COLLEGIUM OF ECONOMIC ANALYSIS
WORKING PAPER SERIES

Measuring the uncertainty of shadow economy estimates using Bayesian and frequentist model averaging

Piotr Dybka, Bartosz Olesiński, Marek Rozkrut and Andrzej Torój

SGH KAE Working Papers Series Number: 2020/046 March 2020

Measuring the uncertainty of shadow economy estimates using Bayesian and frequentist model averaging

Piotr Dybka[*] Bartosz Olesiński[†] Marek Rozkrut[‡] Andrzej Torój[§]

Abstract

Economic literature provides little discussion on the uncertainty around the macroeconometric shadow economy estimates. We fill this gap by deriving the measurement error of the shadow economy estimates stemming from the model uncertainty by using frequentist and Bayesian model averaging techniques. This allows us to make useful insights into the optimal selection of regressors within the Currency Demand Analysis (CDA) framework, basing on the marginal probabilities that the selected variables are included in the "true" model. Hence, we provide the CDA researchers with an additional guidance with respect to the selection of shadow economy determinants that makes CDA-based shadow economy measurements less arbitrary. Our results show that the selection of regressors can have a material and highly country-specific impact on the estimated level of the shadow economy. In consequence, one cannot attribute the same level of uncertainty to every country across the panel. We use our results to demonstrate the average shadow economy estimates as of 2014 for 64 countries, along with the confidence intervals.

Keywords: Shadow economy, Currency Demand Approach, Measurement error, Confidence intervals

JEL: C10, C51, C59, E26, H26, O17

[*]SGH Warsaw School of Economics, Institute of Econometrics; Corresponding author (pdybka@sgh.waw.pl)
[†]SGH Warsaw School of Economics, Collegium of Economic Analyses and EY Economic Analysis Team
[‡]EY Economic Analysis Team
[§]SGH Warsaw School of Economics, Institute of Econometrics

1 Introduction

The issue of measuring the shadow economy has attracted much attention of economists and national statistical offices. The latter have access to detailed microdata from the entire economy and can conduct focused analyses for different subgroups of consumers or firms, making it possible to identify and interpret various discrepancies indicative of the shadow economy. Other researchers, in turn, apply a wide array of econometric and economic methods that use somewhat less detailed datasets, but - at the same time - are much more cost-effective and offer results much faster than in the case of the official measurements of the statistical offices.

Different econometric methods have been discussed in the literature, with one of the most prominent strands being related to modelling the demand for cash (Currency Demand Analysis – CDA), starting with an early contributions of Cagan (1958), followed by Gutmann (1977) and Feige (1979) and with important developments provided by Tanzi (1980, 1983). Later, the important contributions were provided by Giles and Tedds (2002), Embaye (2007), Ahumada et al (2008), Thießen (2010) and Ardizzi et al (2014), to name the few. In this paper, a specific version of the CDA model is applied that addresses many issues encountered in the previous literature (see Dybka et al, 2019, for a detailed discussion of those issues and the proposed improvements over the previous literature).

An important alternative to the CDA approach is the multiple-cause multiple-indicators model (MIMIC), with early contributions provided by Frey and Weck (1983) and Frey and Weck-Hannemann (1984), which has been greatly popularised in multiple works of Giles (1999, 2000); Giles and Tedds (2002) and Schneider (2005, 2006, 2007); Schneider et al (2010); Schneider (2016). In a more recent study of Dybka et al (2019), many well-known but previously unsolved weaknesses of the MIMIC model are addressed and a hybrid CDA-MIMIC approach is proposed.

While the literature discussing the ways how to calculate the point estimates of the shadow economy is quite extensive, it provides little discussion on the uncertainty related to the selection of the shadow economy determinants and measurement error of those estimates. There are some studies that provide confidence intervals based on the survey results (see, e.g. Putniņš et al, 2018; Putnins and Sauka, 2015; Putniņš and Sauka, 2015, 2011), but they are not applicable to studies on the macro level due to methodological differences, since the micro studies focus on the sampling uncertainty. Moreover, as noted by Putniņš and Sauka (2015), a survey-based approach is more costly and time-consuming than in the macro-based methods and the respondents might not be completely honest when asked about their engagement in the shadow economy. The downward bias of the survey-based point estimates is also mentioned by Kirchgässner (2016). In sum, the survey-based estimates have not attracted much attention in the shadow economy measurement literature. Furthermore, Goel and Nelson (2016) provide a discussion on the robustness of various shadow economy determinants; however, the effect of their selection on the measurement of the shadow economy is not quantified. Although Schneider (2016) reports: "I always state that these point estimates have a margin error of $+/-$ 15 %", there is no explanation how those confidence intervals are calculated, as noted by Feige (2016). As a result, we conclude that, to the best of our knowledge, the plausible estimates of uncertainty and measurement error of the shadow economy have never been provided.

In this paper, we aim to fill this gap by proposing a new methodology based on the model averaging procedures, namely Bayesian and Frequentist Model Averaging (BMA and FMA, respectively). The key idea of the methods is to estimate a large number of models and calculate weights/probabilities of each estimated model that allow averaging the results. In our analysis, we estimate all the possible (and economically justified) models for the set of potential determinants of the shadow economy identified on the basis of the literature review. Next, for each of the models, we calculate the appropriate weights (probabilities) that are based on the quality of the model fit to the data. Obtaining weights allows us to calculate the weighted average size of the shadow economy as well as to provide statistics of the shadow economy size distribution across the various models.

Model averaging procedures are gaining popularity in the economic literature, especially in the case of various forms of Bayesian model averaging, that began its expansion from the literature on the economic growth (see, e.g. Sala-I-Martin et al, 2004; Ley and Steel, 2009; Eicher et al, 2011; Amini and Parmeter, 2012). BMA is also gaining popularity in the analyses focusing on current account imbalances (Ca'Zorzi et al, 2012; Moral-Benito and Roehn, 2016; Dybka and Rubaszek, 2017), trade performance (Bierut and Dybka, 2019) and economic forecasting (Koop and Korobilis, 2012; Bork and Møller, 2015; Wang et al, 2016; Montero-Manso et al, 2020). In the context of the shadow economy estimation, BMA was used in Vicente (2019), though only to choose optimal variables in the CDA model for Spain. Our approach constitutes a significant extension of the BMA usage in the context of shadow economy estimation.

The model averaging approach allows us to tackle two issues. Firstly, we discuss the measurement error of the shadow economy estimates stemming from the model uncertainty. Secondly, we draw useful insights on the optimal selection of regressors within the CDA framework (for a large group of countries), providing guidance for other researchers that would like to focus on some single econometric specification, without the need to estimate thousands, or even millions, of models. The shadow economy uncertainty measured in our approach is of course centered around some point estimates, which are related to the "average CDA model" that accounts for a much broader set of regressors than the dataset typically included in the standard CDA literature. The shadow economy point estimates supplemented with different measures of uncertainty (e.g., 95% confidence intervals) offer a complete set of shadow economy results that is more robust and transparent than in the previous literature.

In the econometric analysis, we use a large, quarterly dataset of 26 socioeconomic variables for 64 countries observed over the 2004-2016 timespan. We have decided to present the results obtained for the year 2014, because this is the most recent year for which we have all the data for all the countries included in the dataset.

The remainder of this paper is structured as follows: Section 2 provides the description of our econometric approach, Section 3 contains our results and, finally, Section 4 concludes.

2 Econometric approach

2.1 CDA general specification

The key assumption in the CDA framework is that most of the unregistered transactions are settled with cash[1]. The CDA approach aims to decompose the demand for cash, measured as the ratio of currency in circulation to the M1 monetary aggregate, into two components: the first component is the cash used in the formal economy and the remaining part is the "excess" cash used to facilitate the unregistered transactions. Our approach is based on the following currency demand equation:

$$\frac{\boldsymbol{Cash}}{\boldsymbol{M1}}_{i,t} = \boldsymbol{\beta^{(1)}_{i,t}} \boldsymbol{x_{1,i,t}} + \boldsymbol{\beta^{(2)}_{i,t}} \boldsymbol{x_{2,i,t}} + \boldsymbol{\beta^{(3)}_{i,t}} \boldsymbol{x_{3,i,t}} + \alpha_i + \epsilon_{i,t} \tag{1}$$

where i represents the analysed country and t stands for the analysed time period. In this equation, the dependent variable is the share of currency in circulation ("cash") in the M1 monetary aggregate ("total transactional money"). The $\boldsymbol{\beta^{(1)}_{i,t}}$, $\boldsymbol{\beta^{(2)}_{i,t}}$, and $\boldsymbol{\beta^{(3)}_{i,t}}$ represent vectors of the regression coefficients, related respectively to: "typical" cash shadow economy determinants (x_1), payment card system variables (x_2) and other control variables (x_3) – see Tables 2-3 for details and Appendix B for sources and definitions. We assume that these coefficients linearly depend on real GDP per capita (Purchasing Power Parity adjusted, US dollar in 2011), so they differ across countries and periods (which is reflected in the notation by i and t superscripts). For $k = 1, 2, 3$ we have:

$$\boldsymbol{\beta^{(k)}_{i,t}} = \gamma^{(k)}_0 + \gamma^{(k)}_1 \times GDP_{i,t}, \tag{2}$$

in which $\boldsymbol{\gamma^{(k)}_0}$ and $\boldsymbol{\gamma^{(k)}_1}$ are vectors of parameters describing coefficient heterogeneity. Essentially, some coefficients out of $\boldsymbol{\beta^{(1)}_{i,t}}$, $\boldsymbol{\beta^{(2)}_{i,t}}$, and $\boldsymbol{\beta^{(3)}_{i,t}}$ can be constant. In such cases, the respective element of the vector $\boldsymbol{\gamma^{(k)}_1}$ equals 0, i.e., the strength of the coefficient out of $\boldsymbol{\beta^{(1)}_{i,t}}$, $\boldsymbol{\beta^{(2)}_{i,t}}$, and $\boldsymbol{\beta^{(3)}_{i,t}}$ does not vary with GDP per capita. To estimate $\boldsymbol{\gamma^{(k)}_1}$, we use the interaction terms, i.e. products of the respective variables with GDP (demeaned, so that the symbols $\boldsymbol{\gamma^{(k)}_0}$ describe the average marginal effects). As a result the construction of the coefficients α_i, $\boldsymbol{\beta^{(1)}_{i,t}}$, $\boldsymbol{\beta^{(2)}_{i,t}}$, and $\boldsymbol{\beta^{(3)}_{i,t}}$ reflects country heterogeneity. This is crucial in our econometric model, because there are 64 countries included in the analysis.

Furthermore, we have also accounted for the individual country effects, α_i, which represent time-invariant, unobservable country characteristics that affect the demand for cash in each country. The last element of the equation (1), $\varepsilon_{i,t}$, is the error term (that includes factors that were not accounted for in the model).

Finally, to estimate the parameters of the CDA regression models, we have used the panel-corrected standard error (PCSE) estimator[2] (see Beck and Katz, 1995), in which individual effects α_i are estimated as fixed effects. To estimate the parameters we have collected a large macroeconomic dataset

[1] As a result, it does not include the non-monetary shadow economy that is usually related to the production of goods for own use and transactions settled through barter. The non-monetary shadow economy is mostly related to the products of agriculture is relatively high in less developed countries, where agriculture constitutes a significant part of the GDP.

[2] We have used a common serial correlation coefficient of the error term calibrated at the level of $\rho = 0.83$ (based on our preliminary analyses).

containing 26 variables for 64 countries over the 2004-2016 period (1812 common observations). Detailed information on the data used in the analysis can be found in Table 7.

2.2 Application of frequentist and Bayesian model averaging

There are many potential determinants of the shadow economy and the variable selection can significantly affect the results. Therefore, we propose to use the model averaging techniques that allow us to estimate the probability that a given variable should be included in the model. In our analysis, we compare two types of weights, frequentist and Bayesian, that are based on formal criteria describing the quality of fit and parsimony of specification.

The frequentist weights are based on the Akaike Information Criterion (AIC). They are calculated on the basis of formula proposed by Buckland et al (1997):

$$
\begin{aligned}
w_s^{Freq} &= \frac{\exp(-\frac{AIC_s - \min_s(AIC_s)}{2})}{\sum_{s=1}^{S}\exp(-\frac{AIC_s - \min_s(AIC_s)}{2})}, && (3)\\
AIC_s &= N\log(\frac{SSR_s}{N}) + 2K_s, && (4)
\end{aligned}
$$

in which $s = 1, 2, \ldots, S$ indexes the models under consideration, N is the total number of observations, SSR_s is the sum of squared residuals and K_s is the number of coefficients that are not automatically included in all the specifications (we omit country dummies). The obtained weight w_s^{Freq} allows calculation of the weighted mean value of the coefficient and/or the shadow economy.

The Bayesian weights are based on the posterior probability that a given specification reflects the true data generating process. They are based on the marginal likelihood formula described, i.a., in Steel (2017):

$$
\begin{aligned}
w_s^{Bayes} &= \frac{\Pi_s^{posterior}}{\sum_{s=1}^{S}\Pi_s^{posterior}}, && (5)\\
\Pi_s^{posterior} &= \Pi_s^{prior}\exp(\lambda_s - \max_s(\lambda_s)), && (6)\\
\lambda_s &= -\frac{K_s}{2}\ln(1+g) - \frac{N-1}{2}\ln(\frac{1}{1+g} + \frac{g}{1+g}(1-R_s^2)), && (7)\\
R_s^2 &= 1 - \frac{SSR_s}{SST}, && (8)\\
g &= (K_{max})^2, && (9)\\
K_{max} &= \max_s(K_s), && (10)
\end{aligned}
$$

where SST is the variance of $\frac{cash\ outside\ banks}{M1}$ variable (the explained variable), Π_s^{prior} is the prior probability that a given model is true (described later), K_s is the number of variables in the model (except the variables that are always included), R_s^2 is the coefficient of determination summarising the quality of the model fit (the larger the better). The g is the g-prior hyperparameter, set according to the "Risk Inflation Criterion" (RIC) formula for the choice of the optimal g-prior proposed by Foster and George (1994), where K_{max} is the maximum number of variables included in the analysis (on top of the variables that are always included)[3].

As far as the prior probability of the model specification is concerned, we use the following set of assumptions:

$$\Pi_s^{prior} = \theta^{K_s}(1-\theta)^{K_{max}-K_s} \tag{11}$$

$$\theta = \frac{K_{max}-1}{K_{max}}, \tag{12}$$

where θ is the prior probability of inclusion of each variable (except the variables that are always included).

It needs to be pointed out that the full model space contains 2^K specifications. On the one hand, it means that each additional variable multiplies the number of potential models by a factor of two and, therefore, enumerating the whole model space can be computationally challenging. On the other hand, some of the possible combinations might have little economic sense (e.g. an empty model or models containing only some control variables, but no shadow economy determinants). As a result, we reduce the number of combinations by imposing the following conditions:

1. At least one shadow economy determinant ($\boldsymbol{x_{1,i,t}}$) must be included in each model.
2. Three control variables are always included in the specification: real GDP per capita (in PPS) and both dummy variables (cf. Table 2).
3. We have grouped variables with similar information content into clusters. From the following clusters, only a single variable could be included in a given econometric specification:
   - Global Competitiveness Index variables (Ethics of firms, Public trust in politicians, Regulatory burden, Transparency of policymaking, Cost of crime, Cost of organised crime)
   - Measures of self-employment on the labour market (Self-employment persons, Contributing family workers, Own-account workers)
   - Interest rates (Nominal deposit interest rate, Real interest rate)
   - Measures of labour market under-utilization (Unemployment rate, Unemployed+Inactive persons)
   - Measures of attitudes to government and law (Rule of Law, Government Effectiveness)
   - Financial system variables (Domestic credit to private sector, Financial development).

Moreover, different interactions of variables are allowed in the model space. In particular, the shadow economy determinants and the payment card system variable ($\boldsymbol{x_1}$ and $\boldsymbol{x_2}$) are allowed to be interacted with a real GDP per capita in PPS (abbreviated later as GDP). However, the interaction in a given

[3] The original versions of the formulae (3) and (6) are modified by subtracting $\min_s(AIC_s)$ and $\max_s(\lambda_s)$ from AIC_s and λ_s, respectively. Note that this is arithmetically neutral for the computation of w_s^{Freq} and w_s^{Bayes} in (3) and (6). This modification, however, helps to avoid numerical problems.

model is only allowed if the related variable is included in this model (for instance, a model in which there is no $\boldsymbol{x_2}$ variable, but a product of this variable with GDP is included, is not allowed). Out of other control variables ($\boldsymbol{x_3}$), only the GDP variable is interacted with itself.

In total, $S = 4,913,280$ models meet all the criteria specified above. Note that there are different data points missing for different variables, so that a given selection of variables implies the maximum sample size. To avoid the differences between the models resulting from this implicit sample selection, the same minimum sample of 1812 is used (for 64 countries), common to all the models under consideration[4]. In each case, all the variables (including the $\frac{\boldsymbol{Cash}}{\boldsymbol{M1}}$ variable) are transformed using the Prais-Winsten transformation in order to tackle the issue of estimation efficiency under error autocorrelation.

Last but not least, the models with economically unjustified coefficient signs are removed, but only for the variables ($\boldsymbol{x_{1,i,t}}$) and ($\boldsymbol{x_{2,i,t}}$) and not for the respective interactions with GDP. Table 1 summarizes the imposed sign restrictions.

Table 1: Restrictions on coefficient signs imposed in the CDA model

Regressor	**The direction of impact on cash demand and shadow economy***
Time to prepare and pay taxes (World Bank)	+
Ethics of firms (GCI)	-
Public trust in politicians (GCI)	-
Regulatory burden (GCI)	-
Transparency of policymaking (GCI)	-
Cost of crime (GCI)	-
Cost of organised crime (GCI)	-
Rule of Law (World Bank)	-
Government Effectiveness (World Bank)	-
Unemployed+Inactive persons (% of population aged 15-64)	+
Unemployment rate (% of total labour force)	+
Self-employed persons (% of population aged 15-64)	+
Contributing family workers (% of population aged 15-64)	+
Own-account workers (% of population aged 15-64)	+
Number of active cards per capita	-

* - The impact is allowed to be zero in each case.

[4]Model averaging techniques are based on the quality of fit measures that are not able to establish whether improvement in the quality of fit occurred due to better selection of variables or the change in the sample size so the sample cannot vary depending on the variable selection.

3 Shadow economy estimates

In this section, the shadow economy estimates are demonstrated and discussed. The model averaging exercise allows us to obtain 3 kinds of results (for frequentist and Bayesian weights):

- The marginal probability that a given variable is included in the true data generating process.
- The weighted average value of individual coefficients.
- The distribution of the shadow economy estimates.

The description of distribution of shadow economy estimates in this section includes histograms, quantiles, standard deviations, point estimates (average mean) and 95% confidence intervals. The 95% confidence intervals are based on the weighted population of models implying various shadow economy measurements (using frequentist and Bayesian weights). We start with the marginal probabilities that a given variable is included in the true data generating process and then we discuss the weighted average coefficients. Finally, we analyse the distribution of the shadow economy estimates, which allows us to assess the model uncertainty.

3.1 Key determinants of the shadow economy

The first set of results comprises the posterior inclusion probabilities (PIP) for the shadow economy determinants ($\boldsymbol{x_{1,i,t}}$), the card payment variable ($\boldsymbol{x_{2,i,t}}$) and other control variables ($\boldsymbol{x_{3,i,t}}$), presented in Table 2. We consider the frequentist and Bayesian weights with restricted and non-restricted coefficient signs. In terms of the probability of inclusion, the restricted variants do not differ substantially from the unrestricted one.

There are three major shadow economy determinants with PIP exceeding 90%: (i) Time to prepare and pay taxes (that measures the tax system complexity level), (ii) Rule of Law (the general measure of attitudes towards the law which should be accompanied with the interaction with GDP per capita in PPS), (iii) Unemployed and Inactive persons (that proves to be a better measure than unemployment alone). The high inclusion probability of Unemployed and Inactive persons variable, compared to Unemployment rate, indicates that accounting for the people discouraged from searching for a job is also important. Thus, including also the inactive people allows us to better approximate the state of the labour market.

A relatively high probability can also be observed for the measures of self-employment on the labour market (variables: Self-employment persons, Contributing family workers, Own-account workers), where we imposed a restriction that only one of the variables from that group can be in one model. We can observe that the sum of PIP for those three variables exceeds 80%. At first glance, it seems that Contributing family workers is the best variable, however, there is a problem with its sign (discussed later), so the own-account workers can also be considered (that should also be accompanied with interaction term with GDP per capita in PPS).

Ethics of firms from the Global Competitiveness Index and the Number of active payment cards (with GDP interaction) also have a relatively high posterior inclusion probability and, therefore, should be considered in the estimated CDA model.

Table 2: Marginal probabilities of variable inclusion

	Frequentist variant		**Bayesian variant**	
	Unrestricted	*Restricted*	*Unrestricted*	*Restricted*
I. "Typical" cash shadow economy determinants				
Time to prepare and pay taxes (hours)	94.94%	93.62%	98.71%	98.36%
interaction with GDP	28.80%	28.28%	51.91%	51.62%
Ethics of firms (GCI)	47.60%	49.35%	39.30%	39.22%
interaction with GDP	13.51%	14.77%	19.39%	20.24%
Public trust in politicians (GCI)	16.55%	16.07%	25.06%	24.96%
interaction with GDP	13.72%	13.32%	22.94%	22.80%
Regulatory burden (GCI)	8.08%	8.73%	10.75%	11.61%
interaction with GDP	4.80%	5.23%	8.36%	8.82%
Transparency of policymaking (GCI)	4.20%	4.11%	4.99%	5.24%
interaction with GDP	1.50%	1.54%	2.95%	3.19%
Cost of crime (GCI)	11.35%	10.54%	12.29%	11.68%
interaction with GDP	4.88%	4.54%	7.70%	7.29%
Cost of organised crime (GCI)	5.40%	4.90%	5.58%	5.34%
interaction with GDP	1.53%	1.41%	2.76%	2.70%
Rule of Law	91.39%	94.56%	89.98%	95.26%
interaction with GDP	51.98%	55.20%	70.50%	78.02%
Government Effectiveness	7.19%	4.53%	9.62%	4.56%
interaction with GDP	4.04%	2.46%	7.35%	3.54%
Unemployed+Inactive (% of pop. aged 15-64)	98.03%	95.77%	97.75%	92.82%
interaction with GDP	29.47%	27.78%	51.76%	48.83%
Unemployment rate (% of total labour force)	1.20%	2.56%	1.86%	5.91%
interaction with GDP	0.34%	0.71%	0.93%	2.93%
Self-employed persons (% of pop. aged 15-64)	8.34%	4.39%	6.50%	3.92%
interaction with GDP	2.38%	0.32%	3.26%	0.82%
Contrib. family workers (% of pop. aged 15-64)	50.63%	14.33%	62.96%	8.82%
interaction with GDP	44.15%	0.00%	59.39%	0.00%
Own-account workers (% of pop. aged 15-64)	25.22%	45.77%	27.18%	78.75%
interaction with GDP	17.84%	45.77%	23.35%	78.75%
II. Payment card system variables				
Number of active cards per capita	65.89%	27.21%	91.65%	51.51%
interaction with GDP	52.18%	3.54%	83.13%	9.44%
III. Other control variables				
Real GDP per capita (in PPS)	Always included	Always included	Always included	Always included
Real GDP per capita (in PPS, squared)	35.78%	43.71%	57.35%	70.65%

Nominal deposit interest rate	99.94%	99.98%	99.93%	99.98%
Real deposit interest rate	0.05%	0.02%	0.06%	0.02%
CPI	63.42%	62.63%	80.41%	79.69%
Domestic credit to private sector (as % of GDP)	36.89%	32.27%	51.94%	47.40%
Financial Development (based on IMF)	19.20%	20.32%	25.31%	27.30%
Urban population (% of total population)	57.05%	48.36%	76.11%	64.23%
Agriculture empl. (% of total labour force)	29.17%	29.57%	50.75%	50.93%
People with internet access (% of total pop.)	90.98%	86.58%	94.94%	83.25%
Dummy variable for Demonetization in India	Always included	Always included	Always included	Always included
Dummy variable for a credit boom in Romania	Always included	Always included	Always included	Always included
Additional information				
Number of models	4 913 280	1 345 367	4 913 280	4 913 280
The avg. number of coefficients	79.40	78.05	82.37	81.00

Notes: Interaction with GDP refers to the Real GDP per capita (in PPS). Fixed effects α_i are omitted for brevity of presentation, but included in each specification, as well as Real GDP per capita (in PPS) variable, dummy variable for Demonetization in India and dummy variable for a credit boom in Romania (see Appendix B for details). The restricted variants require that each specification meet the sign restrictions as enumerated in Table 1.

In order to verify which variables should be included in the model, the average coefficients also need to be analysed - they are presented in Table 3. The results clearly depend on the imposition of the sign restrictions (summarised in Table 1) – this is particularly important in the case of the Contributing family workers variable which has a relatively high PIP. This shadow economy determinant should have a positive impact on demand for cash and it is the only case where the restriction leads to a change of the sign. In the case of the remaining variables, the restrictions lead to an increase of the coefficient value (in absolute terms). This result is consistent with the intuition - removing the models with the opposite sign should result in the increase of the weighted mean of the coefficient.

Table 3: Weighted average CDA coefficients

	Frequentist variant		**Bayesian variant**	
	Unrestricted	*Restricted*	*Unrestricted*	*Restricted*
I. “Typical” cash shadow economy determinants				
Time to prepare and pay taxes (hours)	0.004	0.004	0.005	0.005
interaction with GDP	0.000	0.000	0.000	0.000
Ethics of firms (GCI)	-0.387	-0.397	-0.314	-0.300
interaction with GDP	0.004	0.008	0.002	0.010
Public trust in politicians (GCI)	-0.027	-0.032	-0.040	-0.050

interaction with GDP	-0.026	-0.025	-0.044	-0.043
Regulatory burden (GCI)	-0.019	-0.027	-0.024	-0.038
interaction with GDP	-0.007	-0.008	-0.013	-0.014
Transparency of policymaking (GCI)	-0.004	-0.005	-0.005	-0.007
interaction with GDP	-0.001	-0.001	-0.002	-0.003
Cost of crime (GCI)	-0.035	-0.031	-0.036	-0.031
interaction with GDP	0.004	0.004	0.006	0.007
Cost of organised crime (GCI)	-0.011	-0.009	-0.011	-0.009
interaction with GDP	0.000	0.001	0.000	0.001
Rule of Law	-2.367	-2.552	-2.322	-2.626
interaction with GDP	0.179	0.186	0.265	0.296
Government Effectiveness	-0.104	-0.064	-0.142	-0.065
interaction with GDP	0.012	0.007	0.023	0.011
Unemployed+Inactive (% of pop. aged 15-64)	0.259	0.234	0.255	0.209
interaction with GDP	-0.002	0.001	-0.005	0.005
Unemployment rate (% of total labour force)	0.001	0.002	0.002	0.005
interaction with GDP	0.000	0.000	0.000	0.000
Self-employed persons (% of pop. aged 15-64)	-0.001	0.001	0.000	0.000
interaction with GDP	0.000	0.000	0.000	0.000
Contrib.family workers (% of pop. aged 15-64)	-0.210	0.006	-0.293	0.004
interaction with GDP	-0.083	0.000	-0.116	0.000
Own-account workers (% of pop. aged 15-64)	0.021	0.065	0.034	0.123
interaction with GDP	0.017	0.044	0.023	0.081
II. Payment card system variable				
Number of active cards per capita	0.311	-0.066	0.537	-0.125
interaction with GDP	-0.174	-0.010	-0.290	-0.029
III. Other control variables				
Real GDP per capita (in PPS)	0.119	-0.607	0.309	-1.244
Real GDP per capita (in PPS, squared)	0.008	0.019	0.011	0.037
Nominal deposit interest rate	0.149	0.152	0.148	0.154
Real deposit interest rate	0.000	0.000	0.000	0.000
CPI	-0.015	-0.015	-0.020	-0.019
Domestic credit to private sector (as % of GDP)	0.005	0.003	0.007	0.005
Financial Development (based on IMF)	-0.235	-0.246	-0.313	-0.326
Urban population (% of total pop.)	0.106	0.073	0.144	0.083
Agriculture empl. (% of total labour force)	-0.006	-0.002	-0.008	0.005
People with internet access (% of total pop.)	-0.033	-0.030	-0.035	-0.028
Dummy variable for Demonetization in India	-20.223	-20.261	-20.268	-20.334
Dummy variable for a credit boom in Romania	-16.525	-16.556	-16.520	-16.559

Additional information

Number of models	4 913 280	1 345 367	4 913 280	1 345 367
The avg. number of coefficients	79.40	78.05	82.37	81.00

Notes: Interaction with GDP refers to the Real GDP per capita (in PPS). Fixed effects α_i are omitted for brevity of presentation, but included in each specification, as well as Real GDP per capita (in PPS) variable, dummy variable for Demonetization in India and dummy variable for a credit boom in Romania (see Appendix B for details). The restricted variants require that each specification meet the sign restrictions as enumerated in Table 1.

The reason why we impose such sign restrictions is that the economic literature provides insights regarding the proper sign of the considered variables. Consequently, a different sign indicates that there is a problem with the estimation of the model (e.g. due to omitted variables). Moreover, removing the counter-intuitive models results in a significant reduction in the number of potential models - we keep only 1.3 million out of 4.9 million models. To verify the credibility of our sign restrictions, we have estimated a single model with crucial variables identified above[5], and we have obtained results consistent with the restrictions discussed in Table 1.

3.2 Weighted shadow economy estimates

The presented shadow economy distribution statistics and histograms are as of 2014, when the data for all the countries in the sample is available[6]. Such statistics can be obtained for any other year, given the availability of the necessary data for the analysed country.

The results are expressed in terms of % of **total** GDP, that incorporates both registered and the shadow economy. Depending on the value of the non-observed (shadow)[7] economy included in the official GDP calculated by the statistical offices, the total GDP can substantially or just marginally differ from the officially published GDP. If a statistical office does not include the whole shadow economy, the result expressed in terms of the officially published GDP would be higher than that figure expressed in % of total GDP and vice versa. Furthermore, the CDA framework does not take into account the non-monetary shadow economy, which may be a substantial part of the overall shadow economy in countries with a considerable share of agriculture in GDP. Consequently, providing specific results for a single country might require a country-focused approach.

Table 4 presents statistics of the obtained shadow economy distribution using the frequentist weights: the 25th, 50th, 75th, 99th percentiles, maximum and mean values as well as the standard deviation. To obtain the distribution of the shadow economy, we estimated each of the possible models, calculated

[5] We do not discuss this model for brevity of presentation.

[6] We used unbalanced panel dataset and observations for some countries in 2015 and 2016 were unavailable.

[7] In this study, our definition of shadow economy is the same as the definition of the "non-observed economy", that is used in the national accounts methodology. The definition of the non-observed economy (=shadow economy) can be found in the ESA 2010 methodology. `https://ec.europa.eu/eurostat/documents/3859598/5925693/KS-02-13-269-EN.PDF/44cd9d01-bc64-40e5-bd40-d17df0c69334` [online, accessed 12.02.2020]

the shadow economy on the basis of each model and then we used the frequentist (and Bayesian) weights to obtain the mean shadow economy value.

Our results indicate the lowest level of the shadow economy in Switzerland, United Arab Emirates (UAE), Norway, Sweden, Denmark and New Zealand where the mean estimate is below 5% of the total GDP. Such findings are not surprising, because the UAE has one of the lowest levels of taxation, whereas the remaining countries rely heavily on the electronic payments. In contrast, the highest value of the shadow economy can be observed in Algeria, Nigeria and Brazil, where the mean estimate of the shadow economy ranges from 21 to 24% of the total GDP. The standard deviation of the shadow economy ranges from 0.99% of the total GDP in Japan to 3.63% of the total GDP in Singapore.

Table 4: Shadow economy (% of total GDP) distribution: frequentist weights, 2014

	Weighted quantile of order				**Max**	**Weighted Mean**	**Weighted Std. dev.**
	0.25	0.5	0.75	0.99			
Switzerland	2.11	2.75	5.68	8.75	18.41	3.81	2.00
United Arab Emirates	2.62	3.65	5.86	8.46	12.11	4.16	1.78
Norway	3.12	3.79	5.49	8.27	21.73	4.32	1.48
Sweden	3.71	4.19	5.49	7.64	14.60	4.59	1.17
Denmark	3.95	4.53	5.41	8.04	16.56	4.75	1.11
New Zealand	3.84	4.32	5.91	7.90	9.59	4.86	1.29
United Kingdom	4.98	5.46	7.55	9.98	12.50	6.21	1.68
Japan	5.56	6.02	6.93	8.63	12.05	6.23	0.99
Canada	4.91	5.44	7.75	10.29	12.04	6.28	1.78
Australia	4.84	5.64	7.82	10.55	12.79	6.31	1.83
Singapore	4.03	4.88	10.63	15.11	17.58	7.06	3.63
Bahrain	6.76	7.92	9.02	10.87	15.53	7.83	1.47
Rwanda	7.29	8.90	9.98	12.73	21.12	8.68	1.80
Israel	8.46	9.45	10.10	12.42	15.78	9.32	1.25
Kuwait	7.43	9.96	11.32	15.29	35.78	9.46	2.63
Czechia	8.41	9.62	10.41	13.10	14.92	9.55	1.47
Chile	8.73	9.90	10.53	12.76	15.33	9.70	1.35
Uruguay	9.05	9.86	10.51	13.13	15.78	9.84	1.18
Malaysia	9.96	10.86	11.48	13.70	17.51	10.68	1.34
Thailand	10.59	11.41	12.20	14.67	20.54	11.26	1.49
Poland	10.79	12.03	12.79	15.01	18.39	11.77	1.47
Saudi Arabia	11.01	12.89	14.24	16.72	23.62	12.45	2.48
Hungary	11.40	12.57	13.88	15.72	19.53	12.53	1.69
China	12.10	12.82	13.57	15.76	20.39	12.60	1.61
Tanzania	11.45	12.90	14.08	17.16	26.30	12.69	2.03
Kazakhstan	11.52	12.59	14.10	17.36	19.74	12.83	2.01

Nepal	11.70	13.36	14.79	18.01	25.23	13.15	2.25
Russia	12.45	13.55	14.17	15.91	19.69	13.16	1.54
Romania	12.40	13.73	14.89	16.95	22.55	13.56	1.79
Peru	13.03	13.76	14.64	17.30	23.45	13.64	1.64
Croatia	13.35	14.63	15.85	17.50	21.79	14.35	1.91
Colombia	13.62	14.59	15.47	18.02	24.63	14.40	1.70
Indonesia	13.87	14.79	15.61	18.52	25.57	14.59	1.76
Vietnam	13.41	14.79	16.03	19.70	24.75	14.64	2.16
Azerbaijan	13.94	15.01	15.84	18.43	25.52	14.71	1.82
Myanmar	13.24	15.14	16.52	19.78	27.24	14.82	2.40
Bulgaria	14.23	15.32	16.37	18.78	22.47	15.16	1.74
Philippines	14.30	15.57	16.57	19.58	26.20	15.29	2.01
Mongolia	14.38	15.68	16.57	19.27	26.51	15.34	1.98
Mexico	14.84	16.00	16.87	19.27	24.00	15.66	1.85
Jamaica	15.39	16.43	17.27	20.38	27.33	16.17	1.89
Sri Lanka	15.44	16.79	17.73	20.68	27.65	16.37	2.22
Dominican Rep.	15.60	16.81	17.95	20.84	27.29	16.66	1.97
India	15.73	17.07	18.00	21.48	29.26	16.73	2.18
Turkey	15.74	17.01	18.38	20.29	25.82	16.81	2.17
Argentina	16.04	17.56	18.53	20.99	24.42	17.14	2.01
Honduras	16.34	17.91	19.32	22.77	32.09	17.66	2.35
Serbia	16.60	17.83	19.29	22.13	28.02	17.69	2.27
Ukraine	16.73	18.00	19.08	22.29	26.63	17.73	2.11
Armenia	16.74	18.27	19.21	22.72	29.08	17.85	2.25
Jordan	16.96	18.31	19.51	22.62	32.91	17.89	2.71
Tunisia	17.51	18.85	20.25	23.29	30.38	18.55	2.58
Angola	17.91	19.28	20.77	24.39	31.86	19.06	2.50
Bangladesh	17.79	19.35	20.78	24.84	32.42	19.12	2.49
Albania	18.11	19.53	20.57	24.06	30.95	19.12	2.37
Moldova	18.34	20.17	21.28	25.08	35.60	19.59	2.80
Lebanon	18.61	19.98	21.30	24.37	31.76	19.65	2.59
Bolivia	18.28	19.86	21.31	25.63	30.54	19.66	2.57
Pakistan	18.75	20.38	21.66	25.67	32.96	20.05	2.54
Egypt	20.05	21.52	22.43	26.37	32.55	21.07	2.48
Bosnia and Herz.	20.02	21.39	23.16	26.74	33.26	21.26	2.71
Algeria	20.74	22.13	23.40	27.03	33.31	21.74	2.72
Nigeria	22.16	23.82	25.24	30.10	35.40	23.58	2.67
Brazil	23.08	24.22	25.18	30.74	34.52	23.87	3.42

Notes: The results are expressed in terms of % of **total** GDP. We do not include the non-monetary shadow economy.

The results obtained with the use of Bayesian weights are presented in Table 5. Application of the Bayesian weights does not lead to considerable changes in the results, although it slightly increases the estimated level of the shadow economy - on average, the weighted mean value of the shadow economy is by 0.38% of the total GDP higher than in the case of using the frequentist weights. The same countries have the lowest/highest mean shadow economy value (although, in some cases, minor changes in ranks are observed).

Table 5: Shadow economy (% of total GDP) distribution: Bayesian weights, 2014

	Weighted quantile of order				**Max**	**Weighted Mean**	**Weighted Std. dev.**
	0.25	0.5	0.75	0.99			
United Arab Emirates	2.34	3.09	4.76	8.29	12.11	3.61	1.70
Switzerland	4.35	5.89	6.72	9.58	18.41	5.48	2.03
Sweden	4.64	5.59	6.36	8.23	14.60	5.50	1.26
Norway	4.45	5.60	6.78	8.98	21.73	5.52	1.65
Denmark	4.71	5.58	6.37	8.51	16.56	5.62	1.27
New Zealand	4.85	5.99	6.79	8.35	9.59	5.80	1.32
Japan	6.00	6.90	7.59	8.94	12.05	6.82	1.03
Bahrain	6.48	7.31	8.31	10.52	15.53	7.38	1.44
United Kingdom	6.34	7.78	8.71	10.49	12.50	7.46	1.69
Canada	6.59	8.03	8.98	10.80	12.04	7.67	1.77
Australia	6.72	8.16	9.12	11.15	12.79	7.82	1.84
Kuwait	7.60	9.47	11.03	15.69	35.78	9.40	2.62
Israel	8.82	9.59	10.59	12.54	15.78	9.64	1.31
Singapore	7.65	10.66	12.22	16.04	17.58	9.75	3.67
Rwanda	9.02	10.18	10.92	13.06	21.12	9.86	1.62
Chile	9.03	9.94	11.07	12.99	15.33	9.92	1.48
Czechia	9.22	10.02	11.29	13.44	14.92	10.15	1.49
Uruguay	9.44	10.24	11.14	13.36	15.78	10.29	1.23
Malaysia	10.29	11.09	12.01	13.89	17.51	10.98	1.48
Thailand	10.85	11.61	12.53	14.83	20.54	11.59	1.47
Saudi Arabia	10.11	12.33	13.90	16.57	23.62	11.83	2.75
Poland	10.95	11.86	13.05	15.10	18.39	11.86	1.56
Hungary	11.29	12.00	13.73	15.53	19.53	12.26	1.74
China	12.50	13.11	13.98	15.91	20.39	13.00	1.54
Russia	12.45	13.20	14.23	15.82	19.69	13.14	1.51
Romania	12.35	13.24	14.99	16.99	22.55	13.44	1.90
Tanzania	12.83	13.99	14.87	17.33	26.30	13.74	1.80
Kazakhstan	12.59	13.96	15.43	17.85	19.74	13.87	2.00
Croatia	12.99	13.86	15.43	17.19	21.79	13.91	1.98
Peru	13.52	14.15	15.15	17.43	23.45	14.16	1.54
Colombia	13.72	14.55	15.60	18.01	24.63	14.50	1.68

Nepal	13.72	15.00	15.90	18.48	25.23	14.69	1.97
Azerbaijan	14.03	15.05	16.11	18.46	25.52	14.87	1.86
Indonesia	14.20	15.10	15.95	18.49	25.57	14.90	1.73
Bulgaria	14.12	15.01	16.35	18.67	22.47	15.06	1.77
Mongolia	14.42	15.56	16.58	19.20	26.51	15.30	1.98
Philippines	14.71	15.87	16.85	19.50	26.20	15.57	2.00
Mexico	14.84	15.72	17.04	19.24	24.00	15.65	1.94
Sri Lanka	15.28	16.49	17.57	20.43	27.65	16.11	2.32
Vietnam	15.22	16.28	17.24	20.15	24.75	16.14	1.88
Myanmar	15.37	16.61	17.53	20.08	27.24	16.24	2.11
Jamaica	15.62	16.64	17.53	20.31	27.33	16.38	1.87
Turkey	15.56	16.38	18.25	20.06	25.82	16.43	2.34
Dominican Rep.	15.64	16.67	17.94	20.76	27.29	16.63	1.96
India	15.77	16.98	18.11	21.42	29.26	16.70	2.26
Jordan	16.10	17.37	18.74	22.15	32.91	16.98	2.93
Serbia	16.23	17.55	18.84	21.78	28.02	17.30	2.33
Argentina	16.23	17.31	18.88	21.08	24.42	17.31	2.06
Armenia	16.85	18.17	19.32	22.52	29.08	17.87	2.26
Tunisia	16.89	18.29	19.62	22.77	30.38	17.93	2.69
Ukraine	17.36	18.52	19.47	22.22	26.63	18.21	2.04
Honduras	17.29	18.85	19.87	22.86	32.09	18.44	2.21
Albania	17.93	19.19	20.39	23.81	30.95	18.87	2.45
Moldova	18.11	19.53	21.08	24.77	35.60	19.20	2.95
Lebanon	18.29	19.64	20.93	24.02	31.76	19.24	2.71
Angola	18.73	20.15	21.34	24.45	31.86	19.84	2.34
Bangladesh	18.60	20.14	21.38	24.95	32.42	19.86	2.38
Pakistan	19.14	20.65	21.98	25.69	32.96	20.34	2.54
Bosnia and Herz.	19.39	21.01	22.41	26.09	33.26	20.67	2.80
Egypt	20.00	21.29	22.39	26.02	32.55	20.91	2.56
Bolivia	20.07	21.42	22.48	25.97	30.54	21.18	2.28
Algeria	20.34	21.77	23.02	26.48	33.31	21.30	2.86
Nigeria	23.14	24.71	25.93	30.31	35.40	24.42	2.57
Brazil	23.80	24.70	25.96	31.42	34.52	25.03	2.58

Notes: The results are expressed in terms of % of **total** GDP. We do not include the non-monetary shadow economy.

The weighted distributions imply 95% confidence intervals, summarised in Table 6. According to the obtained results, the shadow economy estimates can vary substantially across the models, resulting in rather wide confidence intervals. The coefficients of variation are generally higher in the case of countries where the estimated level of the shadow economy is relatively low - less than 10% of the total GDP.

Table 6: Confidence intervals (CI) of the shadow economy estimates

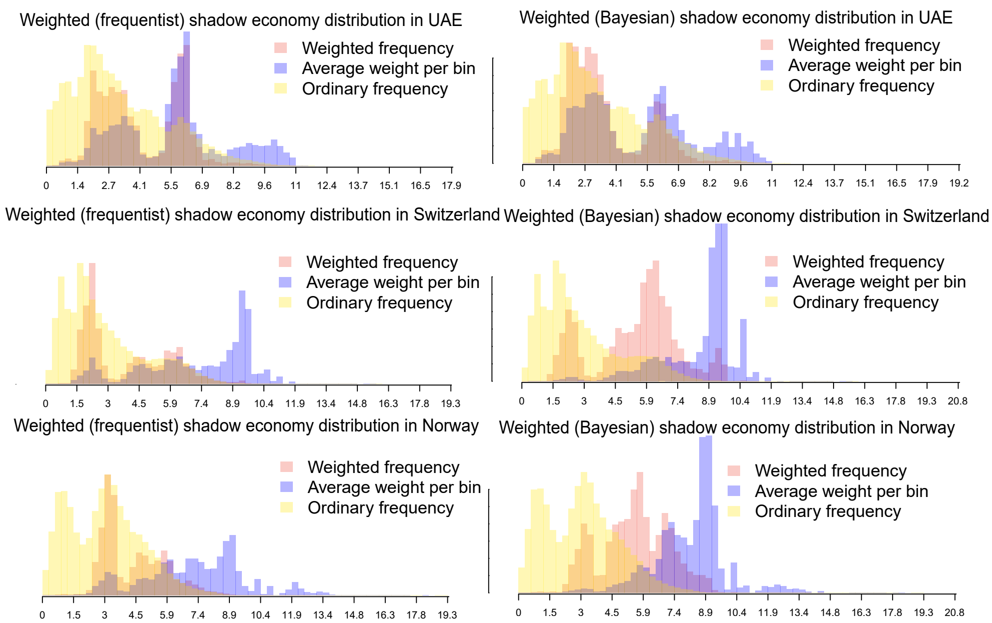
	Frequentist weights: 95% CI		Bayesian weights: 95% CI	
	lower bound	upper bound	lower bound	upper bound
Switzerland	1.32	7.30	1.81	9.40
United Arab Emirates	1.51	6.88	1.17	6.97
Norway	2.31	7.25	2.39	8.40
Sweden	2.79	6.85	3.17	7.88
Denmark	3.03	7.12	3.45	8.26
New Zealand	3.14	7.34	3.29	7.85
United Kingdom	3.80	9.37	4.09	10.03
Japan	4.68	8.29	4.94	8.59
Canada	3.79	9.69	4.03	10.28
Australia	3.70	9.74	4.50	11.04
Singapore	2.08	13.62	3.03	15.25
Bahrain	4.92	10.41	3.96	9.99
Rwanda	5.96	12.06	6.44	12.20
Israel	6.83	11.68	7.28	12.43
Kuwait	4.51	13.88	4.51	14.86
Czechia	6.98	12.28	7.34	13.07
Chile	7.41	12.46	6.72	12.60
Uruguay	7.69	12.40	8.06	12.69
Malaysia	7.48	13.04	7.59	13.38
Thailand	7.78	14.05	8.54	14.60
Poland	8.96	14.62	8.13	14.62
Saudi Arabia	7.65	16.57	5.15	16.11
Hungary	9.21	15.51	8.12	15.12
China	8.66	15.46	9.12	15.61
Tanzania	8.99	16.45	9.61	16.68
Kazakhstan	9.13	17.49	10.37	17.56
Nepal	9.25	17.52	10.29	18.03
Russia	9.43	15.75	9.48	15.35
Romania	9.83	16.65	9.02	16.54
Peru	10.13	16.75	10.67	16.86
Croatia	10.06	17.47	9.14	17.06
Colombia	10.56	17.51	10.74	17.36
Indonesia	10.75	18.13	11.06	17.79
Vietnam	10.04	18.88	12.09	19.76
Azerbaijan	10.39	17.90	10.54	17.92
Myanmar	10.77	19.30	11.49	19.47
Bulgaria	10.93	18.18	10.96	18.18

Philippines	10.77	19.14	11.17	18.89
Mongolia	10.51	18.57	10.79	18.65
Mexico	10.75	18.54	10.81	18.70
Jamaica	11.45	19.49	12.17	19.65
Sri Lanka	10.30	19.90	10.08	19.62
Dominican Rep.	12.07	20.11	12.32	20.06
India	11.03	20.54	10.89	20.29
Turkey	12.05	20.63	10.13	19.83
Argentina	11.71	20.15	12.87	21.16
Honduras	13.32	22.83	13.82	22.04
Serbia	11.93	21.38	11.48	21.08
Ukraine	13.08	21.75	13.70	21.66
Armenia	12.45	21.65	12.96	21.87
Jordan	13.55	23.33	8.17	21.08
Tunisia	13.07	24.05	10.44	21.93
Angola	14.15	23.77	15.19	23.97
Bangladesh	14.28	24.13	14.89	23.99
Albania	12.71	23.00	12.89	22.96
Moldova	14.09	25.84	11.63	23.82
Lebanon	13.72	24.88	12.10	23.34
Bolivia	14.52	25.08	16.26	25.52
Pakistan	14.21	24.74	14.48	24.51
Egypt	14.34	25.45	14.21	25.13
Bosnia and Herz.	16.66	27.87	13.30	24.99
Algeria	15.44	27.67	13.39	25.45
Nigeria	17.70	29.04	18.88	29.29
Brazil	13.77	30.34	22.11	30.97

Notes: Estimates for the year 2014. The results are expressed in terms of % of **total** GDP. We do not include the non-monetary shadow economy.

Figure 1 shows the weighted histograms of the shadow economy estimates across the analysed models for the three countries with the lowest frequentist-weighted mean value of the shadow economy. We can observe that the largest number of models (yellow bars) are grouped along the lowest values of the shadow economy, however the models with higher values of the shadow economy have higher weights. This indicates that models explaining to a lesser extent the changes in the ratio of currency in circulation to M1 money aggregate lead to lower shadow economy estimates (larger part of the variance is explained by the error term and country fixed effects). Such a result indicates that a failure to include the crucial variables into the CDA model results in an underestimation of the shadow economy level.

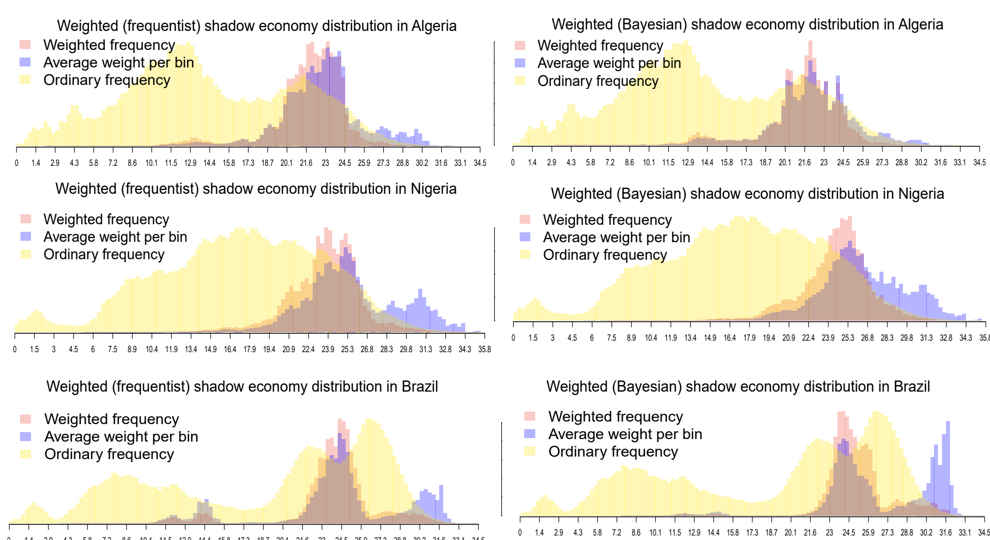
Figure 1: Histogram of the shadow economy estimates in 3 countries with the lowest mean



Notes: Estimates for the year 2014.

To better understand the distribution of the shadow economy we also present the histograms for three countries with the highest frequentist-weighted mean value of the shadow economy in Figure 2. We can observe very similar patterns as in the case of countries with the lowest shadow economy levels, i.e. models that poorly explain the changes in the ratio of currency in circulation to M1 money aggregate also underestimate the shadow economy level. Histograms of the remaining countries can be found in Appendix B.

Figure 2: Histogram of the shadow economy estimates in 3 countries with the highest mean



Notes: Estimates for the year 2014.

4 Conclusions

We propose a novel strategy for quantifying the model uncertainty around shadow economy estimates in the CDA model, based on frequentist and Bayesian model averaging techniques. Our approach allows for more informed selection of regressors entering the CDA equation with a high posterior inclusion probability, including: Time to prepare and pay taxes (measure of the tax system complexity level), Rule of Law (general measure of attitudes towards the law), sum of Unemployed and Inactive persons, Contributing family workers or Own-account workers, Ethics of firms (from the Global Competitiveness Index) and Number of active payment cards. In addition to this, the inclusion of the interaction terms with the above-mentioned variables is necessary to achieve the required model flexibility. By doing so, we are able to better account for the differences across countries that can be crucial while estimating the model on a large macroeconomic data panel.

Bottom line, we provided the CDA-based estimates of the shadow economy level (as % of total GDP) as of 2014, for 64 countries, with the accompanying 95% confidence intervals related to the model uncertainty. The results obtained with both frequentist and Bayesian model weighting schemes are largely consistent.

The uncertainty measures around the estimated shadow economy level turned out to vary across countries to a considerable extent. Countries with a higher value of the shadow economy generally exhibit slightly higher standard deviations of the shadow economy estimate. Yet, the relative (to the mean) standard deviations are smaller for countries with high levels of the shadow economy. Last but not least, the models that poorly explain the changes in the ratio of currency in circulation to M1 money aggregate also tend to underestimate the shadow economy level.

In this paper, we focused on the model uncertainty alone. A promising area of future research is accounting also for the uncertainty related to estimation of the coefficients that can be done in a full Bayesian analysis of CDA model.

Acknowledgements: We thank participants (especially to Prof. Lubomir Cingl) of the conference 'Tax Evasion and Economic Inequality' in July 2019 in Trento for their useful suggestions. We are also grateful to Michał Kowalczuk for his valuable comments and the priceless inspirations from our previous joint research projects.

References

Ahumada H, Alvaredo F, Canavese A (2008) The monetary method to measure the shadow economy: The forgotten problem of the initial conditions. Economics Letters 101(2):97–99, DOI 10.1016/j.econlet.2008.06.001

Amini SM, Parmeter CF (2012) Comparison of model averaging techniques: Assessing growth determinants. Journal of Applied Econometrics 27(5):870–876, DOI 10.1002/jae.2288, URL `https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.2288`, `https://onlinelibrary.wiley.com/doi/pdf/10.1002/jae.2288`

Ardizzi G, Petraglia C, Piacenza M, Turati G (2014) Measuring the underground economy with the currency demand approach: A reinterpretation of the methodology, with an application to italy. Review of Income and Wealth 60(4):747–772, URL http://EconPapers.repec.org/RePEc:bla:revinw:v:60:y:2014:i:4:p:747-772

Beck N, Katz JN (1995) What to do (and not to do) with time-series cross-section data. The American Political Science Review 89(3):634–647, URL http://www.jstor.org/stable/2082979

Bierut BK, Dybka P (2019) Institutional determinants of export competitiveness among the EU countries: evidence from Bayesian model averaging. Working Papers 2019-043, Warsaw School of Economics, Collegium of Economic Analysis, URL https://ideas.repec.org/p/sgh/kaewps/2019043.html

Bork L, Møller SV (2015) Forecasting house prices in the 50 states using dynamic model averaging and dynamic model selection. International Journal of Forecasting 31(1):63 – 78, DOI https://doi.org/10.1016/j.ijforecast.2014.05.005, URL http://www.sciencedirect.com/science/article/pii/S0169207014000958

Buckland ST, P BK, H AN (1997) Model selection: An integral part of inference. Biometrics 53(2):603–618

Cagan P (1958) The Demand for Currency Relative to the Total Money Supply. Journal of Political Economy 66(4):303–328

Ca'Zorzi M, Chudik A, Dieppe A (2012) Thousands of models, one story: Current account imbalances in the global economy. Journal of International Money and Finance 31(6):1319 – 1338

Dybka P, Rubaszek M (2017) What Determines the Current Account: Intratemporal versus Intertemporal Factors. Czech Journal of Economics and Finance (Finance a uver) 67(1):2–14

Dybka P, Kowalczuk M, Olesiński B, Torój A, Rozkrut M (2019) Currency demand and mimic models: towards a structured hybrid method of measuring the shadow economy. International Tax and Public Finance 26(1):4–40, DOI 10.1007/s10797-018-9504-5, URL https://doi.org/10.1007/s10797-018-9504-5

Eicher TS, Papageorgiou C, Raftery AE (2011) Default priors and predictive performance in bayesian model averaging, with application to growth determinants. Journal of Applied Econometrics 26(1):30–55, DOI 10.1002/jae.1112, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.1112, https://onlinelibrary.wiley.com/doi/pdf/10.1002/jae.1112

Embaye A (2007) Underground economy estimates for non-oecd countries using currency demand method, 1984-2005. MPRA Paper 20308, University Library of Munich, Germany, URL https://ideas.repec.org/p/pra/mprapa/20308.html

Feige EL (1979) How big is the irregular economy? Challenge (5):5–13, DOI 10.2307/40719809

Feige EL (2016) Professor Schneider's Shadow Economy (SSE): What Do We Really Know? A Rejoinder. Journal of Tax Administration 2(2)

Foster DP, George EI (1994) The risk inflation criterion for multiple regression. The Annals of Statistics 22(4):1947–1975

Frey BS, Weck H (1983) Estimating the shadow economy: A 'naive' approach. Oxford Economic Papers 35(1):23–44

Frey BS, Weck-Hannemann H (1984) The hidden economy as an 'unobserved' variable. European Economic Review 26(1-2):33–53

Giles DE (1999) Measuring the hidden economy: implications for econometric modelling. The Economic Journal 109:F370–F380

Giles DE (2000) Taxation and the Limits of Government, Kluwer Academic Publishers, chap Modelling the hidden economy and the tax-gap in New Zealand, pp 195–219

Giles DE, Tedds L (2002) Taxes and the Canadian Underground Economy. Canadian Tax Foundation, Toronto

Goel RK, Nelson MA (2016) Shining a light on the shadows: Identifying robust determinants of the shadow economy. Economic Modelling 58:351 – 364, DOI https://doi.org/10.1016/j.econmod.2016.06.009, URL http://www.sciencedirect.com/science/article/pii/S0264999316301729

Gutmann PM (1977) The Subterranean Economy. Financial Analysts Journal 33(6):23–29, DOI 10.2469/ccb.v2007.n6.4818, URL http://www.jstor.org/stable/4478078

Kirchgässner G (2016) On estimating the size of the shadow economy. German Economic Review 18(1):99–111

Koop G, Korobilis D (2012) Forecasting inflation using dynamic model averaging*. International Economic Review 53(3):867–886, DOI 10.1111/j.1468-2354.2012.00704.x, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1468-2354.2012.00704.x, https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1468-2354.2012.00704.x

Ley E, Steel MF (2009) On the effect of prior assumptions in bayesian model averaging with applications to growth regression. Journal of Applied Econometrics 24(4):651–674, DOI 10.1002/jae.1057, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.1057, https://onlinelibrary.wiley.com/doi/pdf/10.1002/jae.1057

Montero-Manso P, Athanasopoulos G, Hyndman RJ, Talagala TS (2020) Fforma: Feature-based forecast model averaging. International Journal of Forecasting 36(1):86 – 92, DOI https://doi.org/10.1016/j.ijforecast.2019.02.011, URL http://www.sciencedirect.com/science/article/pii/S0169207019300895, m4 Competition

Moral-Benito E, Roehn O (2016) The impact of financial regulation on current account balances. European Economic Review 81:148 – 166

Putnins T, Sauka A (2015) Shadow economy index for the baltic states 2009-2014. 4libertyeu Review 3(16 - 28)

Putniņš TJ, Sauka A (2011) Size and determinants of shadow economies in the baltic states. Baltic Journal of Economics 11(2):5–25, DOI 10.1080/1406099X.2011.10840498

Putniņš TJ, Sauka A (2015) Measuring the shadow economy using company managers. Journal of Comparative Economics 43(2):471 – 490, DOI https://doi.org/10.1016/j.jce.2014.04.001, URL http://www.sciencedirect.com/science/article/pii/S0147596714000262

Putniņš TJ, Sauka A, Davidescu AA (2018) Shadow economy index for moldova and romania 2015-2016, URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3171741&download=yes

Sala-I-Martin X, Doppelhofer G, Miller RI (2004) Determinants of Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach. American Economic Review 94(4):813–835

Schneider F (2005) Shadow economies around the world: What do we really know? European Journal of Political Economy 21:598–642

Schneider F (2006) Shadow economies of 145 countries all over the world: What do we really know? Etudes Fiscales Internationales URL http://www.etudes-fiscales-internationales.com/files/ShadEconomyWorld145_2006.pdf

Schneider F (2007) Shadow economies and corruption all over the world: New estimates for 145 countries. Economics - The Open-Access, Open-Assessment E-Journal 1:1–66

Schneider F (2016) Comment on Feige's Paper Reflections on the Meaning and Measurement of Unobserved Economies: What Do We Really Know About the 'Shadow Economy'? Journal of Tax Administration 2(2)

Schneider F, Buehn A, Montenegro CE (2010) Shadow economies all over the world. New estimates for 162 countries from 1999 to 2007. World Bank Policy Research Working Paper

Steel MF (2017) Model averaging and its use in economics. MPRA Paper No 81568

Svirydzenka K (2016) Introducing a New Broad-based Index of Financial Development. IMF Working Paper (January):1–43

Tanzi V (1980) Underground Economy Built on Illicit Pursuits is Growing Concern of Economic Policymakers

Tanzi V (1983) The Underground Economy in the United States: Annual Estimates, 1930-80. Staff Papers (International Monetary Fund) 30(2):283–305

Thießen U (2010) The Shadow Economy in International Comparison: Options for Economic Policy Derived from an OECD Panel Analysis. International Economic Journal 24(4):481–509, DOI 10.1080/10168737.2010.525986, URL http://www.tandfonline.com/doi/abs/10.1080/10168737.2010.525986

Vicente R (2019) New Evidence on the Size and Drivers of the Shadow Economy in Spain: A Model Averaging Approach. MPRA Paper No 97504

Wang Y, Ma F, Wei Y, Wu C (2016) Forecasting realized volatility in a changing world: A dynamic model averaging approach. Journal of Banking & Finance 64:136 – 149, DOI https://doi.org/10.1016/j.jbankfin.2015.12.010, URL http://www.sciencedirect.com/science/article/pii/S0378426615003647

Appendix A: Descriptions and sources of all the variables used in the CDA model

Table 7 presents detailed description of all the variables used in our CDA models, with the respective data sources. Additional materials that can be used to replicate the results are available in the following repository https://doi.org/10.6084/m9.figshare.12000579

Table 7: Definitions of variables used in the analysis

Variable name	Description of the variable	Source(s)
Explained (dependent) variable		
Cash to M1 ratio	The share of the currency in circulation in the sum of the currency in circulation and demand deposits held in financial institutions (M1 monetary aggregate), % of M1, seasonally adjusted. Minor deviations from this definition are possible for some countries. M1 monetary aggregate ("total transactional money") typically includes cash and demand deposits of residents (individuals and firms, excluding public administration), denominated in the national currency, held in local financial institutions. However, in some countries, the definition of M1 monetary aggregate deviates slightly from this standard definition and might include: foreign currency deposits, time deposits, deposits of the central governments and local (e.g., regional) governments. Exclusion of these additional components for all the countries that enter the econometric sample is impossible due to the lack of data on these non-standard components.	International Monetary Fund, Asian Development Bank, local central banks, own calculations
I. "Typical" cash shadow economy determinants		
Time to prepare and pay taxes	Time to prepare and pay taxes in hours**	World Bank – Doing Business project
Ethics of firms	An indicator summarizing the ethical behavior of firms, based on answers to the following question: "In your country, how do you rate the corporate ethics of companies (ethical behavior in interactions with public officials, politicians and other firms)? [1 = extremely poor; 7 = excellent]"**	World Economic Forum – The Global Competitiveness Index
Public trust in politicians	An indicator summarizing the public trust in politicians, based on answers to the following question: "In your country, how do you rate the ethical standards of politicians? [1 = extremely low; 7 = extremely high]"**	World Economic Forum – The Global Competitiveness Index
Regulatory burden	An indicator summarizing the burden of government regulations, based on answers to the following question: "In your country, how burdensome is it for companies to comply with public administration's requirements (e.g., permits, regulations, reporting)? [1 = extremely burdensome; 7 = not burdensome at all]"**	World Economic Forum – The Global Competitiveness Index

Transparency of policymaking	An indicator summarizing the transparency of government policymaking, based on answers to the following question: "In your country, how easy is it for companies to obtain information about changes in government policies and regulations affecting their activities? [1 = extremely difficult; 7 = extremely easy]"**	World Economic Forum – The Global Competitiveness Index
Cost of crime	An indicator summarizing the perceived business costs of crime and violence, based on answers to the following question: "In your country, to what extent does the incidence of crime and violence impose costs on businesses? [1 = to a great extent—imposes huge costs; 7 = not at all—imposes no costs]"**	World Economic Forum – The Global Competitiveness Index
Cost of organised crime	An indicator summarizing the perceptions of organised crime, based on answers to the following question: "In your country, to what extent does organised crime (mafia-oriented racketeering, extortion) impose costs on businesses? [1 = to a great extent—imposes huge costs; 7 = not at all—imposes no costs]"**	World Economic Forum – The Global Competitiveness Index
Rule of Law	The value of the indicator measuring the rule of law from the Worldwide Governance Indicators; ranges from approximately -2.5 (weak rule of law) to 2.5 (strong rule of law)**	World Bank – Worldwide Governance Indicators
Government Effectiveness	The value of the indicator measuring the government effectiveness from the Worldwide Governance Indicators; ranges from approximately -2.5 (low government effectiveness) to 2.5 (high government effectiveness)**	World Bank – Worldwide Governance Indicators
Unemployed and inactive persons	The % share of unemployed (aged 15+*) and economically inactive (aged 15-64) persons in the total population (aged 15-64)**	International Labour Organization, own calculations
Unemployment rate	Unemployment rate, % of total labour force (economically active population), seasonally adjusted***	International Monetary Fund, World Bank
Self employed persons	The ratio of the total number of self-employed (employers, own-account workers, members of producers' cooperatives, and contributing family workers) to the population aged 15-64**	International Labour Organization, own calculations
Contributing family workers	The ratio of the total number of contributing family workers to the population aged 15-64**	International Labour Organization, own calculations
Own-account workers	The ratio of the total number of own-account workers to the population aged 15-64**	International Labour Organization, own calculations
II. Payment card system variables		
Number of active cards per capita	The number of active payment cards per capita, seasonally adjusted***. The number of active cards is calculated on the basis of additional data from the Global Findex database on the share of people using cards throughout the past year. We use the number of active payment cards per capita instead of payment cards transaction value, because the former variable is more likely to be exogenous (i.e. there are less feedback effects from the explained variable to the explanatory variables; such feedback effects are detrimental to the quality of estimation).	Eurostat, European Central Bank, World Bank (Global Payment Systems Survey, Global Findex database), International Bank for Settlements, national central banks
III. Other control variables		

Real GDP per capita	Real GDP per capita in PPS in constant 2010 prices (purchasing power parity adjusted, US dollar in 2011), seasonally adjusted***	Eurostat, International Monetary Fund, World Bank, own calculations
Nominal deposit interest rate	Households deposit rate per annum, in %***. for some countries simplified assumptions are made in order to translate interbank offer rates or central bank policy rates into household deposit rates	International Monetary Fund, local central banks.
Real deposit interest rate	Households deposit rate per annum adjusted by yoy CPI inflation rate, in %***. for some countries simplified assumptions are made in order to translate interbank offer rates or central bank policy rates into household deposit rates	International Monetary Fund, local central banks, own calculations
CPI rate	yoy CPI inflation rate, in %. For Argentina GDP deflator is used	Eurostat, International Monetary Fund, local central banks and statistical offices
Domestic credit to private sector	Domestic credit to private sector [% of GDP], seasonally adjusted***	International Monetary Fund, World Bank, Bank for International Settlements, African Development Bank, local central banks and statistical offices
Financial Development	Index [0-1, 1=best development], aggregate of financial institutions, financial depth and financial market development indices**. IMF data available till 2014, for the 2015-2016 period, a forecast based on the CREDIT GDP variable (see above) is formulated	International Monetary Fund – Svirydzenka (2016), own calculations based on the CREDIT TO GDP variable (see above)
Urban population	The % share of urban population in the entire population**	World Bank
Agriculture employment	The % share of people employed in agriculture in the overall employment**	International Labour Organization
People with internet accessS	The share of the population with Internet access, % of population**	International Telecommunication Union (United Nations)
Dummy variable for Demonetization in India	Binary variable controlling for the effect of demonetization in India in Q4 2016	Own elaboration
Dummy variable for a credit boom in Romania	Binary variable controlling for the credit boom in Romania starting in Q1 2007	Own elaboration

Notes: The historical data on Global Competitiveness Index was provided by the courtesy of World Economic Forum representatives.

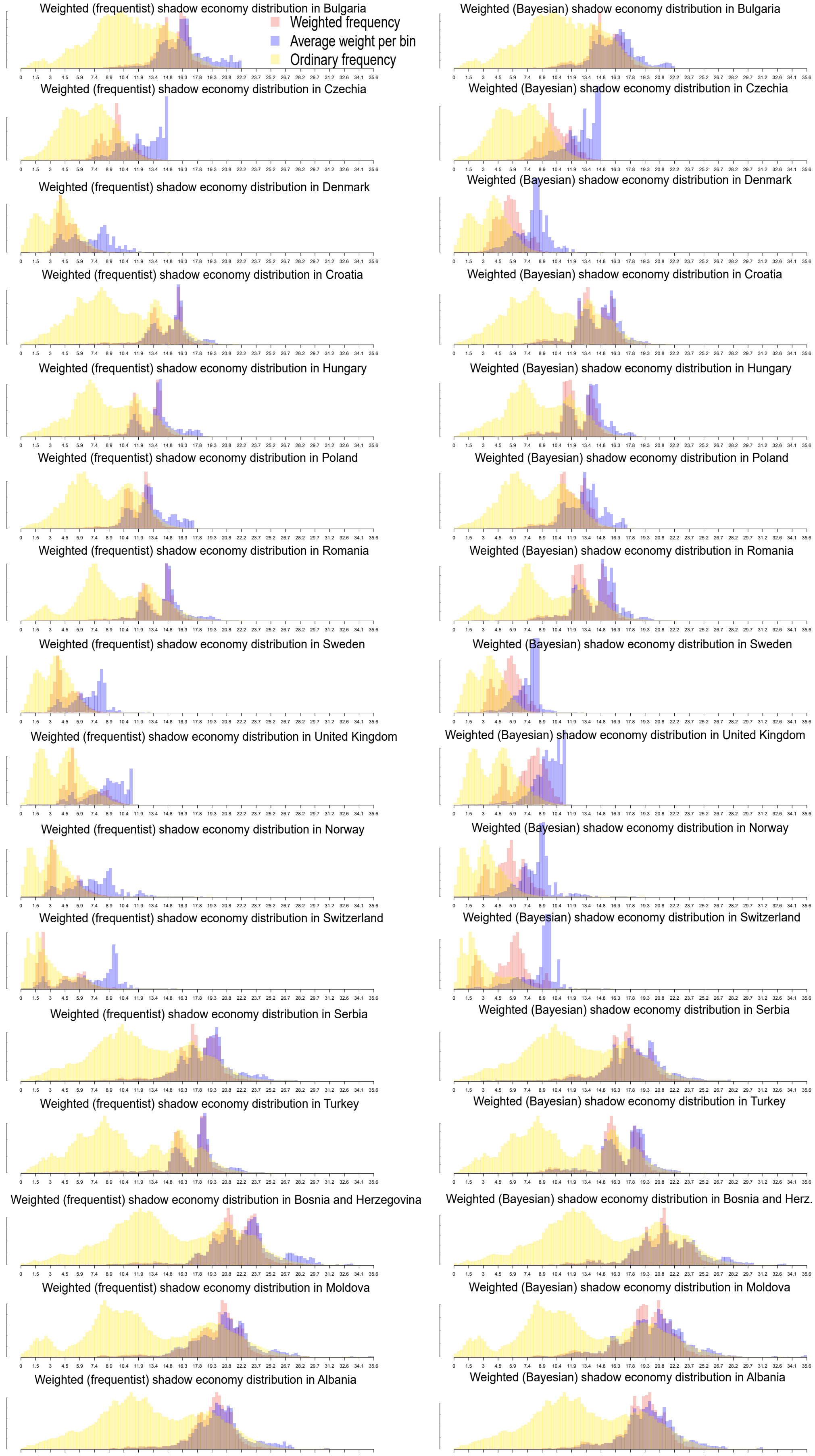
* - Data for unemployed aged 15-64 is unavailable, but unemployed persons are most likely less than 65 years old.

** - interpolated from annual to quarterly.

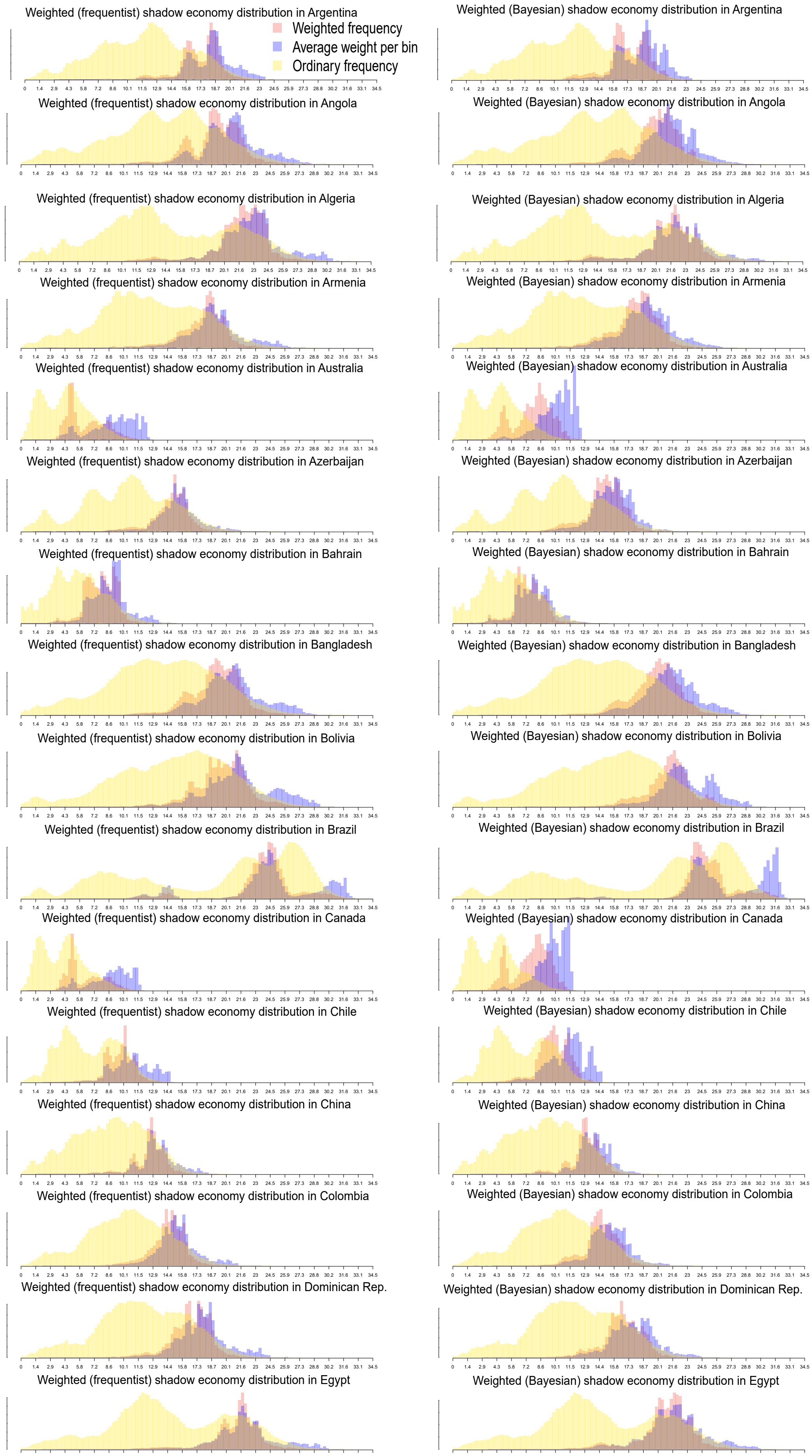
*** - for some countries interpolated from annual to quarterly.

Countries included in the analysis: Albania, Algeria, Angola, Argentina, Armenia, Australia, Azerbaijan, Bahrain, Bangladesh, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Canada, Chile, China (Mainland), Colombia, Croatia, Czech Republic, Denmark, Dominican Republic, Egypt, Honduras, Hungary, India, Indonesia, Israel, Jamaica, Japan, Jordan, Kazakhstan, Kuwait, Lebanon, Malaysia, Mexico, Moldova, Mongolia, Myanmar, Nepal, New Zealand, Nigeria, Norway, Pakistan, Peru, Philippines, Poland, Romania, Russia, Rwanda, Saudi Arabia, Serbia, Singapore, Sri Lanka, Sweden, Switzerland, Tanzania, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, Uruguay, Vietnam.

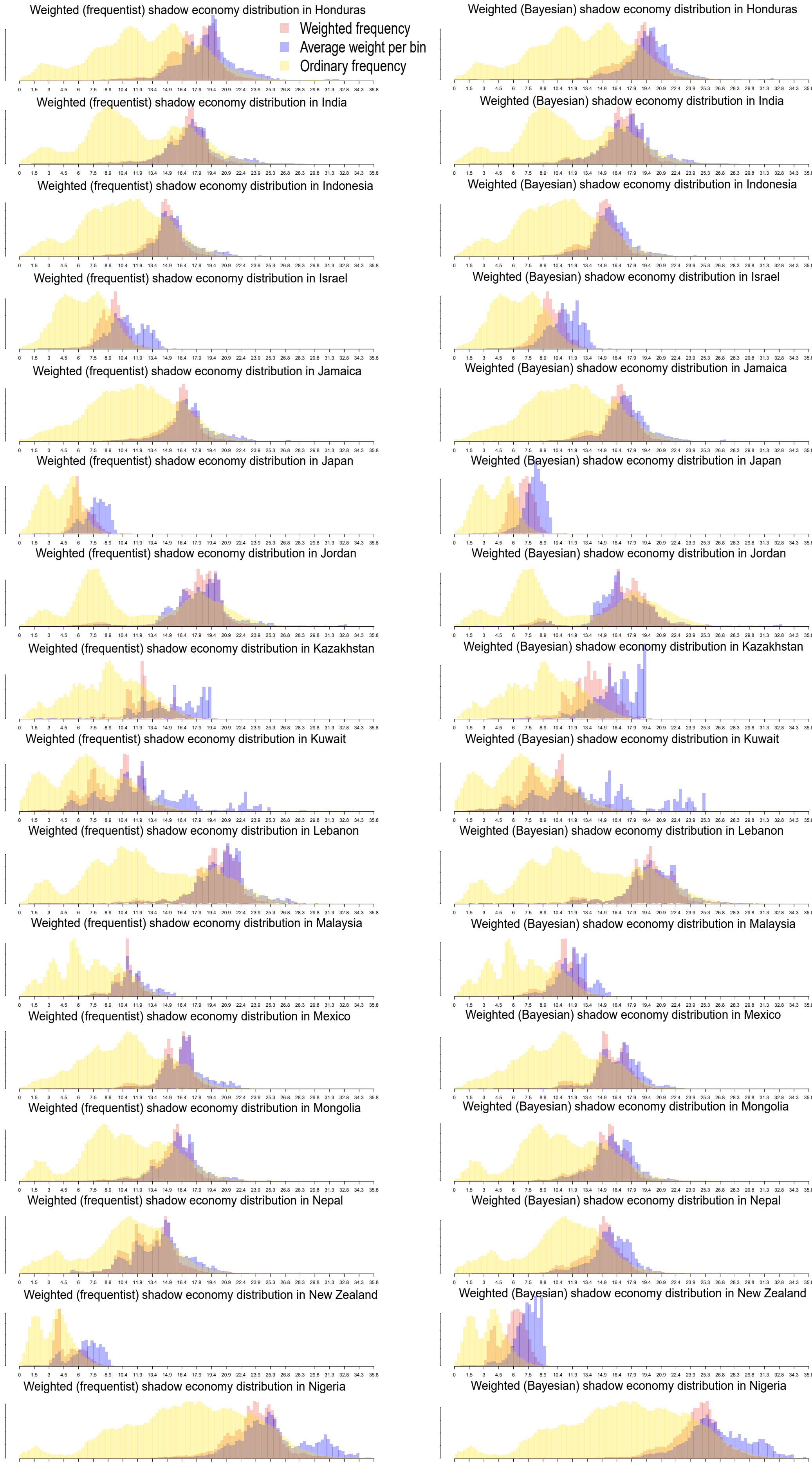
Appendix B: Shadow economy estimates histograms



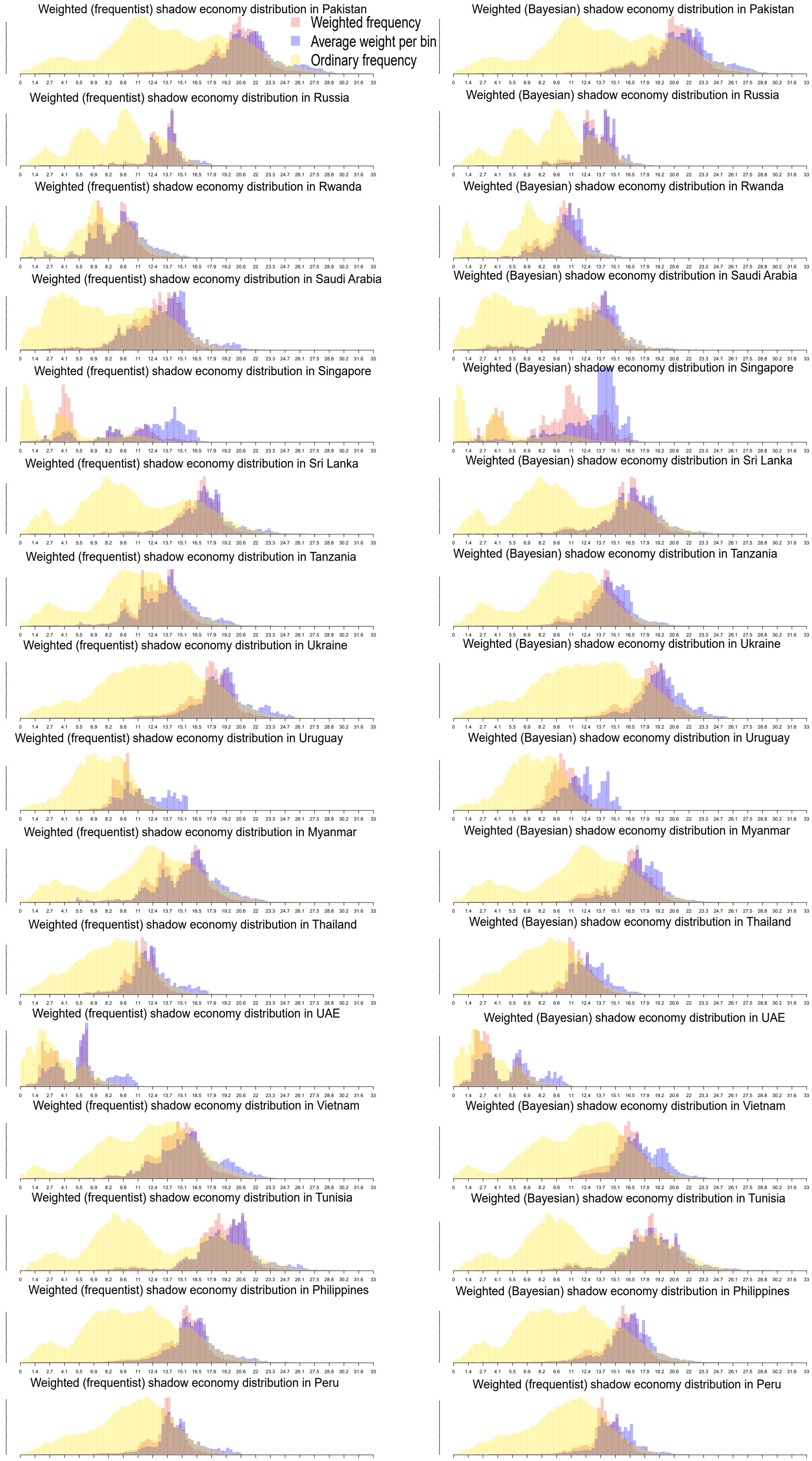
Source: Own calculations. Estimates for the year 2014.



Source: Own calculations. Estimates for the year 2014.



Source: Own calculations. Estimates for the year 2014.



Source: Own calculations. Estimates for the year 2014.