption can be compared, showing which variable leads and which lags behind the other. Moreover, the relation between the noncyclical and non-trend part of the government's expenditure – or discretionary, if we interpret it as mostly intentional and conscious actions of the government – and the analogous part of the GDP is studied. In the particular case under investigation, the government's policy turns out to be mostly procyclical and reactive, having little impact on the GDP's evolution.

This differentiation of a government's expenditure into cyclical and noncyclical parts allows to assess the effectiveness of government policy in shaping economic growth and smoothing cycles, or to determine whether it is in fact procyclical and reactive. Thus, the framework and its possible extensions have the potential to become a valuable policy assessment tool.

Finally, the study of a small open economy allows also to shed light on the dependence of the patterns of various sectors' fluctuations on the foreign, or the 'rest-of-the-world', business cycle. By comparing the net exports' cyclicality and other macroeconomic aggregates' fluctuation patters, we can see that little of such a transmission actually occurs. The only exceptions are large crises like the one of 2008 or the Covid pandemic. The estimated models, however, suggest that such crises seem to be rare, specific shocks to economic activity stemming from some structural problems, uncommon government policies (such as lockdowns), or chance rather than repeatable cyclical movements.

This article is constructed as follows. In section 2, the discussion of the developments in econometric modelling of economic cycles is presented. The following section 3 contains the proposed approach and research design. In the next section, 5, the author briefly describes the data used in estimation. Section 3.6 contains the results and their discussion; it is followed by conclusions in section 6, and an appendix justifying a few claims made in section 2.

2. Extracting economic cycles from the data

Business-cycle fluctuations are one of the core research areas both in empirical and theoretical economics. Harding and Pagan (2005) claimed that the term 'cycle' had rarely been precisely defined in empirical studies, and had in fact been applied to describe different entities. They argued that what is understood by a cycle depends on what type of time series is examined, and pointed to the fact that the existing studies can be divided into three groups. Their classification remains valid, except for minor rephrasings, therefore it is repeated here – the cycles investigated in econometric literature are derived from:

(i) The level of a macroeconomic aggregate, or a monotonic transformation of the data, such as the logarithm.

(ii) The level of a macroeconomic aggregate less a permanent component, i.e., a cycle is obtained by decomposing the data series into trend, cycle and noise (or possibly also some other) components.

(iii) The growth rate of a macroeconomic aggregate.

As for what the term 'cycle' means, Harding and Pagan (2005) identified two definitions that were widespread in the literature, depending on the purpose of the study. One is by the means of spectral analysis, or simple trigonometric functions. The second definition is that a cycle is identified by the consecutive turning points in a studied series, interpreting them as peaks and troughs of an economic business cycle.

In terms of modelling approach, econometric studies on economic cycles can be differentiated with respect to the used method of representing cycles. Three different approaches have been used in the literature for this purpose: Markov chains, sinusoidal functions, using trigonometric functions or rotation matrices in conjunction with spectral analysis, and nonlinear filters such as wavelet filters.

The first two methods have started to be used in about the same time by economists (e.g., Hamilton (1989), A. C. Harvey (1985), and Harrison and Akram (1983)). However, modelling cycles by the means of trigonometric functions has become much more prevalent than the Markov-chains models of economic fluctuations. First, because they are relatively easier to implement. The second reason is that they require either much less assumptions on the structure of the business cycle process, or provide a more detailed description of the business cycle. Models based on Markov switching regression, such as the one developed by Hamilton (1989), basing on Goldfeld and Quandt (1973), often measure only the turning points of business cycles, and differentiate between recession and lack of thereof, or between a fast growth or slow growth phase. The switch between the two is governed by the outcome of a Markov process.

In general, in Markov switching models of business cycles, economic growth is represented as an autoregressive process subject to parameter shifts. This process is modelled as a Markov chain. This allows to recreate the asymmetries in the depth and duration of output growth phases (Smith and Summers 2005). Kim et al. (2005) have improved the original model of Hamilton (1989) so that it exhibits a rebound after a recession period. Smith and Summers (2005) based their model o Paap and Van Dijk (2003) Paap and van Dijk (2003), decomposing the vector of the logarithms of real GDP levels into a trend component and a VAR process. There is also a sizeable related literature that has been published in the 1990s; nonetheless, the sinusoidal, that is, using explicit trigonometric functions or rotation matrices, representations of business-cycle fluctuations have become much more popular in the last two decades.

The most common approach to modelling cycles is by the use of the cyclical signal model. It can be divided into two general groups of variants. The first one consists of those featuring the sinusoidal signal model – as described, e.g., by Kay (1993), that is, a model in which the signal has the form of a sum of cyclical functions of different frequencies, driving the original series. Mathematically, it can be thought as of an extension of the Discrete Fourier Transform to incorporating elements of different frequencies. The second group features models in which a series is decomposed into components such as trend and cycle, and the latter evolves according to a rotation matrix (and an auxilliary term). This division roughly corresponds to dividing econometric studies of business cycle fluctuations into those featuring deterministic cycles within a stochastic model, and those in which stochastic cycles are present.

One of the main reasons why deterministic cycles have been rarely used in the field of econometric business-cycle studies is that the research practice has been to search for and use only a single frequency of a cycle. This remains true for all of the versions of the stochastic cycle model.

One of the earliest formulations and applications of stochastic cycles, using economic time series, were Harrison and Akram (1983) and A. C. Harvey (1985). This modelling method is characterised by assuming that the cyclical element evolves according to a rotation matrix, altered in each period by stochastic innovations drawn from a distribution of presupposed type:

$$\begin{aligned}
\psi_t &= \rho\cos(\lambda)\psi_{t-1} + \rho\sin(\lambda)\psi^*_{t-1} + \xi_t, &(1)\\
\psi^*_t &= -\rho\sin(\lambda)\psi_{t-1} + \rho\cos(\lambda)\psi^*_{t-1} + \xi^*_t, &(2)\\
\xi_t, \xi^*_t &\sim NID(0, \sigma^2_\xi), &(3)
\end{aligned}$$

where λ denotes the single frequency, ρ is the damping factor, ψ_t is the cyclical component's value in time t, and ψ^*_t is an artificial variable, needed for the construction of the stochastic cycle ψ_t. ξ_t and ξ^*_t are random shocks.

A. Harvey and S. Koopman (1997) have extended this model into a multivariate version, but, as argued by S. J. Koopman and Valle E Azevedo (2008), various specifications for the multivariate cycle component. In their paper, they have used the following specification:

$$\begin{aligned}
\psi_{t+1} &= \phi\cos(\lambda)\mathbf{I}_N\psi_t + \phi\sin(\lambda)\mathbf{I}_N\psi^*_t + \kappa_t, &(4)\\
\psi^*_{t+1} &= -\phi\sin(\lambda)\mathbf{I}_N\psi_t + \phi\cos(\lambda)\mathbf{I}_N\psi^*_t + \kappa^+_t, \quad t = 1, \dots, n. &(5)
\end{aligned}$$

where ϕ is the damping factor and κ_t, κ^+_t are the shocks.

Another specification is the n-th order stochastic cycle Andrew C. Harvey and Trimbur (2003):

$$\begin{aligned}
\psi_{1,t} &= \rho\cos\lambda_c\,\psi_{1,t-1} + \rho\sin\lambda_c\,\psi^*_{1,t-1} + \kappa_t, &(6)\\
\psi^*_{1,t} &= -\rho\sin\lambda_c\,\psi_{1,t-1} + \rho\cos\lambda_c\,\psi^*_{1,t-1}, &(7)\\
\psi_{i,t} &= \rho\cos\lambda_c\,\psi_{i,t-1} + \rho\sin\lambda_c\,\psi^*_{i,t-1} + \psi_{i-1,t}, &(8)\\
\psi^*_{i,t} &= -\rho\sin\lambda_c\,\psi_{i,t-1} + \rho\cos\lambda_c\,\psi^*_{i,t-1}, \quad i = 2, \dots, n. &(9)
\end{aligned}$$

note that this formulation presumes the same damping factor ρ and frequency λ_c on every level.

A third variant of representing cycles within frequency-domain group of approaches, the similar cycles model, shares these constraints with the stochastic cycle model; although this formulation consists of many cycles, the sinusoidal components' frequency and the damping factor of the rotation matrices are the same across series, and the error terms of different cycles are allowed to be correlated (A. Harvey and S. Koopman 1997). The common cycles model (also described by A. Harvey and S. Koopman (1997)), on the other hand, is even more restrictive: it imposes the same linear combinations of the same, possibly many, cycles for each of the modelled variables.

The k-th order common cycle model created by Valle E Azevedo et al. (2006) shares these limitations, as can be seen from its formulation (Valle E Azevedo et al. 2006):

$$\psi_{t+1}^{(j)} = \phi\cos(\lambda)\psi_t^{(j)} + \phi\sin(\lambda)\psi_t^{(j)*} + \psi_t^{(j-1)}, \tag{10}$$
$$\psi_{t+1}^{(j)*} = -\phi\sin(\lambda)\psi_t^{(j)} + \phi\cos(\lambda)\psi_t^{(j)*} + \psi_t^{(j-1)*}, \quad j = 1, \ldots, k, \tag{11}$$

with

$$\psi_t^0 \sim \kappa_t \sim N(0, \sigma_k^2), \tag{12}$$
$$\psi_t^{0+} \sim \kappa_t^+ \sim N(0, \sigma_k^2). \tag{13}$$

Another multivariate version of the stochastic cycle model has been constructed by Carvalho and Andrew C. Harvey (2005). In their variant, all of the cycles evolve according to the same rotation matrix, but are subject to different shocks. Convergence components were combined with a common trend and similar cycles, with the former formulated as a second-order error correction mechanism.

The stochastic cycle approach represents a significant advancement in the modelling of cyclical dynamics of real-world economies. These methods offer simple, but very flexible models for representing a cyclical component of a time series, helping to uncover the business cycle and differentiate it from non-cyclical movements in the data. However, there are situations where the underlying assumption of this type of model may be too stringent and thus it might be less effective in uncovering the true cyclical properties of a stationary or non-stationary time series compared to an alternative approach. The alternative is to decompose the series of interest into a sum of sinusoidal signals, in the fashion similar to the Discrete Fourier Transform extended to different frequencies.

A key limitation of the stochastic cycle approach is its reliance on the single frequency assumption. This rules out the presence of multiple, overlapping cycles of distinct frequencies in the studies series. In effect, we risk fitting a model which does not accurately represent the underlying data-generating process, and we forego the possibility to discover the true cyclical processes that drive the data's dynamics.

Moreover, the assumption of a single frequency precludes the comparison of strength of cycles associated with different periods. Similarly, filtering or seasonally adjusting the series before fitting a model may make or makes impossible the assessment of the actual role of relatively high-frequency fluctuations of periods of 2-5 quarters. The regularities in the data that are usually treated as 'seasonality' understood as cyclical noise of little interest to the researcher, may actually reflect structural economic activity that shapes growth. Instead of including seasonal indicator variables, seasonal cycles can be explicitly modelled as additional cyclical components.

Another challenge with stochastic cycles lies in their implicit treatment of amplitudes and phases as stochastic. While this flexibility can adapt to time-varying cycles, it can also lead to blurred identification of specific frequencies and obscure the role of cyclical patterns induced by cycles of different periods. This may happen if the true data-generating process contains stable and repeating cycles that can be successfully modelled as deterministic.

Stochastic cycle models risk overfitting noise if cycles are poorly identified or the model assumes unnecessary complexity (e.g., imposing a cycle on noise-dominated data). Methods based on the sinusoidal signal model can better avoid spurious cycles by explicitly inspecting the spectral properties of the data and selecting significant frequencies. The flexibility of stochastic cycles, while a strength in some contexts, can lead to potential overfitting. This is especially true when cycles are poorly identified, or when the data includes significant noise that the model may interpret as shocks to the cyclical pattern.

In summary, while the stochastic cycle approach has made invaluable contributions to time-series analysis, the sinusoidal signals model, based on deterministic cycles, presents an opportunity for uncovering multiple cycles that drive the data-generating process. By explicitly modelling variables as a sum of sinusoidal functions, each with its own frequency, amplitude, and phase, this method enables a more detailed exploration of the cyclical properties of time series, and can provide both better fit and greater insights into the underlying dynamics of the data.

2.1. The potential distortions of seasonal and calendar adjustment procedures

As for the pre-processing of data to be used in econometric modelling of economic cycles, the most commonly used seasonal and calendar adjustment procedures are the TRAMO-SEATS – Time Series Regression with ARIMA Noise – and X-13-ARIMA-SEATS. Some institutions still use the predecessors of X-13-ARIMA-SEATS, i.e., the X-12- or X-11-ARIMA procedures (Eurostat 2024).

The TRAMO-SEATS (Time Series Regression with ARIMA Noise, Missing Observations, and Outliers - Signal Extraction in ARIMA Time Series) procedure is widely used for seasonal adjustment in economic and statistical applications. It operates as a two-step process: TRAMO handles the modeling and preprocessing of the series, while SEATS decomposes the series into components using the ARIMA model provided by TRAMO. The TRAMO-SEATS procedure consists of several steps.

The first two constitute the TRAMO sub-procedure: initially, the identification of an ARIMA model for the processed time series, handling missing data, outliers, and calendar effects is performed. The second step is the preparation of the series for decomposition by removing nonseasonal and calendar-related distortions.

In the SEATS part of the procedure, the time series is decomposed into components: trend-cycle, seasonal, irregular, and potentially calendar effects.The ARIMA model estimated by TRAMO is used to extract these components (Gómez and Maravall 1996).

Seasonal adjustment in the TRAMO-SEATS procedure relies on spectral decomposition based on the ARIMA model estimated for the time series. While it effectively removes seasonal components, it can introduce distortions in the spectrum beyond the targeted seasonal frequencies. These distortions arise from differencing operators, the frequency-domain filtering process, and any mis-specifications in the ARIMA model or seasonal decomposition. These distortions can severely affect the spectrum of the series, particularly when the true data-generating process (DGP) consists mostly of cycles with periods of 2, 4, and 5 quarters. The TRAMO-SEATS seasonal adjustment procedure can have disastrous consequences for the DGP described, particularly due to the suppression of the 4-quarter cycle. This frequency is indistinguishable from seasonal variation and is fully removed by seasonal differencing. Additionally, the 2-quarter cycle is partially attenuated, and the the 5-quarter cycle is moderately affected as well. Due to the use of the SEATS procedure, similar dangers are associated with the use of the X13-ARIMA-SEATS. Moreover, the older versions, X11- and X12-ARIMA use ARIMA models, and thus face the same dangers as the two previously mentioned procedures.

The trading-days and holiday-related corrections may be problematic as well. The assumption that firms produce or sell the same amount every day, regardless of the number of working days in a quarter, can introduce systematic biases into the seasonal adjustment and forecasting process. This assumption neglects strategic behaviour by firms, such as intensifying production or sales efforts during specific periods (e.g., days within shorter quarters) to achieve quarterly or annual targets. If firms disproportionately increase sales on specific days (e.g., weekends, holidays), the model will systematically underestimate quarterly sales in shorter quarters.

In this paper, it is proposed to investigate the cyclical characteristics of macroeconomic time series in an agnostic manner, that is, by handling fluctuations at frequencies typically interpreted as seasonal directly, using the same sinusoidal signal model as for the lower-frequency components, and without distorting the data by trading-day and holiday adjustments. The details of the proposed procedure are described in the following section.

3. The agnostic approach to modelling cycles

3.1. Overview

Presented research was designed to answer the questions of what cycles are driving economic growth of various macroeconomic aggregates, whether they are common or differ substantially, and how much of their growth can be explained when identifying the cycles using a single periodogram for the entire sample. Because cycles of short periods, usually deemed as seasonals or not true business-cycle frequencies, are modelled explicitly, not-seasonally-adjusted data is used. Its use has been justified also in the previous section. The results show that the models are highly successful in explaining the evolution of the data.

The analysis is performed on gross growth rates of various macroeconomic aggregates. The first reason is that for such data, we do not need to model the trend and cycle components as nonstationary. The second is that most estimation techniques would require transforming the data in levels by first differencing, which would cause a phase shift. Thirdly, we have a correspondence between the two models. Consider the following trend-plus-cycle model for some variable Y:

$$Y_t = \mu_t^Y + \psi_t^Y + \epsilon_t^Y, \tag{14}$$

where μ_t^Y is a stochastic trend, ψ_t^Y is the nonstationary cycle component – nonstationary, because to preserve its mean impact over time, its amplitude(s) has to increase – and ϵ_t^Y is an error term. Now, assuming that we can represent the gross growth rate of Y as featuring a constant, exponential trend, a cycle, and error:

$$G_t^Y = \mu^{G^Y} + \psi_t^{G^Y} + \epsilon_t^{G^Y}, \tag{15}$$

then we have

$$Y_{t+1} = (\mu^{G^Y} + \psi_t^{G^Y} + \epsilon_t^{G^Y}) \cdot Y_t, \tag{16}$$

or:

$$\mu_{t+1}^Y + \psi_{t+1}^Y + \epsilon_{t+1}^Y = \mu^{G^Y} \cdot Y_t + \psi_t^{G^Y} \cdot Y_t + \epsilon_t^{G^Y} \cdot Y_t. \tag{17}$$

Thus, we can easily connect the corresponding trend, cycle and error components of the two models:

$$\begin{aligned} \mu_{t+1}^Y &= \mu^{G^Y} \cdot Y_t, & (18) \\ \psi_{t+1}^Y &= \psi_t^{G^Y} \cdot Y_t, & (19) \\ \epsilon_{t+1}^Y &= \epsilon_t^{G^Y} \cdot Y_t. & (20) \end{aligned}$$

One of the crucial assumptions underlying research design is to use not seasonally-adjusted data. This is because most of the deseasoning techniques utilise linear filters, which significantly alter, or even distort the spectrum (especially if they are not symmetric). Most of all, they are designed to remove the 'seasonal' components, a procedure which removes the cyclicality at high frequencies, which is shown in this paper to be a key component of fluctuations of most of macroeconomic aggregates and their growth rates. What is crucial is that their inclusion in the model allows to achieve very good fit to the data.

The steps of the presented research procedure are as follows. First, the periodograms of the gross growth rates of each macroeconomic aggregate are computed. K=12 frequencies with the highest periodogram values are selected. Next, cyclical signal models of the gross growth rates are computed, for decreasing sets of frequencies, with and without intervention variables, until there is no more improvement in the mean squared error of the fit. That is, the amplitudes and phases of the sinusoidal signal's components are estimated, if the matrix of explanatory variables (the signals) is well-conditioned. The fit to the actual data series is calculated and assessed in terms of the mean squared error.

Procyclicality of government's consumption is assessed by comparing phases of cyclical components of the same frequency, shared by GDP and government consumption. The noncyclical parts of a variable are defined as the best-fit model series subtracted from the data. Some of these are used in the regressions explaining the growth of noncyclical parts of GDP or government consumption, which are aimed at assessing whether these discretionary components of government policy were reactive or active. The total of 100, Newey-West robust, regressions were calculated, in the spirit of Leamer's approach to credible estimation (Leamer 1983).

3.2. Periodogram calculation

Calculation of the periodogram is standard and follows Kay (1993). First, the Toeplitz autocorrelation matrix is computed. Each of its elements depends only on the time lag, defined by the absolute difference between the values of its coordinates.

$$R = [r(k)], \tag{21}$$

$$r(k) = \frac{1}{T\hat{\sigma}^2} \sum_{n=0}^{T-k-1} (x_n - \hat{\mu}_x)(x_{n+k} - \hat{\mu}_x), \tag{22}$$

where $\hat{\mu}_x$ is the sample mean, and $\hat{\sigma}^2$ is the sample variance.

$$\begin{aligned} \hat{\mu}_x &= \frac{1}{T}\sum_{n=0}^{T-1} x_n, & (23)\\ \hat{\sigma}^2 &= \frac{1}{T}\sum_{n=0}^{T-1} (x_n - \mu_x)^2. & (24) \end{aligned}$$

For convenience we may define

$$r_{xx}(k) = \begin{cases} R(0,k) & \text{if } k > 0, \\ R(k,0) & \text{if } k < 0, \\ R(k,k) & \text{if } k = 0. \end{cases} \tag{25}$$

The periodogram, denoted below as $PRG(f)$ estimates the power of the time series at a given frequency f, for a chosen set of frequencies F. The values of the periodogram are calculated as follows:

$$PRG(f) = \frac{1}{T}\sum_{k=-(T-1)}^{T-1}\left(1-\frac{|k|}{T}\right) r_{xx}(k)\cos(2\pi f k) \tag{26}$$

$$+ \frac{1}{T}\sum_{k=-(T-1)}^{T-1}\left(1-\frac{|k|}{T}\right) r_{xx}(k)\sin(2\pi f k). \tag{27}$$

3.3. Cyclical components' parameters estimation

The approach to the selection of cycles for the signal taken in this paper is to attempt to fit the sum of cyclical functions (one function per included frequency) to the data, for various subsets of included frequencies, starting from those that correspond to the twelve highest periodogram values. For some of these subsets the estimation will be infeasible due to collinearity in the sinusoidal regressors matrix – in such cases, the given subset of frequencies is rejected. In some other cases, even though estimation is numerically feasible, the matrix of regressors will have high condition number; such cases correspond to the high or very high mean squared errors of regression fit. They are displayed to show how the inclusion of too many or too few frequencies deteriorates the fit.

The procedure for the estimation of amplitudes and phases of follows the method of estimating the sinusoidal signal model, described by Kay (1993), which has been extended in this paper to include many cycles of different frequencies. Each variable of interest (a macroeconomic aggregate; below denoted as x) is decomposed into cyclical signals, a constant, optionally also intervention variables, representing non-cyclical shocks that could distort the estimation, and a residual:

$$x_t = \sum_f \alpha_f \cos(2\Pi\cdot\lambda_f\cdot t+\phi_f)+\sum_{n=0}^{N^{INT,x}}(\mathbf{1}(t=Int_n^x)\cdot\gamma_n)+c_x+\epsilon_t; \quad t=0,1,...,T-1. \tag{28}$$

where λ_f denotes frequency f, ϕ_f is the cycle-f-specific phase shift, α_f is the amplitude of the cyclical component associated with frequency λ_f. The second summation (optional) contains variable-x-specific intervention variables; c_x is a constant term, while ϵ_t is time-t error. We will denote the constituent part of the signal, associated with this frequency, as $s_{f,t}$, so that we can write

$$x_t = \sum_f (s_{f,t}) + \sum_{n=0}^{N^{INT,x}} (\mathbf{1}(t = Int_n^x) \cdot \gamma_n) + c_x + \epsilon_t; \qquad t = 0, 1, ..., T-1. \quad (29)$$

The objective function for the least-squares estimate is nonlinear, but Kay (1993) notes that we can transform it into linear form by using the fact that

$$\alpha_f \cos(2\Pi \cdot \lambda_f \cdot t + \phi_f) = \alpha_f \cos(\phi_f) \cos(2\Pi \cdot \lambda_f \cdot t) - \alpha_f \sin(\phi_f) \sin(2\Pi \cdot \lambda_f \cdot t) \quad (30)$$

and defining $\eta_{1,f} := \alpha_f \cos(\phi_f)$, $\eta_{2,f} := \alpha_f \sin(\phi_f)$, so that the problem becomes linear in parameters. In matrix form,

$$\mathbf{s} = H\eta. \quad (31)$$

H can be decomposed as

$$H = [H_{f_0} H_{f_1} ... H_{f_{F-1}} G], \quad (32)$$

where G contains the interventions and the constant term. H_f is defined as

$$H_f = \begin{pmatrix} \cos(2\Pi \cdot \lambda_f \cdot (0+59)) & \sin(2\Pi \cdot \lambda_f \cdot (0+59))] \\ \cos(2\Pi \cdot \lambda_f \cdot (1+59)) & \sin(2\Pi \cdot \lambda_f \cdot (1+59)) \\ \vdots & \vdots \\ \cos(2\Pi \cdot \lambda_f \cdot ((T-1)+59)) & \sin(2\Pi \cdot \lambda_f \cdot ((T-1)+59)) \end{pmatrix} \quad (33)$$

for every frequency f in the set of frequencies chosen to be included in the cyclical signal model. The phase shift of 59 is imposed on purpose; starting with $t = 0$ forces all cosine terms to equal $cos(\psi_f)$, making the model sensitive to initial conditions and artificially amplifying discrepancies between the model and the data. Shifting the time index by adding around half of the series' length to all time indices of functions in the H_f matrix, effectively balances the cosine terms around their average contributions. It reduces the large initial contributions from $cos(\psi_f)$ at $t = 0$ and spreads them more evenly across the series. This prevents the phase shift misalignment and produces a much better fit. The shift of the time index has been chosen as 59 rather than 58.5 (the mean and the median are 58.5 for periods $0 - 117$), in order to preserve integer values of periods in the functions of H_f.

Of course, the formula 33 requires modification if frequency $\frac{1}{2}$ is among the ones chosen to be included in the model (because $\forall_{k \in \mathbb{Z}} sin(\Pi \cdot k) = 0$).

In such a case, the corresponding H_f matrix consists of only one column, containing the cosine terms.

The least-squares estimate of the transformed coefficients is

$$\hat{\eta} = (H^T H)^{-1} H^T x. \tag{34}$$

Inverse transformation for obtaining the estimates of frequency-f-related amplitude, phase, coefficients on the intervention variables and the constant (where f is the index from 0 to F) can be derived as presented in the equations (35)-(38). The notation of the indices of η parameters has been changed to match the coordinates of the entries of the $\hat{\eta}$ matrix.

$$\begin{aligned}
\hat{\alpha}_f &= ((\hat{\eta}_{2f})^2 + (\hat{\eta}_{2f+1})^2)^{\frac{1}{2}}, && (35)\\
\hat{\phi}_f &= \arctan(\frac{-\hat{\eta}_{2f+1}}{\hat{\eta}_{2f}}), && (36)\\
\hat{\gamma}_n^{INT} &= \hat{\eta}_{1+2(F-1)+n}, n = 0, ..., N^{INT,x} - 1 && (37)\\
\hat{\gamma}^{CONST} &= \hat{\eta}_{1+2(F-1)+N^{INT,x}}. && (38)
\end{aligned}$$

when frequency $\frac{1}{2}$ is present; otherwise, in the last two equations $1+2(F-1)$ is replaced by $2F$.

After estimating the parameters, the estimated model series is

$$\hat{s}_t = \sum_f \hat{s}_{f,t} = \sum_f (\hat{\alpha}_f \cos(2\Pi \cdot f \cdot t + \hat{\phi}_f)). \tag{39}$$

3.4. Comparison across models: feasibility and a model's fit

In some cases, mostly for the larger sets of included frequencies (10-12), the H matrix is either not invertible due to very little differences between the frequencies of the lowest power, or ill-conditioned. In these cases, either the given version of the model could not have been estimated, or the estimation produced a very high mean squared error due to the ill-conditioned matrix of regressors. These two cases can be seen in the results' tables as 'X' or very high mean squared error values.

After the amplitudes and phases of cycles with a given set of frequencies have been estimated, the model values are constructed, and the mean squared error of their fit to the data is calculated. The version of the model (the set of included frequencies) with the smallest mean squared error is chosen as the best estimate, and in the case of some variables, it is the one used in further

regressions, serving the evaluation of activeness of a small open economy's government policy.
The fit of each version of the model (as above, the variants differ in the number of included frequency-specific cycles) is evaluated on the basis of the mean squared error (MSE) of its fit for the entire sample. The MSE is standard,

$$MSE^{x,version} = \frac{1}{T}\sum_{t}(x_t - \hat{s}_t)^2. \tag{40}$$

3.5. Robustness check: comparison with models built on winsorized data

As a robustness check, it has been verified whether using winsorized data instead of crude data paired with intervention variables can provide better fit. Winsorization has been conducted very conservatively: only the observations of the extreme 2 percent were winsorized (i.e., those above the 99th or below the 1st percentile). The implemented procedure was the same, but conducted only for the GDP series. The best version's fit was quite good in terms of the MSE, but worse than the one of the most successful model using crude data and intervention variables; these results are located in the Online Appendix. Moreover, as can be seen from the periodograms of winsorized data in the Online Appendix, winsorizing the data leads to the loss of some of the information carried by the less pronounced frequencies. This is likely the reason for the slightly worse fit. This is not surprising, given that winsorizing is a simple, theory-free mechanical procedure.

3.6. Evaluation of pass-band filters performance with the cyclical signal model: the example of the Baxter-King filter

Band-pass filters have been widely used in business cycle analysis in the last thirty years (Nachane and Chaubal 2022; Pollock 2016; Creal et al. 2010; Christiano and Fitzgerald 2003; Baxter and King 1999). This widespread practice often reduced to identifying a single peak in the spectrum and interpret it as corresponding to 'the' business cycle. Another approach has been to accept more than one frequency, classify them according to sets of intervals of cycles' periods, e.g., 2-8, 8-20 and 20-40 years, and then to apply a bandpass filter for extracting the cycles from a given set, as in Christiano and Fitzgerald (2003).

However, in neither of the two procedures has the fit of the cycle, or of the model it was embedded in, to the data been included. Instead, researchers have compared the output of various filters, or computed correlations between the filtered and raw data, or evaluated how close is a filter to the optimal one; the work of Christiano and Fitzgerald (2003) is an example of both approaches. Thus, it usually remained unknown how good a fit to the data is provided by the extracted cycle or their linear combination, except for calculating simple correlations.

Christiano and Fitzgerald (2003) concluded that what they have termed the trigonometric regression filter performs worse as a bandpass filter than the one they have proposed. The measure of performance, however, is the distance from the optimal bandpass filter, not the fit of the model of the signal to the data. Moreover, as in virtually all econometric studies of business cycles, Christiano and Fitzgerald or Baxter and King dismiss all frequencies corresponding to periods below two years as potential relevant for the economic fluctuations process ((Christiano and Fitzgerald 2003; Baxter and King 1999)). In this paper it is demonstrated that it is a severe omission, negatively affecting both the discovery of the economic cycles that are dominant in shaping growth of macroeconomic aggregates and accurate modelling (fitting) the data.

3.7. Evaluating procyclicality/reactiveness of a small open economy's government policy

By assumption, the cyclical signal model used in this paper allows more than one frequency to be included, therefore the term 'procyclicality' will be understood in two ways. The first one is assessing the alignment of periods of the cycles driving these two variables' growth rates. The second is evaluating the phase shift between common cycles of the two macroeconomic aggregates; if, for a given common cycle of a certain frequency, the cycle of GDP precedes the one of government consumption, the latter is deemed to be reactive.

4. Data

Due to the availability of long, quarterly not-seasonally-adjusted data and that one of the applications of the devised frameworks is the test of whether small open economies' cycles are always driven by foreign cycles, transmitted through net exports, the used data concerns the economy of Poland. It is one of the largest economies of the European Union, roughly among the top twenty world economies, but nonetheless considerably smaller than those that can be considered large open economies. The source of data used in this article is Eurostat's 'GDP and main components' database spreadsheet, containing quarterly data that span from the first quarter of 1995 to the second quarter of 2024. The set of considered variables consists of gross domestic product (GDP), final consumption expenditure of households, which will be termed household consumption from now on, consumption of non-profit institutions serving households, gross capital formation, gross capital formation – fixed assets, inventories, exports of goods, imports of goods, exports of services, imports of services, compensation of employees, wages and salaries,, operating surplus and mixed income, taxes on production and imports less subsidies, subsidies, final consumption expenditure of general government. Net exports of goods and services were constructed on the basis of the four series of exports and imports.

Raw quarterly gross growth rates were calculated from levels of each of the considered variables, and used for periodogram analysis and estimation of all the models, both the cyclical signal models and regressions of non-cyclical parts of variables.

5. Results: unbalanced macroeconomic growth, procyclical and reactive government economic policy

5.1. Potential cycles

For each variable, representing a macroeconomic aggregate, a periodogram had been calculated, from which the frequencies corresponding to the twelve highest periodogram values were chosen. The cycle periods that are associated to these frequencies are shown in Table 1 in a descending order with respect to their periodogram value.

Looking at the results of the calculation of the periodograms of the gross growth rates for each variable we can group the variables according to specific criteria. First, considering which period-of-a-cycle has the highest periodogram value (which period is the dominant one), we can see that the periodograms of most variables share a dominant period of 2. These include Gross Domestic Product (GDP), final consumption expenditure of households (CONS_HSH), gross capital formation (GCF), gross capital formation of fixed assets (GCF_FA), Exports of goods (EXPR_GOODS), Exports of services (EXP_SRVCS), Imports of goods (IMP_GOODS), Compensation of employees (CMPNS_EMPLYEES), operating surplus and mixed income

Table 1 – Periods of potential cycles identified using the periodograms, in a descending order according to the corresponding periodogram values

Variable	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
GDP (GDP)	2	4	5	6	23	24	22	25	7	13	15	14
Household consumption (CONS_HSH)	2	4	5	23	24	117	116	115	114	113	112	111
Consumption of NPISH (CONS_NPISH)	4	2	5	23	10	24	11	59	58	60	57	61
Gross capital formation (GCF)	2	4	5	6	7	3	8	15	14	10	9	11
Gross capital formation – fixed assets (GCF_FA)	2	4	5	6	7	3	8	9	10	11	14	23
Inventories (Inventories)	15	23	24	14	22	25	16	17	117	116	115	114
Exports of goods (EXPR_GOODS)	2	5	10	4	15	14	23	20	19	24	11	22
Imports of goods (IMP_GOODS)	2	15	14	19	20	4	18	10	21	13	16	8
Exports of services (EXP_SRVCS)	2	3	4	5	8	10	14	15	38	39	40	41
Imports of services (IMP_SRVCS)	4	2	5	23	24	3	6	25	10	22	26	27
Net exports (NX_GDS_SRVCS)	3	9	12	109	110	111	112	113	114	115	116	117
Compensation of employees (CMPNS_EMPLYEES)	2	4	23	24	5	22	25	59	58	60	57	61
Wages and salaries (WAGES_SALARIES)	4	2	23	24	5	22	25	13	39	40	38	41
Op. surplus and mix. income (OP_SRPL_MIX_INC)	2	4	5	6	7	15	14	8	23	24	13	25
Taxes (prod., imp.) less subsidies (TX_LSS_SBS)	4	2	5	3	7	13	19	20	11	6	14	15
Subsidies (SBSDS)	2	4	117	116	115	114	113	112	111	110	109	108
Consumption of gen. gov. (CONS_GOV)	2	4	5	6	7	23	24	10	22	25	13	11

Table 2 – Periods of potential cycles corresponding to the 12 highest periodogram values for each considered macroeconomic aggregate.

(OP_SRPL_MIX_INC), subsidies (SBSDS), and final consumption expenditure of general government (CONS_GOV). Among the exceptions in terms of the period having the dominant value in a periodogram, there are final consumption expenditure of non-profit institutions serving households (CONS_NPISH), wages and salaries (WAGES_SALARIES) and taxes less subsidies (TX_LSS_SBSDS). Both of these variables' periodograms suggest period 4 as the dominant one. Moreover, changes in inventories (Inventories) seem to be dominated by period 15, and net exports of goods and services (NX_GDS_SRVCS) by period 3.

When looking at the period-of-a-cycle with the second highest periodogram value, for most variables these are periods 4 or 2 (if the highest value was achieved for period 4).

Except for net exports and subsidies, the cycles corresponding to top six periodogram values have periods less than 24 quarters. This is in line with the usual view and definitions of business cycles, limiting business-cycle frequencies to frequencies $\frac{1}{6} - \frac{1}{32}$ or $\frac{1}{6} - \frac{1}{40}$. Nonetheless, for each of the majority of the studied macroeconomic aggregates, the three most pronounced – as indicated by the periodogram – cycles have periods of 2, 4, 5 quarters.

For many variables, the first cyclical component – of frequency 0.5 in these cases – has an initial phase shift equal to $\Pi = 3.14159265...$. This group includes GGR_GDP, GGR_CONS_NPISH, GGR_GCF, GGR_GCF_FA, GGR_EXPR_GOODS, GGR_IMP_GOODS, GGR_CMPNS_EMPLYEES, GGR_OP_SRPL_MIX_INC, and GGR_CONS_GOV. First, this is because they share a perfectly synchronised common cycle of a short period (2 quarters), which turns out to be a dominant one for many of these macroeconomic aggregates, as can be later seen in Table 17. Another, small group of variables – GGR_CONS_HSH, GGR_EXP_SRVCS – lags behind exactly one period of this short cycle. The behaviour of households' consumption can be treated as evidence of consumption tracking current rather than permanent income.

As for the dominant, four-quarters-period cycles of IMP_SRVCS, WAGES_SALARIES and TX_LSS_SBS, their phases suggest that these cycles are somehow related to the two-quarters periods of at least some other macroeconomic aggregates. This is because the three aforementioned variables' dominant cycles are lagging exactly one period – i.e., four quarters – behind the two-quarter cycles of most of the other variables. Given that economic theory and evidence suggest that wages and salaries are often rigid and infrequently changed, and that the hiring processes may be concentrated at specific parts of a year (due to tax reasons), then the four-quarters phase shift is not surprising. As for imports of services, it might result from rigid contracts.

There is much greater diversity in the phase shifts of other cyclical components of the considered macroeconomic aggregates, showing that this economy – and likely economies in general – does not undergo a single business cycle. Thus, macroeconomic growth is not balanced in this case. Notably, the net exports of goods and services have a cyclical pattern that does not correspond to other aggregates, even though this is a small open economy. These observations suggest that, first, the considered economy undergoes a constant sectoral change – at least in the considered period – and is not driven by the 'rest-of-the-world' cycle, only by individual shocks, which sometimes translate into recessions, such as the 2008-crisis of the COVID pandemic.

5.2. Model fit

Since it has been assumed that we do not know a priori which frequencies are the correct ones to include in the cyclical growth rate models, many variants of the model have been estimated and their fit to the data compared in order to uncover the patterns of fluctuations driving each variable of interest. The comparison has been conducted on the basis of mean squared error of the fit of a given model to the data.

Estimation and analysis of results shows that most of the considered macroeconomic aggregates are characterised by more than one or two cycles, and, crucially, the best fit usually involves also cyclical functions of frequencies characterised by not very high periodogram values. This means that not only the peaks

For raw, unprocessed data of GDP gross growth rates, almost every version of the model, characterised by the set of included cycles, shows a consistent pattern: including the intervention variables dramatically reduces the MSE relative to their counterpart models without interventions. For many of the other variables, the inclusion of interventions, representing non-cyclical shocks, also results in a lower MSE, indicating a better fit. However, there are exceptions. For example, wages and salaries and government consumption achieve their best fit without intervention variables. In the case of government consumption this is shows that including interventions does not suffice, and computing two or three different periodograms for sub-series of the data series would provide a better fit in the subsequent estimation. In this paper, however, the object of interest is the best performance of the approach using a single periodogram for the entire set of observations.

Surprisingly, the model for the full gross capital formation aggregate yields slightly worse fit than its counterpart limited to fixed assets – the MSE approaches 0.01, but does not fall below this value. Such a difference in cyclical patterns in both variables shows that the non-fixed assets part of firms' investment is less cyclical and more idiosyncratic, or undergoes dif-

Table 3 – The fit of cyclical models to the data: gross domestic product

	No intervention variables	Intervention variables
12 frequencies	0.6465865128840329	X
11 frequencies	0.5668572901267667	X
10 frequencies	0.6781370280189686	0.4301394667869738
9 frequencies	0.6363355667316284	0.35046383905261036
8 frequencies	0.6535249835937924	0.31246297329962247
7 frequencies	0.0019306698480624658	0.005599133022028306
6 frequencies	0.0020410544226771602	0.0013604930406125805
5 frequencies	0.0020234473958530507	0.0013343151275881
4 frequencies	0.0018175024611940592	0.0012249006219877349
3 frequencies	0.0017727031215221502	0.001239103873549717

Table 4 – Model fit expressed as mean squared error: gross domestic product (GDP) for the versions with and without intervention variables across different frequencies; X denotes a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles..

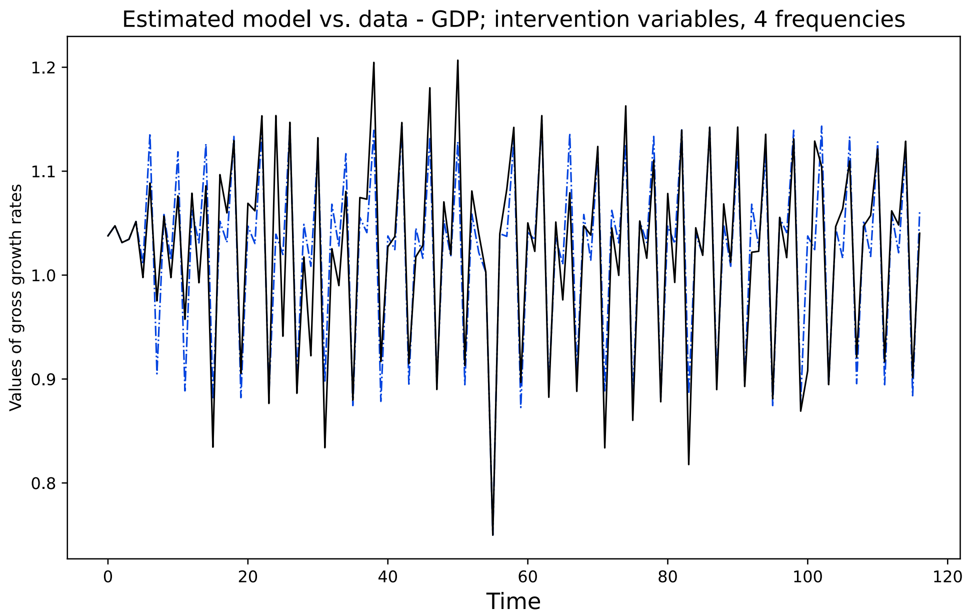

Fig. 1 – The cyclical signal model's fit to the data: gross domestic product (the version of the model yielding the smallest MSE).

Table 5 – The fit of cyclical models to the data: household consumption

	No intervention variables	Intervention variables
12 frequencies	X	X
11 frequencies	X	X
10 frequencies	23468062467.204937	299231019830.8492
9 frequencies	83142439227.9757	87477330512.3857
8 frequencies	73632.01609922417	1162753.858745442
7 frequencies	4.884070734187999	6.090685345639438
6 frequencies	0.005616548711442633	0.005422123109094857
5 frequencies	0.0054304290699519215	0.0056799707347809725
4 frequencies	0.005338933088955422	0.0054973138159728685

Table 6 – Model fit expressed as mean squared error: household consumption (CONS_HSH) for the versions with and without intervention variables across different frequencies; X or a very large MSE denote a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles..

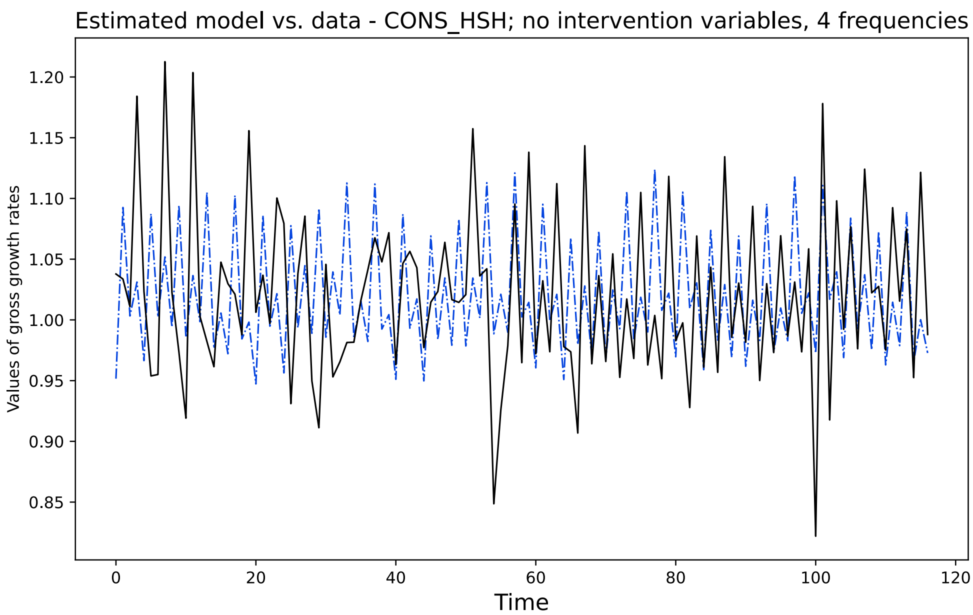

Fig. 2 – The cyclical signal model's fit to the data: household consumption (the version of the model yielding the smallest MSE).

Table 7 – The fit of cyclical models to the data: gross capital formation

	No intervention variables	Intervention variables
12 frequencies	0.014479064947257213	X
11 frequencies	0.015143188878991421	X
10 frequencies	0.015258409810652799	0.014847731556231772
9 frequencies	0.015051263251931239	0.014556434253903943
8 frequencies	0.012655212577586065	0.011306440959131497
7 frequencies	0.01271863090915135	0.011346227206832074
6 frequencies	0.012683710536317285	0.01134528831605751
5 frequencies	0.012671115178029773	0.011354784262483703

Table 8 – Model fit expressed as mean squared error: gross capital formation (GCF) for the versions with and without intervention variables across different frequencies; X denotes a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles.

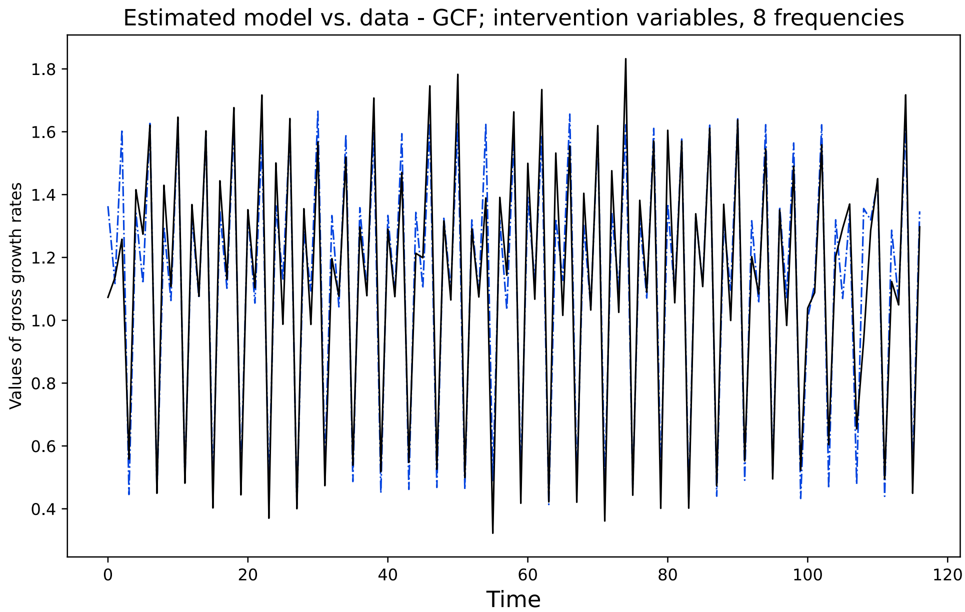

Fig. 3 – The cyclical signal model's fit to the data: gross capital formation (the version of the model yielding the smallest MSE).

ferent patterns of fluctuations which are too weak to be identified by the periodogram of GCF.

Table 9 – The fit of cyclical models to the data: gross capital formation (fixed assets)

	No intervention variables	**Intervention variables**
12 frequencies	0.009270889721893357	X
11 frequencies	X	X
10 frequencies	0.007880004781122437	0.00635846297751168
9 frequencies	0.007986174436737943	0.006441929261429127
8 frequencies	0.008001581343901502	0.006479260505877881
7 frequencies	0.00805424557880507	0.006497848484039994
6 frequencies	0.008058947696863432	0.006554963311609997
5 frequencies	0.008049856332856131	0.006565278274604561
4 frequencies	X	X
3 frequencies	X	X

Table 10 – MSE values by number of frequencies for gross capital formation limited to fixed assets (GCF_FA) for the versions with and without intervention variables across different frequencies; X denotes a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles.

The number of included cycles that leads to the best model fit varies between variables. For the gross capital formation, incorporating 8 cyclical functions and intervention variables results in the closest approximation of the data. Curiously, gross capital formation of fixed assets requires a model incorporating 10 cyclical functions and interventions. For all of the other variables, with five exceptions, including 4 to 6 cycles in the cyclical signal model yields the smallest mean-squared error.

The aforementioned exceptions are interventions, net exports of goods and services, exports of goods, exports of services, and taxes less subsidies. In their cases, the number of included cycles that yields the smallest mean squared error is, respectively, 2, 3, 9, 8, 9. The results for inventories and net exports – with only two and three cycles, respectively, providing the lowest MSE but only weakly reproducing the shape of the series – reveal that, for small open economies, the dynamics of these aggregates can actually be less cyclical than we may have believed. In fact, net exports are much less cyclical than its components, i.e., the series of export of goods, export of services, and the two corresponding types of imports. This suggests that the cyclicality of these four constituent macroeconomic aggregates partially cancels each other out.

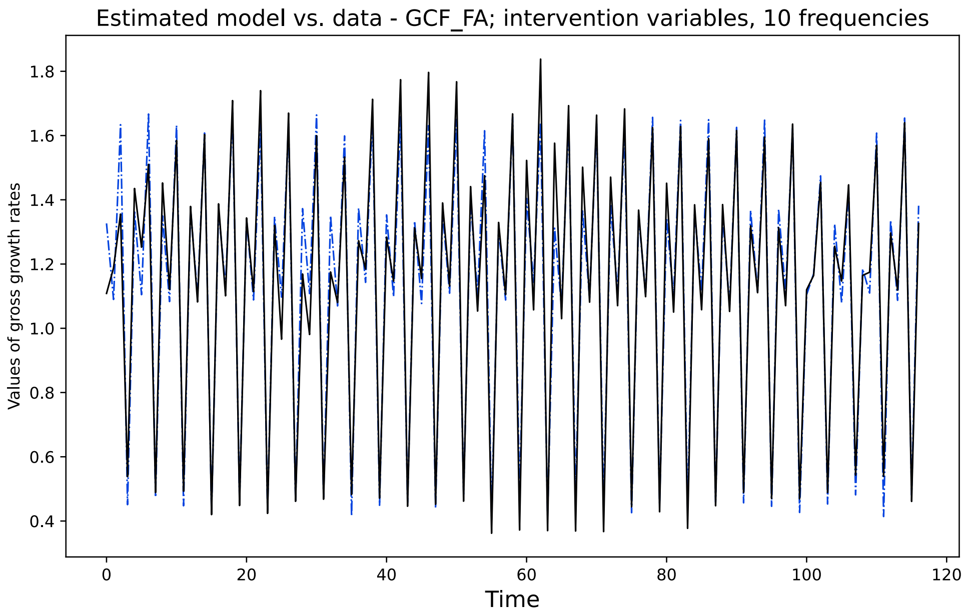

Fig. 4 – The cyclical signal model's fit to the data: gross capital formation – fixed assets (the version of the model yielding the smallest MSE).

Table 11 – The fit of cyclical models to the data: general government consumption

	No intervention variables	Intervention variables
12 frequencies	0.8176206542702383	X
11 frequencies	0.8660678768421327	X
10 frequencies	1.9782428504537526	1.8702964797760782
9 frequencies	0.014130143491535399	0.014140435791072697
8 frequencies	0.00934430625429328	0.009027533859083748
7 frequencies	0.00938559065850931	0.00903920804899793
6 frequencies	0.009370188108798192	0.009012390903239579
5 frequencies	0.009120429471929665	0.008699332668985812

Table 12 – Model fit expressed as mean squared error: government consumption (CONS_GOV) for the versions with and without intervention variables across different frequencies; X denotes a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles.

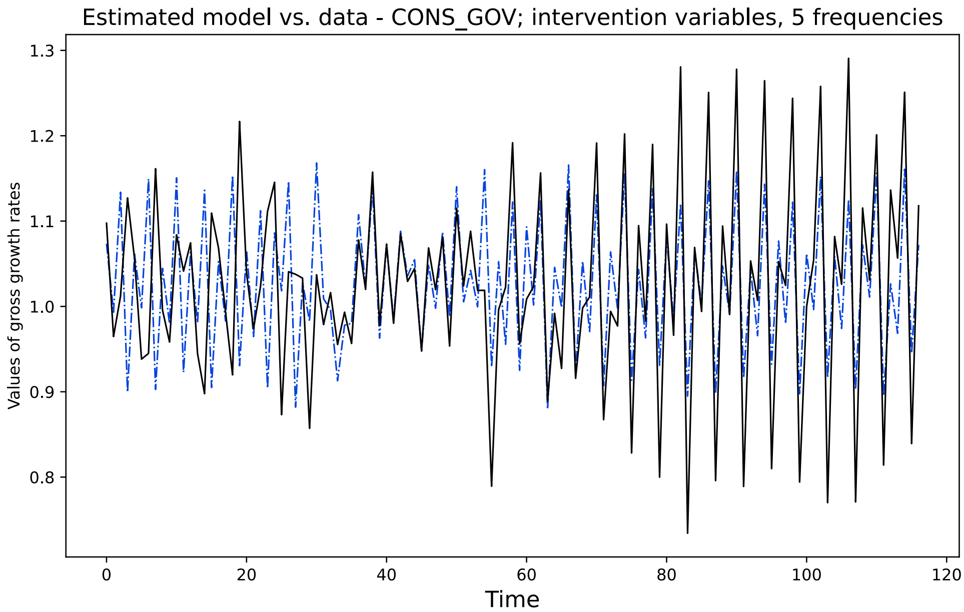

Fig. 5 – The cyclical signal model's fit to the data: final consumption of general government (the version of the model yielding the smallest MSE).

Table 13 – The fit of cyclical models to the data: compensation of employees

	No intervention variables	Intervention variables
12 frequencies	10151086259.356552	X
11 frequencies	383764.1868495384	X
10 frequencies	13.593413882113358	119.82990342890916
9 frequencies	2.4640762727648524	1.6594288259521073
8 frequencies	1.0127225712733972	0.609506964910017
7 frequencies	1.1821178532665295	0.9729984041263424
6 frequencies	0.00656619828789854	0.009166300339745376

Table 14 – Model fit expressed as mean squared error: wages and salaries (CMPNS_EMPLYEES) for the versions with and without intervention variables across different frequencies; X denotes a lack of an estimate due to ill-conditioned matrix of regressors, i.e., multicollinearity caused by the inclusion of non-structural cycles.

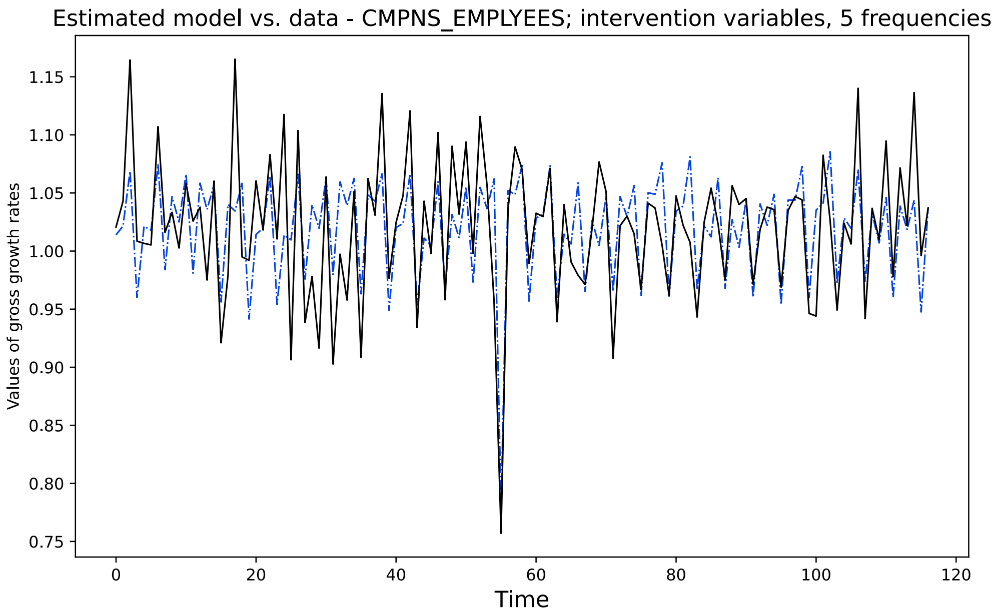

Fig. 6 – The cyclical signal model's fit to the data: compensation of employees (the version of the model yielding the smallest MSE).

This pattern shows that macroeconomic time series are driven by more than just a single cycle or a simple level component, and including all of these cyclical functions, 'seasonal' cycles of 2, 3 and 4 as well, allows explaining a very large part of the volatility of the growth rate of the variable under consideration. Multiple cyclical components are relevant, and capturing a richer frequency structure improves model performance in most cases, which implies that the underlying dynamics are multi-layered. Practically, this means that to explain and forecast macroeconomic data effectively, we likely may to incorporate more than one or two cyclical frequencies—i.e., a model that accounts for multiple types of cycles, and exogenous interventions or structural changes rather than relying solely on a filtered level or a single cycle.

The lack of inclusion of intervention variables and the resulting imperfect fit of rare, likely exogenous shocks to the cyclical patterns, crises in the case of some aggregates, e.g., household consumption, may be caused by full or partial alignment of these events with the cycles whose amplitudes and phases are estimated. That is, if the cycles' troughs happen to be partially or fully synchronised with the rare, exogenous crises, then the inclusion of intervention variables can actually worsen the overall fit (as happened in the case of households' consumption as well as wages and salaries). This is because they dampen the estimate of the amplitudes not only for the periods they control for, but for the entire series, as the amplitudes are constant. Thus, small losses of fit accumulate across the sample. This causes that in the cases of household consumption and wages and salaries the models which reproduce less shape, but yield slightly smaller MSE have been chosen.

5.2.1. Discovered cycles

Results of estimation of the best models of cyclical signals reveal some common features among the considered macroeconomic aggregates, but the estimates also demonstrate substantial differences in the cycles driving each sector.

For many variables, the first cyclical component – of frequency 0.5 in these cases – has an initial phase shift equal to $\Pi = 3.14159265...$ (Table 15). This group includes GDP, CONS_NPISH, GCF, GCF_FA, EXPR_GOODS, IMP_GOODS, CMPNS_EMPLYEES, OP_SRPL_MIX_INC, and CONS_GOV. First, this means that they share a common cycle of a short period (2 quarters), which turns out to be a dominant one for many of these macroeconomic aggregates, as can be seen by comparing the amplitudes of each of the cyclical components, displayed in Table 17. Another, small group of variables – CONS_HSH, EXP_SRVCS, nieIMP_SRVCS, nieWAGES_SALARIES and

Table 15 – The (mis-)alignment of macroeconomic cycles

Variable	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
GDP	3.1416	-0.5377	-1.1287	-0.6646						
CONS_HSH	0.0000	-0.1937	-1.3527	0.4362						
CONS_NPISH	3.1416	0.1141	1.4535	-0.2412	1.1180					
GCF	3.1416	-0.4221	1.0549	0.7938	0.7917	-0.8835	-0.2622	-0.4891		
GCF_FA	3.1416	-0.4109	-1.1349	0.7703	0.9948	0.7868	0.7873	-1.0506	1.3380	0.5510
Inventories	1.6581	1.2935								
EXPR_GOODS	3.1416	-1.1500	0.8817	1.3855	-1.1700	-1.2915	-0.8962	-0.6898	0.3951	
EXP_SRVCS	0.0000	-0.7608	1.2915	-0.2779	-0.0752	-0.5749	-1.1428	-1.2929		
IMP_GOODS	3.1416	1.3811	-1.1420	0.2417	-0.7687	-0.7866				
IMP_SRVCS	0.0000	-0.6168	-0.4032	1.4610	-0.0985	0.8761				
CMPNS_EMPLYEES	3.1416	-0.3991	-0.8536	0.3845	-1.2468	0.5759				
WAGES_SALARIES	0.0000	0.1325	-0.7807	0.5027	-1.2877	0.6718				
OP_SRPL_MIX_INC	3.1416	-0.8616	1.3038	-1.1379	1.3750	1.4123				
TX_LSS_SBS	0.0000	-0.7333	-0.4927	1.4240	0.8464	1.4252	-0.4592	-1.0310	1.2852	
NX_GDS_SRVCS	2.3770	1.5230	-0.7574							
CONS_GOV	3.1416	-0.8398	1.3456	0.4956	0.4697					

Table 16 – Phase shifts of each cyclical component (in radians) from the cyclical signal model that produced the best fit to the data. Columns 1–10 correspond to the first through tenth phase shifts for each variable, with blank cells for variables for which the signal has fewer than ten different cycles. The order of columns corresponds to the order of the associated periodogram values. The presented results concern cycles in the gross growth rates of the displayed macroeconomic aggregates.

TX_LSS_SBS – lags behind exactly one period of this short cycle (the phase of 0). This can be interpreted as follows. In the case of wages and salaries, economic theory provides an explanation: the lag in the 2-period cycle may reflect lagged wage-setting and infrequent hiring processes (e.g., due to tax optimisation). The behaviour of households' consumption can be treated as evidence of consumption tracking current rather than permanent income. As for exports and imports of services, this behaviour may be caused by multiple factors, for example by firms strictly following the budgets planned for a given year, or slow adjustment of their decision rules.

There is much greater diversity in the phase shifts of other cyclical components of the considered macroeconomic aggregates, showing that this economy – and likely economies in general – does not undergo a single business cycle. Thus, macroeconomic growth is not balanced in this case. Notably, the net exports of goods and services have a cyclical pattern that does not correspond to other aggregates, even though this is a small open economy. These observations suggest that, first, the considered economy undergoes a constant sectoral change – at least in the considered period – and is not driven by the 'rest-of-the-world' cycle, only by individual shocks, which sometimes translate into recessions, such as the 2008-crisis of the COVID pandemic.

The results of estimation and selection of the best models reveal that economic growth in the studied small open economy has not been balanced in the period in question. This is evident from the values of the constants in the gross growth rates models, which translate into exponential trends in the levels of the considered macroeconomic aggregates. All of them are different, as displayed in Table 19. We may interpret this result, together with the differences in cycles driving each of the variables, as evidence against the existence of balanced growth in this small open economy.

5.2.2. Little transmission of foreign cycles to the examined small, open economy

The large mean squared errors observed for the estimated models of the cycles in net exports' gross growth rates (Table 21) indicate that this macroeconomic aggregate is not characterised by many cyclical patterns, and most of the variance can be treated as noise. The introduction of intervention variables does not improve the fit. The inability to detect any meaningful cyclical pattern implies that foreign business cycle dynamics, which would presumably manifest as recurring fluctuations in net exports, are not playing a strong or discernible role in shaping these data.

If foreign business cycles were exerting a significant influence on the studied economy's net exports, we would expect to see strong cyclical components

Table 17 – Estimated amplitudes of the best models

Variable	1	2	3	4	5	6	7	8	9	10
GDP	0.0675	0.0791	0.0123	0.0055						
CONS_HSH	0.0386	0.0363	0.0170	0.0127						
CONS_NPISH	0.0538	0.0632	0.0166	0.0090	0.0103					
GCF	0.3474	0.3377	0.0226	0.0233	0.0166	0.0046	0.0006	0.0073		
GCF_FA	0.3622	0.3558	0.0123	0.0105	0.0115	0.0068	0.0118	0.0080	0.0093	0.0137
Inventories	0.1177	0.1511								
EXPR_GOODS	0.0130	0.0131	0.0140	0.0092	0.0079	0.0157	0.0040	0.0142	0.0081	
EXP_SRVCS	0.0645	0.0479	0.0131	0.0075	0.0144	0.0125	0.0024	0.0056		
IMP_GOODS	0.0323	0.0089	0.0181	0.0155	0.0264	0.0116				
IMP_SRVCS	0.0962	0.0475	0.0078	0.0500	0.0366	0.0113				
CMPNS_EMPLYEES	0.0290	0.0374	0.0775	0.0415	0.0159	0.0366				
WAGES_SALARIES	0.0004	0.0452	0.0935	0.0483	0.0143	0.0448				
OP_SRPL_MI_INC	0.1129	0.1362	0.0033	0.0124	0.0060	0.0128				
TX_LSS_SBS	0.1024	0.1099	0.0141	0.0183	0.0175	0.0146	0.0197	0.0263	0.0271	
NX_GDS_SRVCS	0.3460	0.3386	0.3088							
CONS_GOV	0.0763	0.0548	0.0122	0.0141	0.0104					

Table 18 – Amplitudes for various cyclical components in the best cyclical signal models. Each row shows the amplitude values of cyclical components in order of their periodogram values. The calculated values have been rounded to include four decimal places. The presented results concern cycles in the gross growth rates of the displayed macroeconomic aggregates.

Table 19 – Exponential trend components in the best models

Variable	Exponential trend value
GDP	1.02393
CONS_HSH	1.02022
CONS_NPISH	1.02380
GCF	1.12542
GCF_FA	1.13720
Inventories	0.99980
EXPR_GOODS	1.02860
EXP_SRVCS	1.04626
IMP_GOODS	1.02820
IMP_SRVCS	1.03891
CMPNS_EMPLYEES	1.02163
WAGES_SALARIES	1.02010
OP_SRPL_MIX_INC	1.02973
TX_LSS_SBS	1.03169
NX_GDS_SRVCS	1.14307

Table 20 – The presented results concern cycles in the gross growth rates of the displayed macroeconomic aggregates.

of the growth rate, with high amplitudes and producing a good fit of the model to the data. The persistently poor fit is not a failure of the modelling approach, but rather an indication that cyclical patterns of other countries are transmitted only very little, and mostly through the cycles of periods 3, 9, 12 (see Table 1). This does not imply that the effects of rare, infrequent events, such as the 2008 crisis or the pandemic, are not transmitted internationally. Conversely, they affect the growth rates of macroeconomic aggregates of the economy under study, but only as shocks.

Table 21 – Model fit expressed as mean squared error: net exports of goods and services for the versions with and without intervention variables across different sets of cycles included in the cyclical signal model.

	No intervention variables	Intervention variables
8 frequencies	9576709076916498.0000	281423320854737.2500
7 frequencies	40365816429189.8500	13224455119237.8200
6 frequencies	506112501.7692	38093871.0599
5 frequencies	2443.1798	725.1622
4 frequencies	4.4460	1.5947
3 frequencies	4.0688	1.4738

Table 22 – The calculated values have been rounded to four decimal places.

5.3. Do we loose crucial information by winsorising?

For the comparison of the performance of the proposed modelling approach on non-transformed and winsorised data, the GDP series is used. Winsorisation has been conducted by changing the extreme two percentiles, i.e., observations above 99-percentile or below the 1-percentile.

The model estimated on winsorised data performs either almost equally well as the one using crude observations, or better, especially for the larger set of cycles included in the model of the signal. Nonetheless, the approach using crude data and intervention variables is capable of achieving slightly better fit to the data in terms of the mean squared error. This can be seen by comparing Tables 3 and 24 – the lowest mean squared error in the former case equals approximately 0.0012249, while the approach using winsorised data can achieve only around 0.001775.

Thus, in terms of the mean squared error, the crude-data models with intervention variables not only usually outperform their counterparts without interventions but also surpass the best winsorized models evaluated. This demonstrates a clear advantage of using crude data in modelling economic

Table 23 – Comparison of periods of the potential twelve cycles in GDP growth rate – crude and winsorized data.

Importance (periodogram value, descending order)	Crude data – period of a cycle	Winsorized data – period of a cycle
1	2	2
2	4	4
3	5	5
4	6	6
5	23	23
6	24	24
7	22	7
8	25	22
9	7	25
10	13	15
11	15	14
12	14	13

Table 24 – MSE results for the fit of the models estimated using winsorised data on gross domestic product.

	No intervention variables
12 frequencies	0.6080099120308232
11 frequencies;	0.6041297995699723
10 frequencies	0.5007127206630912
9 frequencies	0.5814463019445537
8 frequencies	0.0019891443779818667
7 frequencies	0.001988669849820148
6 frequencies	0.002012187213181529
5 frequencies	0.001993403645115576
4 frequencies	0.0018141833903360565
3 frequencies	0.0017749504492872402
2 frequencies	0.0018736513798625037

fluctuations, paired with intervention-variables corrections, in accurately discovering and reconstructing cyclical patterns present in the data.

The estimates of amplitudes for the three cycles that are common – in terms of their frequency – in the best models estimated using crude and winsorised data slightly differ. For phases, the discrepancy is very visible for the third cycle, of period 5. Additionally, the difference in the MSE results also from the fact that in the crude-data model one more cycle has been included. Even though its amplitude is small, it helps to achieve a better fit, which we may interpret as evidence that this is an actually existing economic cycle. Of course, this conclusion is warranted only because the choice of this cycle is justified by the conducted periodogram analysis.

Table 25 – Comparison of amplitudes and phase shifts for the cycles included in the best models estimated using crude and winsorised data

Amplitudes – best model using crude data	Amplitudes – best model using winsorised data
0.06748505	0.0644794
0.07909576	0.07602261
0.01227046	0.01315138
0.00552389	
Phases – best model using crude data	Phases – best model using winsorised data
3.14159265	3.14159265
-0.53770251	-0.50147333
-1.12865737	-1.30105443
-0.66457624	X

What is perhaps most striking is that the inclusion frequencies that have been traditionally treated as the business-cycle frequencies, i.e., corresponding to periods of $6-32$ or $8-32$ quarters (e.g., Baxter and King (1999), Christiano and Fitzgerald (2003)), has not yielded a better fit to actual GDP data than the inclusion of cycles with periods 2-5. Even though the former appear slightly in the periodogram, it seems that they are either too weak to be meaningfully included in the model, or the corresponding periodogram values represent in fact some artefacts like small structural changes, rather than actual cycles.

In order to better investigate whether the cycles corresponding to frequencies $\frac{1}{32}$-$\frac{1}{6}$ are spurious or obsolete for business cycle modelling for the considered small open economy, Baxter and King's bandpass filter have been considered (Baxter and King 1999). The next subsection contains the discussion of this issue and the use of this filter in general.

5.4. Are business cycles really so short and do we loose crucial information by using the Baxter-King filter?

Table 26 – MSE results for the fit of the models estimated using Baxter-King filtered data on gross domestic product.

	No intervention variables
6 frequencies	50.9870384777554
5 frequencies	29.2721502671299
4 frequencies	0.13607215356464358
3 frequencies	0.009112996792832433
2 frequencies	0.009175888462011818
1 frequency	0.009023828180100425

The periodogram of Baxter-King-filtered GDP data, for the band of 6-32 has a familiar shape (figure 7), seemingly uncovering hidden, but fundamental cycles lying within the expected frequency intervals. However, the comparison of mean squared errors of the fit for the models using crude and filtered data show that filtering actually worsens the fit by excluding crucial frequencies, as can be seen by comparing the MSE of the models fitted to actual data using the cycles identified using periodograms of filtered data, presented in Table 26, with the MSE of crude-data models (Table 3)

5.5. Evaluating government policy's procyclicality/reactiveness and idiosyncratic activeness

As mentioned in subsection 3.7, in this paper, systematic reactiveness – procyclicality – of government consumption is distinguished from the lack of idiosyncratic activeness. The latter is defined as considerable impact of non-cyclical part of the gross growth rate of government expenditure on its GDP counterpart. 'Considerable' is understood as a one percentage point increase in the former leading to more than one percentage point times the share of government consumption in GDP, which is between 0.17 and 0.2 for almost all time periods in the sample. Procyclicality, on the other hand, is defined as perfect alignment of the phases of all or some of the cycles composing the gross growth rates of these variables (and of these aggregates themselves, according to the model presented in this paper – recall equations 14-18), or as government consumption's cycles lagging behind GDP's counterparts. If government expenditure leads, but the phase shifts between its cycles and the period-related counterparts of GDP are not large, then we can interpret this

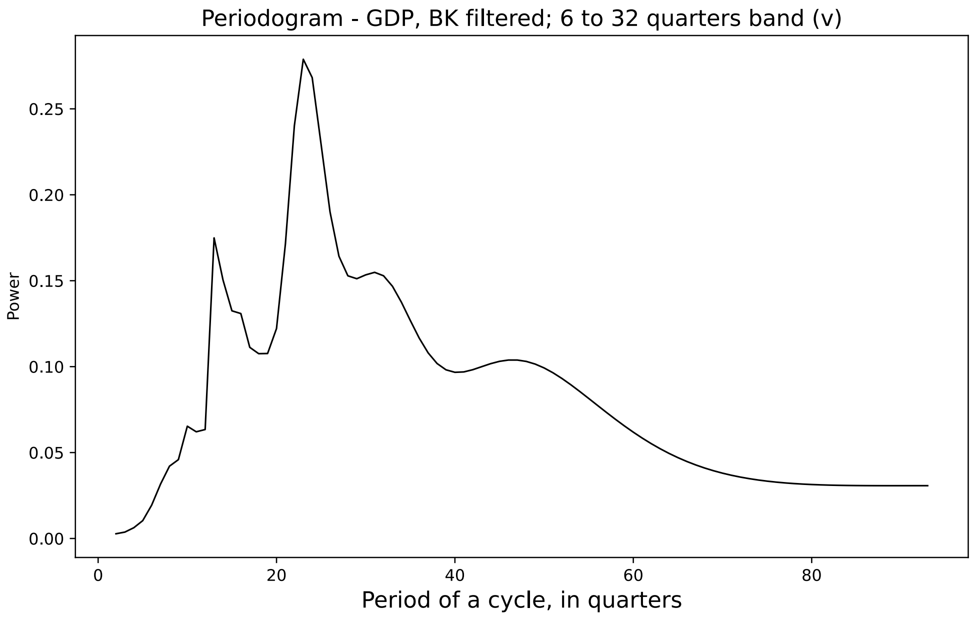

Fig. 7 – The cyclical signal model's fit to the data: household consumption (the version of the model yielding the smallest MSE).

as evidence for the government to have attempted to act countercyclically, but too slow and ineffectively.

5.5.1. Phase shifts of the cycles of the gross growth rates of gross domestic product and government consumption

The procyclicality of general government consumption or lack of thereof is evaluated on the basis of the comparison of phase shifts of the cyclical functions that constitute the signals in the gross growth rates of the government's expenditure and GDP.

Table 27 – Phases of the cycles in the gross growth rates of GDP.

Variable	1	2	3	4
GDP	3.14159265	-0.53770251	-1.12865737	-0.66457624

Table 28 – Phases of the cycles in the gross growth rates of general government consumption.

Variable	1	2	3	4	5
CONS_GOV	3.14159265	-0.83979524	1.34562187	0.49560472	0.46974495

All of the cycles of growth rates of GDP and the four most pronounced of the government's consumption occur at the same frequencies (see Table 1), which correspond to periods of 2, 4, 5 and 6 quarters, with this order corresponding to their importance as measured by the periodogram. The comparison of the phases of estimated cyclical functions of the two variables allows to evaluate whether the government's expenditure was active or reactive. If a given cyclical component leads its counterpart in the growth rate of GDP, it is considered that the government's policy was active, and the size of the phase shift helps to evaluate how much future-oriented the government's expenditure was. In the case of synchronisation of cycles, or GDP cyclical components leading, the government's policy is considered to have been reactive.

Estimation of the cyclical growth rate model has shown that the first, half-year cycle, is perfectly synchronised – the phase shift is exactly equal in the corresponding cyclical growth component is identical for both macroeconomic aggregates (this can be seen by comparing the values of the phases of the two variables, shown in Table 29). Recall that the cycles of periods 2 and 4 were the strongest for both variables; for GDP, the 4-quarter cycle had the highest amplitude, while for GDP it has been the 2-quarter one; in both cases, the differences in amplitudes between these two cycles were not high.

Table 29 – Differences in phases of common cycles of gross growth rates of GDP and general government's consumption

Cycle's period	The leading variable	Time shift value (in quarters)
2	Perfect synchronisation	0
4	GDP leads	0.1923181996988839
5	Government consumption leads	-1.968968857330291
6	Government consumption leads	-1.1078912123803155

In the case of the four-quarters cycle, the corresponding cyclical component of GDP leads the government-consumption counterpart by around 2.5 weeks (approximately 19.23% of a quarter, assuming it consists of exactly 13 weeks).

The timing advance in quarters of each of the four cycles of GDP and the government's consumption are shown in Table 29. We can see that in the only case where the cyclical component of the latter variable leads its GDP's counterpart the phase shift difference in terms of quarters is approximately 1.97 and 1.11 quarters, for cycles of periods 5 and 6 quarters. This can hardly be called an anti-cyclical policy. In the case of other frequencies, the cyclical functions of the government consumption's growth rate either is perfectly synchronised with the analogous components of current GDP or it lags by approximately 0.19 of a quarter (Table 29).

These results may suggest that the government has been attempting to act countercyclically, but due to inadequate or insufficient measures it only managed to do so within one-two quarters horizons. This may mean that the governemnt was too slow to implement its policy effectively, or had too little fiscal space to achieve its goals.

5.5.2. The ineffectiveness of non-cyclical government consumption in shaping GDP growth

In regressions presented and discussed in this subsubsection, the non-cyclical part of the gross growth rates of GDP has been regressed on the corresponding rates of contemporaneous or lagged macroeconomic aggregates. These are the cycles-unrelated remainders of the gross growth rates of general government's consumption, gross capital formation, households' consumption and net exports. Research design follows Leamer's approach to estimation (Leamer 1983): various regressions with different sets of regressors have been estimated, and the estimated coefficients on given variables were compared to judge whether they are consistent across specifications and to evaluate the effect of the non-cyclical government expenditure on the non-cyclical part of GDP growth. For each set of explanatory variables, ten different regressions have been estimated. In each of them, the regressors are lagged by the same

Table 30 – Estimation Results for GGR_GDP on GGR_CONS_GOV

Lag	CONS_GOV	p-value	regSE
L_0	0.10705	0.00211	0.11474
L_1	0.03528	0.35054	0.11918
L_2	-0.07253	0.05339	0.11749
L_3	0.01009	0.79260	0.11967
L_4	0.05172	0.21333	0.11860
L_5	0.03942	0.31078	0.11908
L_6	-0.10444	0.01686	0.11514
L_7	0.03022	0.40362	0.11831
L_8	-0.01162	0.72242	0.11654
L_9	0.04340	0.21918	0.11581

Table 31 – This table presents the regression results for the gross growth rate of GDP (GGR_GDP) explained by government consumption (GGR_CONS_GOV), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

number of periods. This serves the purpose of evaluating both contemporaneous and partial long-run impact on GDP growth, given that multiplier effects might be present.

The first regression relates GDP with general government's consumption (Table 30). Estimates of two-element sets of regressors, including also either household's consumption or gross capital formation, are shown in Tables 32 and 34. Table 36 shows how these estimates change when these three explanatory variables are accounted for in a single regression. Finally, the results of estimation with the specification including also net exports are displayed in Table 38. All regressions were estimated and evaluated using robust Newey-West covariance matrices ((Newey and West 1987)).

The seemingly low standard errors of the single-variable regressions (Table 30) as compared to the multivariate regressions (Tables 34, 32, 36, 38) result from the fact that the non-cyclical parts of the growth of considered macroeconomic aggregates are only weakly correlated with the non-cyclical part of the growth rates of GDP. This causes the error terms to dominate the expression for the Newey-West covariance matrix estimator, inflating its value.

The estimate of the impact of the non-cyclical part of general government's consumption gross growth rates on the non-cyclical part of non-systematic (i.e., excluding the cyclical signal) piece of gross growth rates of GDP is fairly consistent across specifications. Once the non-cyclical consumption of households is included, it falls from around 0.1 to 0.071-0.076.

Table 32 – Estimation Results for GGR_GDP on GGR_CONS_GOV and GGR_CONS_HSH

Lag	CONS_GOV	p-value	CONS_HSH	p-value	regSE
L_0	0.07143	0.00000	0.22485	0.00000	0.10612
L_1	0.03371	0.00000	0.00994	0.03110	0.12561
L_2	-0.07719	0.00000	0.02695	0.00000	0.12365
L_3	0.01560	0.00002	-0.03149	0.00000	0.12589
L_4	0.04613	0.00000	0.03146	0.00000	0.12477
L_5	0.04016	0.00000	-0.00423	0.38314	0.12551
L_6	-0.10654	0.00000	0.01145	0.00790	0.12133
L_7	0.04006	0.00000	-0.05364	0.00000	0.12398
L_8	-0.01285	0.00007	0.00671	0.07240	0.12283
L_9	0.05306	0.00000	-0.05286	0.00000	0.12135

Table 33 – This table presents the regression results for the gross growth rate of GDP (GGR_GDP) explained by the gross growth rates of CONS_GOV (GGR_CONS_GOV) and CONS_HSH (GGR_CONS_HSH), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

Table 34 – Estimation Results for GGR_GDP on GGR_CONS_GOV and GGR_GCF

Lag	CONS_GOV	p-value	GCF	p-value	regSE
L_0	0.10673	0.00000	0.10858	0.00000	0.11355
L_1	0.03561	0.00000	-0.06412	0.00000	0.12321
L_2	-0.07238	0.00000	0.06668	0.00000	0.12119
L_3	0.00978	0.00631	-0.03086	0.00000	0.12559
L_4	0.05203	0.00000	0.03494	0.00000	0.12431
L_5	0.03996	0.00000	-0.05750	0.00000	0.12363
L_6	-0.10503	0.00000	0.04166	0.00000	0.12035
L_7	0.03099	0.00000	-0.05704	0.00000	0.12285
L_8	-0.01246	0.00004	0.05840	0.00000	0.12085
L_9	0.04736	0.00000	-0.11931	0.00000	0.11473

Table 35 – This table presents the regression results for the gross growth rate of GDP (GGR_GDP) explained by the gross growth rates of government consumption (GGR_CONS_GOV) and gross capital formation (GGR_GCF), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

Table 36 – Estimation Results for GGR_GDP on GGR_CONS_GOV, GGR_GCF, and GGR_CONS_HSH

Lag	CONS_GOV	p-value	GCF	p-value	CONS_HSH	p-value	regSE
L_0	0.07289	0.00000	0.09765	0.00000	0.21383	0.00000	0.10537
L_1	0.03288	0.00000	-0.06501	0.00000	0.01732	0.00019	0.13060
L_2	-0.07552	0.00000	0.06561	0.00000	0.01814	0.00006	0.12845
L_3	0.01461	0.00004	-0.02917	0.00000	-0.02744	0.00000	0.13301
L_4	0.04724	0.00000	0.03330	0.00000	0.02685	0.00000	0.13165
L_5	0.03925	0.00000	-0.05776	0.00000	0.00404	0.37583	0.13112
L_6	-0.10610	0.00000	0.04130	0.00000	0.00583	0.14993	0.12764
L_7	0.03944	0.00000	-0.05424	0.00000	-0.04626	0.00000	0.12972
L_8	-0.01225	0.00003	0.05847	0.00000	-0.00111	0.77306	0.12819
L_9	0.05345	0.00000	-0.11647	0.00000	-0.03385	0.00000	0.12136

Table 37 – This table presents the regression results for the gross growth rate of GDP (GGR_GDP) explained by the gross growth rates of government consumption (GGR_CONS_GOV), gross capital formation (GGR_GCF), and household consumption (GGR_CONS_HSH), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

This means that what we could interpret as the countercyclical part of the government's policy is ineffective in shaping GDP growth contemporaneously, given that we would evaluate the impact of changes of explanatory variables in percentage points, and the share of government consumption in GDP (in terms of levels) is within the interval of $[0.17, 0.2]$ for most of the sample.

This implies very small effect of the government's non-cyclical part of policy on actual growth of the considered economy, which means that either these are actually not countercyclical efforts, but some other discretionary actions, or that these policies usually took the form of transfers or tax cuts, gains from which were not immediately spent. However, the lagged effects are even smaller and in some cases negative, meaning that even if the entirety of the non-cyclical part of general government's consumption were devoted for countercyclical measures, they were not successful, even if all the contemporaneous lagged values of every explanatory variable were not correlated (for a given variable, not between them) and representing different causality (multiplier) channels..

Regarding the previously discussed issue of international transmission of shocks to the studied small open economy, we can reach similar conclusions as we have when discussing the alignment of the cycles of growth rates of GDP and net exports. Here, the non-cyclical parts of the gross growth rates of the two variables are little correlated, again suggesting that the transmission

Table 38 – Estimation Results for GGR_GDP on GGR_CONS_GOV, GGR_GCF, GGR_CONS_HSH, and GGR_NX_GDS_SRVCS

Lag	CONS_GOV	p-value	GCF	p-value	CONS_HSH	p-value	NX_GDS_SRVCS	p-value	regSE
L_0	0.07607	0.00000	0.10003	0.00000	0.21233	0.00000	-0.00089	0.00000	0.11237
L_1	0.03434	0.00000	-0.06394	0.00000	0.01665	0.00031	-0.00041	0.00005	0.13957
L_2	-0.08384	0.00000	0.05954	0.00000	0.02219	0.00000	0.00228	0.00000	0.13585
L_3	0.01520	0.00002	-0.02874	0.00000	-0.02773	0.00000	-0.00016	0.13520	0.14219
L_4	0.03658	0.00000	0.02577	0.00000	0.03208	0.00000	0.00276	0.00000	0.13864
L_5	0.04209	0.00000	-0.05580	0.00000	0.00267	0.55511	-0.00073	0.00000	0.14003
L_6	-0.11064	0.00000	0.03848	0.00000	0.00833	0.04603	0.00108	0.00000	0.13613
L_7	0.04413	0.00000	-0.05134	0.00000	-0.04883	0.00000	-0.00111	0.00000	0.13834
L_8	-0.02083	0.00000	0.05326	0.00000	0.00373	0.32456	0.00201	0.00000	0.13591
L_9	0.05501	0.00000	-0.11495	0.00000	-0.03495	0.00000	-0.00038	0.00001	0.12970

Table 39 – This table presents the regression results for the gross growth rate of GDP (GDP) explained by the gross growth rates of government consumption (CONS_GOV), gross capital formation (GCF), household consumption (CONS_HSH), and net exports of goods and services (NX_GDS_SRVCS), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

Table 40 – Estimation Results for CONS_GOV on GDP

Lag	Beta Coeff.	p-value	regSE
L_0	0.76030	0.00064	0.30577
L_1	-0.01056	0.96350	0.31894
L_2	-0.39102	0.10152	0.31538
L_3	0.24155	0.35958	0.31516
L_4	0.25002	0.32959	0.30674
L_5	-0.16339	0.46091	0.30756
L_6	-0.48431	0.03264	0.30215
L_7	0.00527	0.98482	0.30071
L_8	0.46341	0.20891	0.28415
L_9	-0.37188	0.12982	0.28561

Table 41 – This table presents the regression results for the gross growth rate of government consumption (CONS_GOV) explained by the gross growth rate of GDP (GDP)), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

of foreign shocks has only a moderate impact on the economy, unless it is a large, world-wide crisis, which is clearly visible in raw data, as alignment of falls of GDP in time.

5.5.3. The reactiveness of non-cyclical government consumption

In regressions presented and discussed in this subsubsection, the non-cyclical part of the gross growth rates of general government's consumption has been regressed on the contemporaneous or lagged macroeconomic aggregates. These include the counterpart remainders of the gross growth rates of GDP, gross capital formation, households' consumption and net exports. As in the previous subsubsection, Leamer's approach to estimation has been applied, by estimating regressions with different sets of regressors and evaluating the stability of estimated coefficient values (Angrist and Pischke 2010; Leamer 1983).

The first regression relates general government's consumption with GDP (Table 40). Estimates of two-element sets of regressors, including also either household's consumption or gross capital formation, are shown in Tables 42 and 44. Table 46 shows how these estimates are altered when GDP, household's consumption and gross capital formation are accounted for in a single regression. The results of estimation with the full specification are shown in Table 48.

The analysis of the impact of non-cyclical GDP growth on non-cyclical government consumption reveals the reactive nature of fiscal policy in the

Table 42 – Estimation Results for CONS_GOV on GDP and CONS_HSH

Lag	GDP	p-value	CONS_HSH	p-value	regSE
L_0	0.72162	0.00000	0.08542	0.00000	0.32826
L_1	-0.01184	0.64341	0.17377	0.00000	0.33963
L_2	-0.37133	0.00000	-0.04927	0.00056	0.33871
L_3	0.22492	0.00000	0.02135	0.09044	0.33911
L_4	0.29144	0.00000	-0.03554	0.06115	0.33194
L_5	0.00214	0.92987	0.06193	0.00000	0.33327
L_6	-0.45743	0.00000	-0.02664	0.14372	0.32805
L_7	0.05566	0.08932	0.04806	0.00277	0.32721
L_8	0.53726	0.00000	-0.08938	0.00010	0.31115
L_9	-0.34463	0.00000	0.07710	0.00000	0.31410

Table 43 – This table presents the regression results for the gross growth rate of government consumption (CONS_GOV) explained by the gross growth rates of GDP (GDP) and household consumption (CONS_HSH), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

Table 44 – Estimation Results for CONS_GOV on GDP and GCF

Lag	GDP	p-value	GCF	p-value	regSE
L_0	0.85106	0.00000	-0.09035	0.00000	0.32065
L_1	0.01216	0.60270	-0.02271	0.01053	0.33609
L_2	-0.49864	0.00000	0.10644	0.00000	0.33021
L_3	0.26749	0.00000	-0.02577	0.00604	0.33208
L_4	0.42379	0.00000	-0.17135	0.00000	0.31741
L_5	-0.07974	0.00029	-0.08218	0.00000	0.32288
L_6	-0.70384	0.00000	0.21678	0.00000	0.30907
L_7	-0.01181	0.67968	0.01685	0.13882	0.31692
L_8	0.57628	0.00000	-0.10911	0.00000	0.29702
L_9	-0.24496	0.00000	-0.12455	0.00000	0.29831

Table 45 – This table presents the regression results for the gross growth rate of government consumption (CONS_GOV) explained by the gross growth rates of GDP (GDP) and gross fixed capital investment (GCF_FA), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

Table 46 – Estimation Results for CONS_GOV on GDP, GCF, and CONS_HSH

Lag	GDP	p-value	GCF	p-value	CONS_HSH	p-value	regSE
L_0	0.75684	0.00000	-0.08438	0.00000	0.08320	0.00000	0.33952
L_1	-0.27336	0.00000	-0.00490	0.58067	0.25114	0.00000	0.35140
L_2	-0.31944	0.00000	0.09624	0.00000	-0.16034	0.00000	0.34819
L_3	0.26270	0.00000	-0.02550	0.00547	0.00429	0.75057	0.35222
L_4	0.52793	0.00000	-0.17726	0.00000	-0.09235	0.00003	0.33596
L_5	-0.21117	0.00000	-0.07508	0.00000	0.11697	0.00000	0.34136
L_6	-0.59579	0.00000	0.21074	0.00000	-0.09677	0.00000	0.32703
L_7	-0.13567	0.00000	0.02380	0.02920	0.11080	0.00000	0.33514
L_8	0.72931	0.00000	-0.11826	0.00000	-0.13307	0.00000	0.31352
L_9	-0.39763	0.00000	-0.11675	0.00000	0.13390	0.00000	0.31487

Table 47 – This table presents the regression results for the gross growth rate of government consumption (CONS_GOV) explained by the gross growth rates of GDP (GDP), gross capital formation (GCF), and household consumption (CONS_HSH), showing estimated beta coefficients, their p-values, and the regression standard error (regSE) for each lag.

studied economy. The coefficients on the contemporaneous part of GDP not associated with cycles are consistently high, ranging between 0.72-0.852, with 0.7731 in the regression with the largest set of explanatory variables (Tables 40, 44, 42, 46, 48). This indicates that this part of general government's consumption in the studied small open economy is shaped in over 72% by current inflow of collected funds, or that the government's decisions are taken mostly on the basis of contemporary economic changes.

These results lead to the conclusion that the non-cyclical part of the government's policy in this small open economy is reactive. It does not appear to actively shape the path of economic growth. As for the other explanatory variables, while the contemporaneous inverse relation between investment and government's consumption could have been expected, i.e., higher growth of investment serving as an information of good condition of the economy to the policy makers, the positive contemporaneous coefficient of household consumption may partly show the impact of indirect taxation. Almost no impact of net exports of government expenditure is not surprising, as this variable largely depends on factors outside of the government control.

Overall, these findings suggest that government consumption is highly reactive to current changes of GDP rather than being an independent force shapining economic growth. The contemporaneous response is strong, but the long-term trajectory is marked by instability, with significant negative

Table 48 – Estimation Results for CONS_GOV on GDP, GCF, CONS_HSH, and NX_GDS_SRVCS

Lag	GDP	p-value	GCF	p-value	CONS_HSH	p-value	NX_GDS_SRVCS	p-value	regSE
0	0.7731	0.0000	-0.1032	0.0000	0.0844	0.0000	0.0066	0.0000	0.3582
1	-0.2714	0.0000	-0.0070	0.4286	0.2512	0.0000	0.0007	0.0000	0.3756
2	-0.3119	0.0000	0.0880	0.0000	-0.1599	0.0000	0.0029	0.0000	0.3713
3	0.2362	0.0000	0.0044	0.5812	0.0027	0.8279	-0.0104	0.0000	0.3651
4	0.5518	0.0000	-0.2061	0.0000	-0.0902	0.0000	0.0101	0.0000	0.3479
5	-0.2081	0.0000	-0.0788	0.0000	0.1172	0.0000	0.0013	0.0000	0.3647
6	-0.5939	0.0000	0.2085	0.0000	-0.0964	0.0000	0.0008	0.0000	0.3495
7	-0.1538	0.0000	0.0451	0.0000	0.1070	0.0000	-0.0076	0.0000	0.3520
8	0.7380	0.0000	-0.1307	0.0000	-0.1300	0.0000	0.0045	0.0000	0.3328
9	-0.3944	0.0000	-0.1212	0.0000	0.1348	0.0000	0.0010	0.0000	0.3365

Table 49 – This table presents the regression results for the gross growth rate of government consumption (CONS_GOV) explained by the gross growth rates of GDP (GDP), gross capital formation (GCF), household consumption (CONS_HSH), and net exports of goods and services (NX_GDS_SRVCS), showing estimated beta coefficients, their p-values (in the directly neighbouring column), and the regression standard error (regSE) for each lag.

coefficients emerging at later lags. This pattern indicates that fiscal policy adjusts over time, potentially in response to new information about economic conditions, budgetary constraints, or stabilization policies. The fragility of these estimates across different model specifications suggests that government consumption is reactive with respect to GDP fluctuations, and it does not manage to shape effectively the growth rates of gross domestic product of the studied small open economy. Instead, government spending not only depends to a large extent on the current income flows, as indicated by the large coefficients on GDP-term in contemporaneous regressions (first rows of tables 40, 42, 44, 46, 48), but also reacts to lagged economic information concerning GDP growth four and eight quarters ago. Given that statistical offices often supply information on yearly growth of output, this seems a robust result; the high positive coefficient on the eight-quarters lag may reflect a similar mechanism or, e.g., the role of the number of lags included in the models used by the government for planning or forecasting expenditure and tax revenues. Negative coefficients on the non-cyclical part of GDP growth may indicate that given this particular information the government attempted to counteract forecast adverse conditions or reduce deficit.

6. Conclusions

Expanding the existing body of research on economic cycles, it is shown in this paper that the frequencies traditionally treated as shorter than those associated with the business-cycle, or even as representing seasonal noise, have in fact much more impact on the growth rates of GDP and most of the other macroeconomic aggregates than the 'business-cycle' frequencies. Moreover, the latter are not present at all for some of the studied variables, gross capital formation among them. The devised sinusoidal signals model of economic cycles provides very close fit to the data, as evaluated by the mean squared error criterion, for most of macroeconomic aggregates, showing not only that the model enables to discover actual cycles in real-world data, but also that it is a useful research tool, which can help guide and shape government policy.

A simple yet novel, periodogram-guided representation of economic cycles has been proposed. It exploits direct correspondence between a form of the trend-plus-cycle model and the sinusoidal signals plus exponential trend representation of the gross growth rates of macroeconomic aggregates to avoid the problem of nonstationarity of data in levels, while retaining information about its cycles. The implementation of the model to discover the combination of highest periodogram frequencies that allow the representation to match the data the most is straightforward and does not require the use of sophisticated techniques. Nonetheless, the success in terms of the discovery of the cycles in the data is due to the research design.

Results demonstrate that the most pronounced cycles for almost all macroeconomic aggregates of the studied small open economy are those of periods of 2 and 4 quarters, contrary to the usual assumptions regarding which frequencies matter. It was possible to make this discovery only by assuming that fluctuations at the frequencies of 0.5 and 0.25 are not noise to be erased, but that they represent meaningful economic activity. Of course, this necessitates using not seasonally adjusted data, and making the assumption that firms adjust their marketing activity and capacity utilisation so that trading day and holiday effects do not matter or can be ignored.

The success of the devised model suggests that seasonal adjustment is unnecessary for business cycle analysis – by applying it, we loose the information on the cycles which constitute the core of growth rates of the majority of macroeconomic aggregates. Thus, we overlook the crucial part of the process and forego the possibility of fitting the growth path of GDP and other macroeconomic aggregates accurately.

Strikingly, the cycles that shape the evolution of various macroeconomic aggregates are different in terms of the composition of the entire sinusoidal signal, i.e., the values of its components' periods, amplitudes and phases. Moreover, the exponential trend (of the levels of aggregates) components are unequal for every pair of considered variables, which indicates that economic growth is unbalanced even in terms of these trends.

The devised framework has been applied to assessing two other theoretical concepts. First, the estimation of cyclical signals demonstrated that small open economies' business cycles are not necessarily aligned with or driven by the foreign ones, at least when the transmission of the latter is measured by net exports of goods and services. Additionally, the analysis performed in this paper gives credence to the claim that large international crises like the 2008 crash or the COVID pandemic can be treated as exogenous or very rare but random events.

The second theory-related question concerned evaluating whether the government's consumption was pro- or countercyclical – in terms of cycles in the data – and whether it was active or reactive when it comes to its non-cyclical, 'discretionary' parts. In terms of the former, the cycles of the best model of government expenditure's sinusoidal signals were mostly either perfectly synchronised or lagging behind their counterparts from the GDP series, which suggests procyclicality. As for the non-cyclical parts and the results of 100 Newey-West-robust regressions, contemporary GDP strongly shapes government's policy, while government expenditure had little success in changing the path of GDP growth. Large dependence on the contemporary conditions indicates that the government does not actively attempt to smooth fluctuations, but rather spends all or a significant part of its current income flow.

The constructed framework for the discovery of economic cycles can be applied to data for any country, as long as not-seasonally-adjusted data is available and long enough. The discoveries presented in this article open new research avenues for economic theorists. It may be that the multiplicity of cycles of macroeconomic aggregates will be easier to explain in agent-based models, rather than in frameworks based only on analytical systems of difference equations and intertemporal optimisation.

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Potential losses of information associated with using TRAMO-SEATS or other seasonal adjustment procedures

In this appendix, it is shown that the TRAMO-SEATS seasonal adjustment procedure severely distorts and removes crucial information from a data-generating process (DGP) dominated by cycles with periods of 2, 4, and 5 quarters. Specifically, seasonal differencing fully removes cycles of 2 and 4 quarters, while distorting the 5-quarter cycle. Assume for simplicity the true DGP consists of three cyclical components with periods of 2, 4, and 5 quarters:

$$Y_t = \alpha_2 \cos(2\pi\lambda_2 t + \phi_2) + \alpha_4 \cos(2\pi\lambda_4 t + \phi_4) + \alpha_5 \cos(2\pi\lambda_5 t + \phi_5) + \varepsilon_t \quad (41)$$

where $\alpha_2, \alpha_4, \alpha_5$ are the amplitudes of the cycles, ϕ_2, ϕ_4, ϕ_5 are the phase shifts, and ε_t is a white noise process. The corresponding frequencies are: $\lambda_2 = 0.5$, $\lambda_4 = 0.25$, $\lambda_5 = 0.2$.

TRAMO-SEATS applies seasonal differencing using the seasonal operator:

$$(1 - L^4)^D \quad (42)$$

where L is the lag operator ($LY_t = Y_{t-1}$), and D is the order of seasonal differencing. Applying $(1 - L^4)$ to a single sinusoidal component:

$$(1 - L^4) \cos(2\pi\lambda t + \phi) \quad (43)$$

Using the identity:

$$L^4 \cos(2\pi\lambda t + \phi) = \cos(2\pi\lambda(t - 4) + \phi), \quad (44)$$

we obtain:

$$(1 - L^4)\cos(2\pi\lambda t + \phi) = \cos(2\pi\lambda t + \phi) - \cos(2\pi\lambda(t-4) + \phi). \tag{45}$$

Using the trigonometric identity:

$$\cos(A) - \cos(B) = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \tag{46}$$

we get:

$$(1 - L^4)\cos(2\pi\lambda t + \phi) = -2\sin(2\pi\lambda(t-2) + \phi)\sin(4\pi\lambda). \tag{47}$$

Thus, the amplitude of the cycle is scaled by a factor of $2\sin(4\pi\lambda)$.

For $\lambda_4 = 0.25$:

$$\sin(4\pi \cdot 0.25) = \sin(\pi) = 0. \tag{48}$$

Thus, the seasonal differencing completely eliminates the 4-quarter cycle:

$$(1 - L^4)\cos(2\pi(0.25)t + \phi_4) = 0. \tag{49}$$

The full removal of the 4-quarter cycle by seasonal adjustment is disastrous if the data-generating process relies on this cycle for periodic dynamics.

For $\lambda_2 = 0.5$:

$$\sin(4\pi \cdot 0.5) = \sin(2\pi) = 0. \tag{50}$$

Thus, the 2-quarter cycle is also completely eliminated:

$$(1 - L^4)\cos(2\pi(0.5)t + \phi_2) = 0. \tag{51}$$

The removal of the 2-quarter cycle causes further distortions to the true DGP, and is very harmful if the cycle of frequency 0.5 is one of the dominant ones for the analysed variable.

For $\lambda_5 = 1/5$:

$$\sin(4\pi \cdot 1/5) = \sin(0.8\pi) \approx 0.5878. \tag{52}$$

Thus, the seasonal differencing alters the 5-quarter cycle by:

$$\alpha_5^{new} = \alpha_5 \cdot 2 \cdot 0.5878 \approx 1.1756\alpha_5. \tag{53}$$

The 5-quarter cycle is not fully removed, but its amplitude is modified.

The 4-quarter and 2-quarter cycles are completely removed, which is disastrous if the DGP relies on them. The 5-quarter cycle is distorted, but not entirely removed. Thus, TRAMO-SEATS seasonal adjustment is highly distorting and removing information for a DGP dominated by 4-quarter and 2-quarter cycles.